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FOAMVIS: A VISUALIZATION TOOL FOR FOAM RESEARCH

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THEME
Visualization

KEYWORDS
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SUMMARY

Foam researchers generate large, complex, time-dependent data sets with hundreds or thousands of individual bubbles and thousands of time steps. However, no foam specific visualization software exists. We briefly describe foam research and its challenges and we present FoamVis, a software application that provides visualization, exploration and analysis techniques for foam research.

1 Introduction and Motivation

Liquid foams have important practical applications in areas such as oil and mineral extraction, food and beverage production, cleaning and fire extinguishing [20]. In oil extraction, foam is pushed through porous rock to displace oil [17]. Domain experts want to understand how the constricted geometry of the rock affects the flow of foam. Foam is used in mineral separation [15] in a process where ground ore is washed with foam. The efficacy of the separation between mineral and rock depends on how objects with different properties interact with foam. When forced to flow through a constricted channel, many complex fluids, show regions in which material circulates in the upstream corners (salient corner vortices). As a consequence, material issuing from the channel can show markedly different ages, and therefore possibly different properties. In the case of foams, such recirculation may lead to particles dropping out of the foam before they can be captured or, in the case of food foams, material becoming unusable because of its age. Scientists are interested if and when such recirculation occurs for various channel geometries.

Liquid foam behavior is not yet well understood. Scientists try to determine foam behavior from measurable properties such as bubble size and distribution, liquid fraction, and surface tension. One way to study this dependence is to simulate foam at the bubble level, which makes it possible to model foam properties and see their influence on general foam behavior. However, it also poses challenges for visualizing and inferring the general foam behavior. Foam is simulated at a small scale, where each bubble is modeled individually, yet the goal is to determine behavior at a large scale, where foam can be described as a continuous medium. Liquid foams require the analysis of an additional dimension, time, which poses challenges to researchers.

Surface Evolver (SE) [2] is the *de facto* standard for simulating foam at bubble scale. Researching SE foam simulations poses special challenges:

- Access to simulation data is difficult and requires domain specific knowledge. Parsing and special processing are required to access the full simulation data. Important bubble attributes are not provided by the simulation but inferred using domain specific knowledge.
- It is challenging to visualize general foam behavior. While bubble-scale simulation makes it possible to investigate the influences that material properties have on general foam behavior, it makes it difficult to visualize the general behavior that is of primary interest. Simulation data is complex (unstructured grid with polygonal cells) and time dependent, with large fluctuations in the values of the parameters determined by changes in the topology of the soap film network.
- Triggers to various foam behaviors are difficult to infer. Multiple attributes have to be examined and foam properties have to be taken into account. Topological changes, in which bubbles swap neighbors, have to be considered.
- Foam scientists work with dozens of simulations with a wide range of simulation parameters. Examples include foam container properties (such as shape), foam attributes (such as bubble size and distribution, liquid fraction and surface tension) or the properties of objects interacting with foam (such as shape, size or position). The large number of existing simulations and the variety of simulation parameters makes it difficult to manage simulation data. Comparing related datasets results in a better understanding of various foam behaviors, however existing tools do not facilitate that.

These challenges make it difficult to use a general purpose visualization tool for foam research. Domain experts' visualizations only partially address these challenges. They may require intervention in the simulation code and potentially recomputing the simulations for summarizing and saving the relevant data. Their standard visualizations do not have the ability to explore and analyze the data and do not facilitate comparison of datasets. They do not have the high level of detail and speed that is achieved using graphics hardware. We address shortcomings of existing visualizations used by domain experts and we provide visualizations to address foam research challenges. To the best of our knowledge, no visualization software exists for foam simulations modeled with SE. FoamVis [11] fills this void by providing a comprehensive solution which facilitates advanced examination, visualization, analysis and comparison of foam simulation data.

The rest of this paper is organized as follows: Sec. 2 presents related work and Sec. 3 describes the simulation datasets used in this work. Then we describe FoamVis interface (Sec. 4), visualization capabilities (Sec. 5) and user interaction (Sec. 6). We end with conclusions and future work (Sec. 7).

