Paper:
Cui, R., Chen, L., Yang, C. & Chen, M. (2017). Extended State Observer-Based Integral Sliding Mode Control for an Underwater Robot with Unknown Disturbances and Uncertain Nonlinearities. *IEEE Transactions on Industrial Electronics*, 1-1.
http://dx.doi.org/10.1109/TIE.2017.2694410

Correction to "Extended State Observer-Based Integral Sliding Mode Control for an Underwater Robot with Unknown Disturbances and Uncertain Nonlinearities"

Rongxin Cui, *Member, IEEE*, Lepeng Chen, Chenguang Yang, *Senior Member, IEEE*, and Mou Chen, *Member, IEEE*

The purpose of this note is to correct the matching condition and stability proof in [1]. While the main results are unchanged, there should be some consequent modifications, which are shown in detail as follows:

Firstly, we define that (A.i) represents the ith equation in the original paper. (A.14) should be modified as

$$-G_3\dot{H}_d/w_0^2 - G_2H_{un}/w_0 = P^{-1}\Theta\rho_t \tag{1}$$

where $\rho_t = [\rho_{t1}^\top, \rho_{t2}^\top, \rho_{t3}^\top]^\top \in \mathbb{R}^{18\times1}$, $\rho_{t1}, \rho_{t2}, \rho_{t3} \in \mathbb{R}^{6\times1}$, and the time-varying matrix Θ can be defined as

$$\Theta = \begin{bmatrix} I_{6\times6} & -r_2 & -r_3 \\ 0_{6\times6} & r_1 & 0_{6\times6} \\ 0_{6\times6} & 0_{6\times6} & r_1 \end{bmatrix} \tag{2}$$

where $r_1 = \mathrm{diag}(\varepsilon_{11}, \cdots, \varepsilon_{16})$, $r_2 = \mathrm{diag}(\varepsilon_{21}, \cdots, \varepsilon_{26})$, $r_3 = \mathrm{diag}(\varepsilon_{31}, \cdots, \varepsilon_{36})$, $\varepsilon_1 = [\varepsilon_{11}, \cdots, \varepsilon_{16}]^\top, \varepsilon_2 = [\varepsilon_{21}, \cdots, \varepsilon_{26}]^\top, \varepsilon_3 = [\varepsilon_{31}, \cdots, \varepsilon_{36}]^\top$ are scaled estimation errors. Due to the added term Θ, (A.23) can be corrected as

$$\begin{aligned}\dot{V}_1 = &- w_0\varepsilon^\top(A_\varepsilon^\top P + PA_\varepsilon)\varepsilon + 2\varepsilon^\top PQ^{-1}\tilde{f} \\ &+ 2\varepsilon^\top PP^{-1}\Theta\rho_t - 2\varepsilon^\top P\varpi\end{aligned} \tag{3}$$

Substituting (A.18) into (3), we have

$$\begin{aligned}\dot{V}_1 = &- w_0\varepsilon^\top\varepsilon + 2\varepsilon^\top PQ^{-1}\tilde{f} \\ &+ 2\varepsilon^\top\Theta\rho_t - 2\varepsilon^\top P\varpi \\ \le &- w_0\|\varepsilon\|^2 + 2\|\varepsilon\|\|P\|\|Q^{-1}\tilde{f}\| \\ &+ 2\varepsilon^\top\Theta\rho_t - 2\varepsilon^\top P\varpi \\ \le &\left[-w_0 + c_2(\zeta_1+\zeta_2)/w_0\right]\|\varepsilon\|^2 \\ &+ 2\varepsilon^\top\Theta\rho_t - 2\varepsilon^\top P\varpi\end{aligned} \tag{4}$$

Furthermore, (A.25) should be updated as follows:

$$\dot{V}_1 \le -\beta\|\varepsilon\|^2 + 2\varepsilon^\top\Theta\rho_t - 2\varepsilon^\top P\varpi \tag{5}$$

Since $\varepsilon^\top\Theta = [\varepsilon_1^\top, 0_{1\times6}, 0_{1\times6}]$, (5) can be rewritten as

$$\begin{aligned}\dot{V}_1 \le &- \beta\|\varepsilon\|^2 + 2\varepsilon_1^\top\rho_{t1} - 2\varepsilon^\top P\varpi \\ = &- \beta\|\varepsilon\|^2 + 2(C\varepsilon)^\top\rho_{t1} - 2\varepsilon^\top P\varpi\end{aligned} \tag{6}$$

where ρ_{t1} is bounded and satisfies $\|\rho_{t1}\| \le \rho_2 \in \mathbb{R}^+$, and which is the same as (A.25), therefore the result is unchanged.