2 Related Work

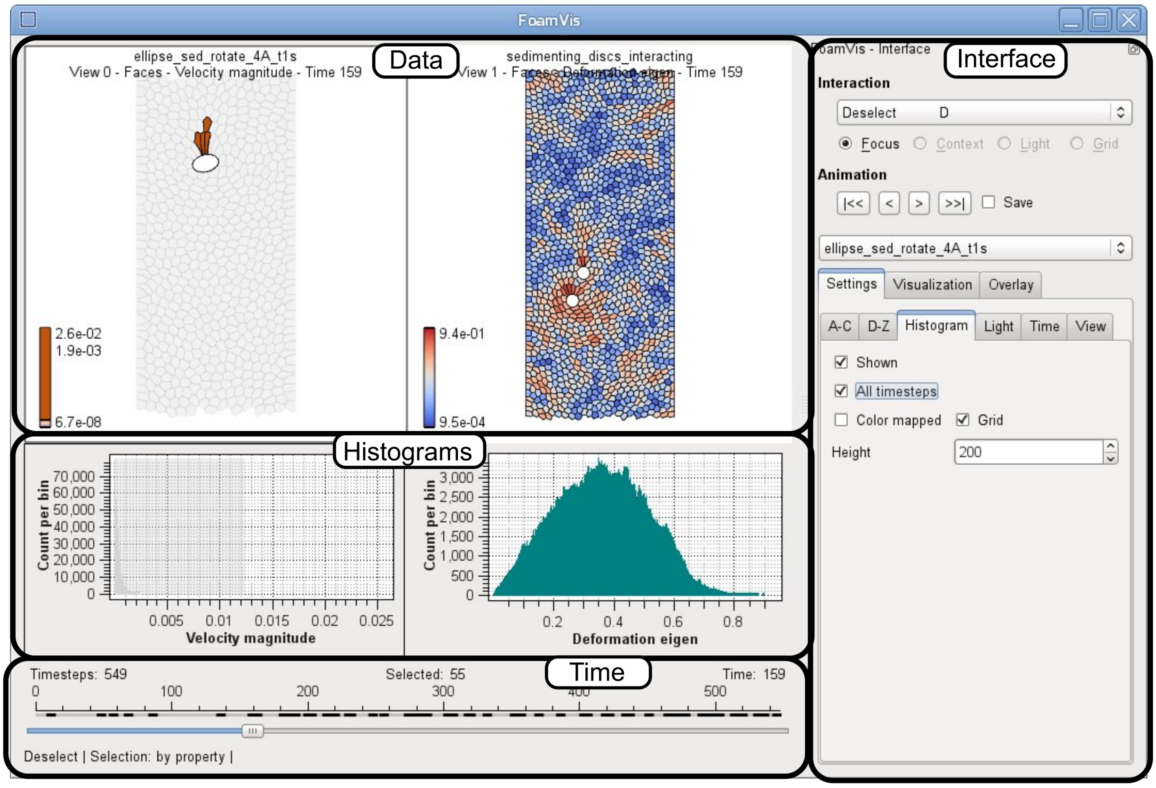
In our survey of the literature [13], very little work in visualization of time-dependent, physically accurate foam simulation data has been published, related works focus on visualization of static foam or foam-like structures [1, 10, 8]. Existing tools to manipulate Evolver data include *evmovie*, which is distributed with Evolver, and the Surface Evolver Fluid Interface

Tool (SE-FIT) [4]. The first scrolls through a sequence of evolver files, without adding any further visualization, while SE-FIT provides an interface for direct interaction with Evolver and display the result, as well as import and export data. In previous case studies we show how our tool helps foam scientists explore and analyze their data [11, 5] and enables them to compare related simulations [12]. Here we provides a comprehensive view on FoamVis interface, visualization capabilities, ways to interact with the data, and possible use scenarios.

3 Foam Simulation Cases

To present our solutions for visualization of foam simulation data, we use two simulation groups containing related simulations: sedimenting objects and constriction. The sedimenting-objects simulation group contains the *sedimenting-ellipse* and the *sedimenting-discs* simulations (Fig. 1). *Sedimenting-discs* simulates two discs falling through a monodisperse (bubbles having equal volume) foam under gravity. It contains 330 time steps and simulates 2200 bubbles. The two discs are initially side-by-side and in close proximity. As they fall, they interact with foam and each other by rotating towards a stable orientation in which the line that connects their centers is parallel to gravity. There are two forces acting on each disc in addition to its weight. A pressure force results from each adjacent bubble pushing against it, while a network force arises because each contacting soap film pulls normal to the circumference with the force of surface tension. Due to the flow, the distribution of films and bubbles pressures around each disc is not uniform (for example, there is a high density of films above each disc, leading to a large, upward, network force there), resulting in a non-zero resultant force. *Sedimenting-ellipse* simulates an ellipse falling through a monodisperse foam under gravity. This group contains 540 time steps and simulates 600 bubbles. The major axis of the ellipse is initially horizontal. As the ellipse falls, it rotates toward a stable orientation in which its major axis is parallel to gravity. As for the sedimenting discs, a network and pressure force act on the ellipse and, due to its shape, they give rise to a non-zero torque that rotates it. Both simulations are a variation of the classic Stokes experiment that is used to probe the rheology of a 2D foam, and for which there is a great deal of experimental data[7, 16].

The constriction simulation group contains two simulations, one with a *square-constriction* and one with a *rounded-constriction* (Fig. 2). They simulate a 2D polydisperse (bubbles with different volumes) foam flowing through a constricted channel, with 725 bubbles and 1000 time steps. The foam container has unit length and the length of the constricted region is 0.148. The width of the container is 0.5 and the width of the constricted region is 0.24. The simulations differ from each other in the geometry of the constriction. The radius of the circles creating the rounded corners of the constriction is 0.014 for the square-constriction and 0.069 for the rounded-constriction. The constriction simulations subject foam to different kinds of stress simultaneously and is therefore a testing ground from which to validate the approximations in the model against experiment. The simulations in both groups are periodic in the direction of motion.



Interaction: Deselect D

● Focus ○ Context ○ Light ○ Grid

☐ Save

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☑ Shown

☑ All timesteps

☐ Color mapped ☑ Grid

Height 200

Figure 1: FoamVis' Main Window showing the data, histograms, time and interface panels. The data panel shows two views: bubble velocity magnitude for a sedimenting ellipse simulation and bubble deformation a sedimenting discs simulation. A selection on velocity magnitude values is performed in the histogram panel and it is reflected in the data and time panels. We can observe in the time panel that only 55 time steps out of 549 contain high velocity bubbles and in the data panel we see those bubbles color mapped, the rest of the bubbles are displayed grayed.

4 Interface

Foam simulations consists of a list of text files stored in a folder. The only information about the simulation available without parsing the simulation files is the name of the folder. While this often encodes important parameters of the simulation, their meaning may be cryptic and only known to the scientist that created the simulation. It is difficult to infer from the folder names if simulations are related. One simulation may be part of several groups of similar simulations, which makes matters even more challenging. Additionally there is an increasing number of parameters that our visualization tool needs which are not encoded in the simulation files. To address these issues we create a simulations database and a browsing interface. The simulations database records for each simulation three pieces of information: a simulation name - usually this is the same as the folder that stores the simulation files, a list of labels, each label is used to group simulations on a specific criteria, and simulation specific visualization parameters. The database is stored as an XML file and is created by the user from a template.