Secondly, (A.29) should be corrected as

$$\dot{s}(t) = K_p\dot{e}(t) + K_ie(t) + K_d\dot{\hat{e}}(t) \tag{7}$$

Based on the observer that presented in (A.10), we have

$$\begin{aligned}\dot{\hat{e}} = \dot{\hat{\dot{\eta}}} - \ddot{\eta}_r = &- \ddot{\eta}_r - C_\eta(\eta,\hat{\nu})\hat{\dot{\eta}} - D_\eta(\eta,\hat{\nu})\hat{\dot{\eta}} \\ &- G_\eta + M_\eta LU + \hat{H}_d - 3w_0^2\tilde{x}_1 - w_0\varpi_2\end{aligned} \tag{8}$$

Using $\dot{e} = \hat{\dot{e}} - w_0\varepsilon_2$ and (8), (A.31) can be rewritten as

$$\begin{aligned}\dot{s} + K_ss = &K_p\hat{\dot{e}} + K_ie + K_ss - w_0K_p\varepsilon_2 + K_d(-\ddot{\eta}_r \\ &- C_\eta(\eta,\hat{\nu})\hat{\dot{\eta}} - D_\eta(\eta,\hat{\nu})\hat{\dot{\eta}} - G_\eta + M_\eta LU \\ &+ \hat{H}_d - 3w_0^2\tilde{x}_1 - w_0\varpi_2)\end{aligned} \tag{9}$$

where $\varpi = [\varpi_1^\top, \varpi_2^\top, \varpi_3^\top]^\top \in \mathbb{R}^{18\times1}$, $\varpi_i \in \mathbb{R}^{6\times1}$, $i = 1, 2, 3$. (A.32) should be updated as

$$\begin{aligned}U_{\mathrm{eq}} = &- (K_dM_\eta L)^{-1}(K_p\hat{\dot{e}} + K_ie + K_ss) \\ &+ (M_\eta L)^{-1}[\ddot{\eta}_r + C_\eta(\eta,\hat{\nu})\hat{\dot{\eta}} + w_0\varpi_2 \\ &+ D_\eta(\eta,\hat{\nu})\hat{\dot{\eta}} + G_\eta - \hat{H}_d + 3w_0^2\tilde{x}_1]\end{aligned} \tag{10}$$

(A.34) should be written as

$$U_{\mathrm{sw}} = -(K_dM_\eta L)^{-1}K_{\mathrm{sw}}\mathbf{sgn}(s) \tag{11}$$

Then, (A.36) can be described as

$$U = U_{\mathrm{eq}} + U_{\mathrm{sw}} \tag{12}$$

Compared with U_{eq} in the original controller, the term $(M_\eta L)^{-1}(w_0\varpi_2 + 3w_0^2\tilde{x}_1)$ are added, which will converge to zero. Then, the main experimental results are unchanged.

Theorem 1: Consider system (A.6) satisfying Assumptions 1, under the designed ESO (A.10), the tracking error and

This work was supported by the National Natural Science Foundation of China under Grant 61472325, Grant 61633002, and Grant 51209174.

R. Cui and L. Chen are with the School of Marine Science and Technology, Northwestern Polytechnical University, Xian 710072, China (e-mail: rongxin.cui@gmail.com; chenlepeng_nwpu@163.com).

C. Yang is with the Zienkiewicz Centre for Computational Engineering, Swansea University, SA1 8EN, UK. (email: cyang@theiet.org).

M. Chen is with the College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China (e-mail: chenmou@nuaa.edu.cn). (Corresponding author: Rongxin Cui).

external disturbance estimation error will converge to zero under the control law (11), and the parameters β, w_0, K_d, K_s, and K_{sw} satisfy following conditions: $\beta > w_0\lambda_{\max}(K_p)$, $\lambda_{\min}(K_s) > w_0\lambda_{\max}(K_p)/2$ and $\lambda_{\min}(K_{\text{sw}}) > 0$.