The browsing feature (Fig. 3) presents all grouping labels from the simulation database in a

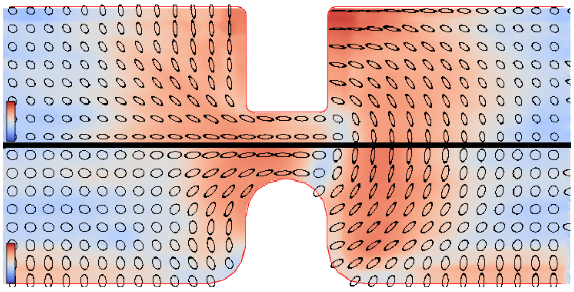
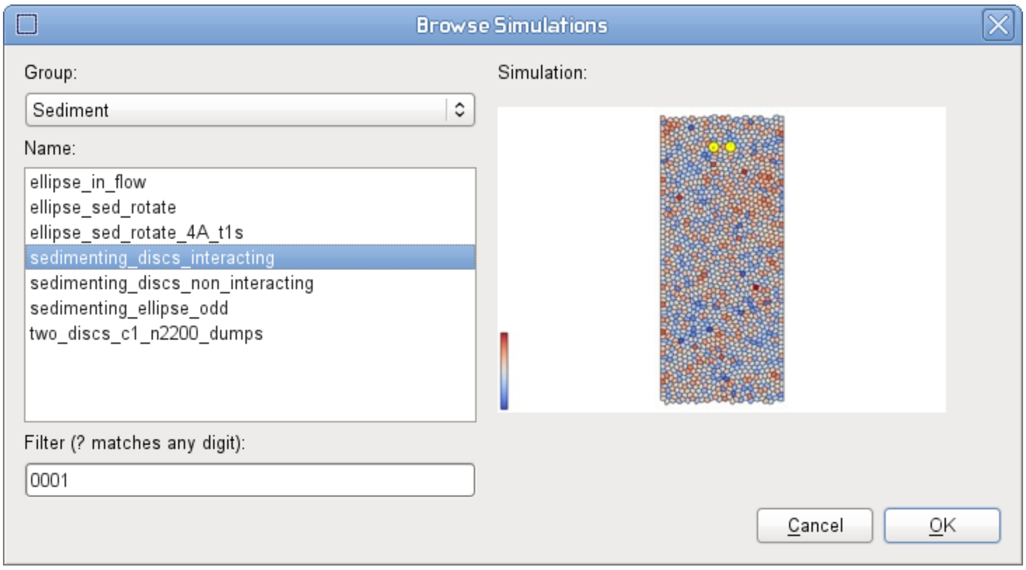


Figure 2: Rounding the corners of the constriction results in reduced elastic deformation of the foam (top versus bottom). Note as well, in both simulations, an area where bubbles are not deformed just downstream from the constriction. We show the square (top) and rounded (bottom) constriction simulations. Deformation size and direction is displayed with ellipses, deformation size is also color-mapped. An average over the entire duration of the simulations is displayed.



Group: Sediment

Name:
ellipse_in_flow
ellipse_sed_rotate
ellipse_sed_rotate_4A_t1s
sedimenting_discs_interacting
sedimenting_discs_non_interacting
sedimenting_ellipse_odd
two_discs_c1_n2200_dumps

Filter (? matches any digit): 0001

Figure 3: The Browse Simulations dialog which allows the user to view related simulations and select simulations of interests for individual analysis or comparison.

list. When a user selects a label, a list with all simulation grouped by that label is presented. When a user selects a simulation name, a picture of the first time step in the simulation is displayed. The image is saved beforehand so no parsing of simulation files is required. This allows a user to explore existing simulations based on similarity criteria encoded in labels and visually select simulations of interest for individual analysis or comparison.

FoamVis' main window (Fig. 1) contains three panels that are used for both visualization and user interaction (data, histograms and time), and one panel (interface) that allows the user to specify the visualizations customized using visualization parameters. The data panel shows multiple views with each view showing a different visualization, a visualization of a different simulation attribute or a visualization of a different simulation. The histograms panel shows histograms for simulation scalars shown in the data panel. The time panel shows the current time step and marks time steps resulting from selections on scalar values.

5 Visualization

Our visualization solutions are driven by the foam research and visualization challenges listed in Sec. 1. Surface Evolver output files are parsed and processed to access the complete data generated by the simulation. Our application works with any SE simulation and no changes to the simulation output are necessary to accommodate the application. This processing addresses challenge one.

We visualize important simulation attributes (Sec. 5.2) which include bubble scalar measures, bubble velocity (a vector), bubble deformation (a tensor), location of topological changes and forces acting on objects in foam. Overall foam behavior is analyzed using the average feature (Sec. 5.3, kernel density estimate for topological changes (Sec. 5.4) and bubble paths (Sec. 5.2). This addresses challenge two.

Foam scientists wish to understand what triggers certain behavior in foam simulations (challenge three). Foam behavior is studied by either examining different attributes that influence it or by comparing simulations (challenge four) where the behavior is varied by modifying simulations parameters. Both these requirements are addressed using multiple linked-views (Section 5.6).

We use consistent color palettes throughout this work: diverging color maps [14]: blue-red for deformation, blue-tan for pressure, purple-orange for velocity and a sequential colormap white-blue[3] for the kernel density estimate.

5.1 Simulation attributes

Scalar bubble attributes include velocity along principal axes, velocity magnitude, edges per face, deformation, pressure, volume and growth rate. Scalar bubble attributes are visualized using color mapping which maps scalar values to colors from a color palette. The user can change the color palette and change the range of scalar values mapped to color through clamping (Sec. 6). Fig. 4-a and Fig. 2 and Fig. 1 show examples of scalar attributes visualized through color mapping.

Bubble velocity, defined as the motion of the center of mass, provides information about foam flow. We visualize bubble velocities using glyphs (Fig. 4-a) and streamlines (Fig. 5).

Bubble deformation magnitude and *direction* are important bubble attributes, because they facilitate validation of simulations and provide information about the force acting on a dynamic object in foam. While visual inspection of individual bubbles provides information about foam deformation, this information is not quantified, and, more importantly, cannot be averaged to obtain the general foam behavior. To address these issues, we define a bubble deformation measure[12] expressed as a tensor. The deformation tensor is visualized using glyphs as shown in Fig. 2.

When foam is subjected to stress, bubbles deform (elastic deformation) and move past each other (plastic deformation). Domain experts are interested in the distribution of the plasticity

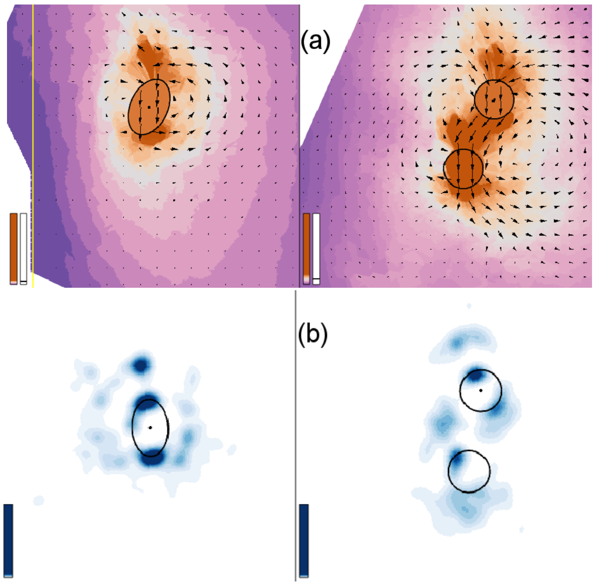


Figure 4: Are the sedimenting discs behaving like the sedimenting ellipse? (a) The foam between the discs moves at high velocity with the discs. Velocity is displayed using glyphs and velocity magnitude is also color-mapped. (b) Few topological changes occur between the discs, so foam behaves mainly as an elastic solid. Topological changes over time visualized using KDE.

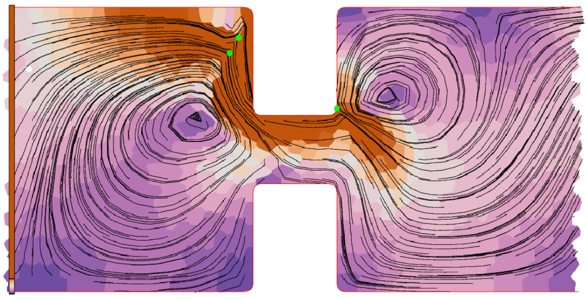


Figure 5: Topological changes causing strong circulation movement, topological changes shown with green dots, velocity field shown with streamlines, $t = 412$. Velocity is shown with streamlines, $t = 417$ and velocity magnitude is color-mapped.

which is indicated by the location of topological changes. A topological change is a neighbor swap between four neighboring bubbles. In a stable configuration, bubble edges meet 3-way at 120° angles. As foam is sheared, bubbles move into an unstable configuration, in which edges meet 4-way, then quickly move back into a stable configuration. Topological changes for the current time step or for all time steps are visualized with dots of configurable color and size showing the location of the topological change (Fig. 5).

The forces and the torque acting on objects are computed by the simulation code and stored in the simulation data. Each force acting on a disc is represented as an arrow that starts in the center of the disc and has length proportional to the magnitude of the force.

For the sedimenting discs simulation, the interplay of the network and pressure forces rotate one disc around the other. We provide an user option that displays the difference between the forces acting on the leading disc and forces acting on the trailing disc. This difference allows

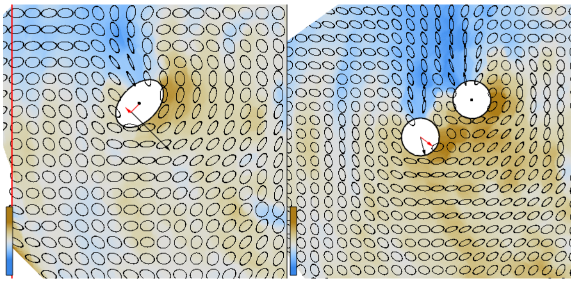


Figure 6: Sedimenting-ellipse versus sedimenting-discs. The *linked time with event synchronization* feature is used to synchronize the rotation of the ellipse and the two discs such that they reach an orientation of 45° in the same time. Attributes (pressure, deformation and forces) are averaged over 52 time steps for the ellipse simulation (resulting in an average over 15 time steps for the two disc simulation). Pressure is color-mapped, deformation is shown using ellipses. The force *difference* between the leading disc and the trailing disc and the torque on the ellipse is indicated. The network force and torque are indicated with a black arrow and the pressure force and torque are indicated with a red arrow.

us to better analyze the causes of the rotating behavior as there is a direct correspondence between the forces displayed on the screen and the movement of the disc (Fig. 6 right).

The torque τ rotating an object around its center is displayed as a force F acting off-center on the object $\tau = r \times F$, where r is the displacement vector from the center of the object to the point of where the force is applied. The distance $|r|$ is a user-defined parameter, FoamVis calculates the appropriate value of F to keep the torque constant (Fig. 6 left).

5.2 Bubble paths

Visualization of bubble paths provides information about the trajectory of individual bubbles in the simulation. The paths are a useful way to compare simulation with experiment. They also provide insight into the overall behavior of the foam. A bubble path is determined by connecting the center of bubbles with the same ID in consecutive time steps. Fig. 7 shows a pattern of bubbles traversing loops revealed by a bubble paths visualization.

5.3 Average

Bubble-scale simulations can be too detailed for observing general foam behavior and topological changes generate large fluctuations in attribute values that hide the overall trends. A good way to smooth out these variations is to calculate the average of the simulation attributes over all time steps, or over a time window before the current time step. This visualization reveals global trends in the data because large fluctuations caused by topological changes are eradicated. This results in only small variations between averaged successive time steps. The time window is a parameter set by the user. We compute the average for the entire simulation (Fig. 2) if there are no dynamic objects interacting with the foam. In this case, at a high level of detail, there is no difference between different time steps in the sim-

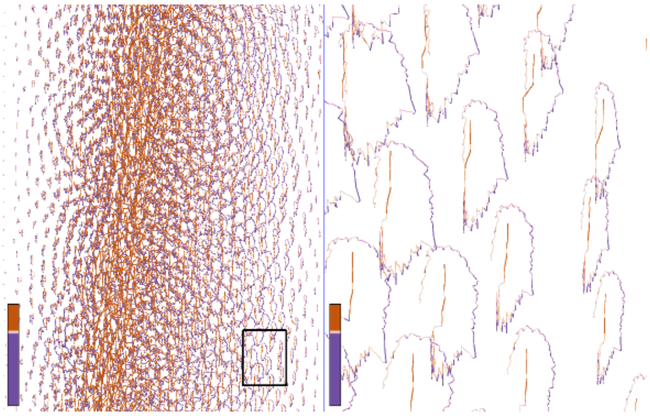


Figure 7: Pattern of bubbles traversing loops visualized using bubble paths, sedimenting discs simulation. The bubbles paths are color-mapped to velocity along Y, with orange indicating descent and purple indicating ascent. The left image shows the bubble paths over the entire simulation. The red area shows paths of the two discs. The black rectangle shows the region that is magnified in the right image.

ulations. For simulations that include dynamic objects interacting with the foam (Fig. 4, 6) a smaller time window is appropriate, as objects may traverse transient states that have to be analyzed independently of each other.

5.4 Topological changes kernel density estimate (KDE)

Foam topological changes, in which bubbles change neighbors, show plasticity in foam. Domain experts expect that their distribution will be an important tool for validating simulations. Simply rendering the position of each topological change suffers from over-plotting, so it may paint a misleading picture of the real distribution. We compute (see Lipsa et al.[12] for details) a KDE for topological changes (Fig. 4). While traditional histograms show similar information and are straightforward to implement they have drawbacks which may prove important depending on the context. Drawbacks of histograms include the discretization of data into bins which may introduce aliasing effects and the fact that the appearance of the histogram may depend on the choice of origin for the histogram bins[6, 18]. Kernel-based methods for computing the probability density estimate eliminate these drawbacks.

5.5 Histograms

We provide both a histogram of bubble attribute values over one time step and over all time steps. To facilitate data analysis, our histogram is configurable. The user can choose a maximum height, logarithmic or linear height scale and uni-color or color-coded display. Histograms are also used in selection and filtering of data based on attribute value and in color-map clamping used for selecting features of interest in the data. These interactions are described in detail in Sec. 6.

5.6 Multiple linked-views

Foam scientists wish to understand what triggers certain behavior in foam simulations, so the ability to see different attributes at the same time and to understand how different attributes relate to one another is very important. In the same time, to understand the influence simulation parameters have on foam behavior, foam scientists want to analyze and compare related simulations. These requirements are both addressed using multiple linked views. We provide up to four different views. For maximum flexibility, each view can depict a different simulation attribute, a different visualization or even a different simulation, uses its own color-bar and can show the navigation context. To set up optimal views to analyze data, users can copy viewing transformations and color mapping between views depicting the same attribute.

The *two halves* option facilitates visual comparison of two related foam simulations (Fig. 2). It visualizes related simulations that are assumed to be symmetric with respect to one of the main axes. While the same information can be gathered by examining the two simulations in different views, the *two halves* view may facilitate analysis as images to be compared are closer together and it is useful for presentation as it saves space. This type of visualization was previously performed manually by domain experts.

We provide three *connection operations* [19] between views: one linked-selection connection and two linked-time connections. The linked-selection connection works by showing data selected in one view in other views. This is used to see, for instance, the elongation of high pressure bubbles or both pressure and elongation for bubbles involved in a topological change.

The first linked-time connection works by having each view linked to the same time step, as foam scientists want analyze several attributes in the same time to understand foam behavior influenced by those attributes. The second linked-time connection, linked-time with event synchronization, is described next. In simulations that involves dynamic objects interacting with foam, we want a similar event in both simulations to happen at the same time so that behavior up to that event can be compared and analyzed together. When comparing the sedimenting discs with the sedimenting ellipse simulations, the ellipse and the discs start in similar configurations. The main axis of the ellipse and the line connecting the center of the two discs are horizontal. We want the ellipse and the discs to reach an intermediate configuration and the stable configuration at the same time. Both these configurations are defined in terms of the angle that major axis of the ellipse and the line connecting the centers of the two discs make with gravity. The angles are 45° for the intermediate configuration and 0° for the stable configuration. This technique splits simulation times in intervals - an interval before each event and an interval after the last event. For each interval before an event, one simulation will run at its normal speed (the simulation with the longest interval), all other simulations will be “slowed down”. Simulations will run at normal speed for the interval after the last event. Using this approach, related events occur at the same *linked time* in all simulations, facilitating their comparison as well as the comparison of their temporal context. Fig. 4, 6 use linked-time with event synchronization feature.

6 Interaction

Interaction with the data is an essential feature of our application.

Navigation is used to select a subset of the data to be viewed, the direction of view, and the level of detail [19]. We provide the following navigation operations: rotation around a bounding box center for specifying the direction of view, and translation and scaling for specifying the subset of data and the level of detail. A navigation context (Fig. 7 left) insures that the user always knows its location and orientation during exploration of the data.

We can **select and/or filter** bubbles and center paths based on three distinct criteria: based on bubble IDs, to enable relating to the simulation files and for debugging purposes; based on location of bubbles, to analyze interesting features at certain locations in the data; and based on an interval of attribute values specified using the histogram tool (Fig. 1). A composite selection can be specified using both location and attribute values.

Selected bubbles or center paths constitute the focus of our visualization, and the rest of the bubbles or center paths provide the context[9]. The context of the visualization is displayed using user-specified semi-transparency, or it can be hidden altogether.

Encoding operations are variations of graphical entities used in a visualization that emphasize features of interest[19]. We provide encoding operations to change the color map used, to specify the range of values used in the color map and to adjust the opacity of the visualization context. Selection of the interval used in color-mapping is guided by the histogram tool (Fig. 8). This provides essential information for selecting an interval that reveals features of interest.

7 Future Work

This paper provides a short introduction into foam research and presents challenges faced by domain scientists. We present FoamVis, an exploration, visualization and analysis tool that helps scientists address some of these challenges and presents their data in a new, more useful way.

For future work, we want to compare simulations with experiments and provide visualizations for 3D foam simulations and experiments.

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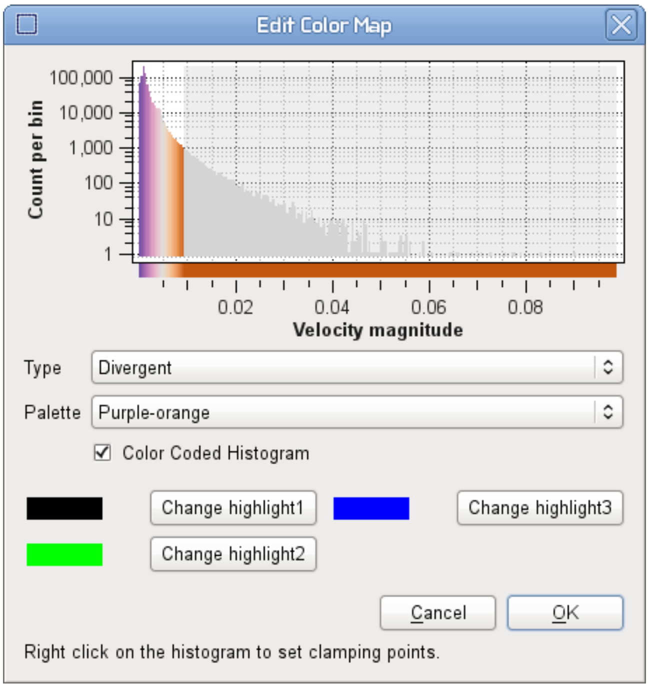

Type: Divergent

Palette: Purple-orange

☑ Color Coded Histogram

Right click on the histogram to set clamping points.

Figure 8: Color-map clamping guided by the histogram tool. This is a histogram of the constriction simulation which uses the logarithmic height scale. The histogram is clamped in the high values region. The dialog also allows the user to choose a different color palette and to change highlight colors used for vector and tensor glyphs and forces acting on objects.

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