Proof: Let us define a Lyapunov function candidate

$$V = \frac{1}{2}V_1 + \frac{1}{2}s^\top s + \frac{1}{2\gamma_2}\tilde{\rho}_2^2 \tag{13}$$

where V_1 is defined in (A.21). Based on Lemma 1, we have

$$\begin{aligned}\dot{V} \leq &-\frac{\beta}{2}(\varepsilon_1^\top\varepsilon_1 + \varepsilon_2^\top\varepsilon_2 + \varepsilon_3^\top\varepsilon_3) + \varepsilon^\top C^\top \rho_{t1} \\ &-\varepsilon^\top P\varpi + s^\top\dot{s} + \frac{1}{\gamma_2}\tilde{\rho}_2\dot{\hat{\rho}}_2\end{aligned} \tag{14}$$

Substituting (A.13) and (A.15) into (14), we have

$$\begin{aligned}\dot{V} \leq &-\beta\varepsilon^\top\varepsilon/2 + s^\top\dot{s} + \|\tilde{Y}\|\rho_2 - \varepsilon^\top P\varpi + \|\tilde{Y}\|\tilde{\rho}_2 \\ = &-\beta\varepsilon^\top\varepsilon/2 + s^\top\dot{s} + \|\tilde{Y}\|\hat{\rho}_2 \\ &-\frac{\|\tilde{Y}\|^2\hat{\rho}_2 - c_1\|\tilde{Y}\|^2\dot{h}_1\hat{\rho}_2^2/\|\tilde{Y}\|}{\|\tilde{Y}\| - c_1\dot{h}_1\hat{\rho}_2} \\ = &-\beta\varepsilon^\top\varepsilon/2 + s^\top\dot{s}\end{aligned} \tag{15}$$

Substituting (12) into (9), we have

$$\dot{s} = -K_s s - w_0 K_p\varepsilon_2 - K_{\text{sw}}\mathbf{sgn}(s) \tag{16}$$

Substituting (16) into (15), we see that the derivative of V can be described as

$$\dot{V} \leq -\frac{\beta}{2}\varepsilon^\top\varepsilon - s^\top K_s s - w_0 s^\top K_p\varepsilon_2 - s^\top K_{\text{sw}}\mathbf{sgn}(s) \tag{17}$$

Since $-w_0 s^\top K_p\varepsilon_2 \leq w_0\lambda_{\max}(K_p)(\varepsilon^\top\varepsilon + s^\top s)/2$, we have

$$\begin{aligned}\dot{V} \leq &-\frac{\beta}{2}\varepsilon^\top\varepsilon - s^\top K_s s - \lambda_{\min}(K_{\text{sw}})\|s\| \\ &+ w_0\lambda_{\max}(K_p)(\varepsilon^\top\varepsilon + s^\top s)/2 \\ \leq &-\xi^\top\Lambda\xi - \lambda_{\min}(K_{\text{sw}})\|s\|\end{aligned} \tag{18}$$

where $\xi = [\varepsilon^\top, s^\top]^\top$, $\Lambda = \begin{bmatrix}\Lambda_1 & 0_{18\times 6} \\ 0_{6\times 18} & \Lambda_2\end{bmatrix}$, $\Lambda_1 = \left(\frac{\beta}{2} - \frac{w_0\lambda_{\max}(K_p)}{2}\right) I_{18\times 18}$, $\Lambda_2 = \left(\lambda_{\min}(K_s) - \frac{w_0\lambda_{\max}(K_p)}{2}\right) I_{6\times 6}$.

Because the parameters β, w_0, K_s and K_{sw} satisfy related conditions mentioned in Theorem 1, we know that $\beta > w_0\lambda_{\max}(K_p)$, $\lambda_{\min}(K_s) > w_0\lambda_{\max}(K_p)/2$, $\lambda_{\min}(K_{\text{sw}}) > 0$, therefore $\lambda_{\min}(\Lambda) > 0$.

Inequation (17) implies that $\dot{V} < 0$ for $\xi \neq 0$, and the signals s, ε and $\tilde{\rho}_2$ are bounded. Based on (18), we have $\dot{V} \leq -\xi^\top\Lambda\xi$. Then, we have $\lim_{t\to\infty}\int_0^t(\xi^\top\Lambda\xi)d\tau \leq V(0) - V(\infty)$. Because $V(0)$ and $V(\infty)$ are bounded, s and ε are square integrable. According to (16) and the boundedness of s, we can conclude that $\dot{s}$ is bounded.

From (A.12), we know that $\|\tilde{f}\| = \|\tilde{\varphi}\|$. Further, we have $\tilde{f}$ is bounded according to (1). The boundedness of $\dot{H}_d$ and H_{un} implies that $\rho_t(t)$ is bounded from (1). Because ε, $\hat{\rho}_2$ and $\dot{h}(t)$ are bounded, from (A.13), we can obtain ϖ is bounded. Then, $\dot{\varepsilon}$ is bounded from (A.16). The boundedness of $\dot{s}$ and $\dot{\varepsilon}$ implies that $\dot{\xi}$ is bounded. According to Lemma 2, we have $\lim_{t\to\infty}\xi(t) = 0$, i.e., $\lim_{t\to\infty}s(t) = 0$ and $\lim_{t\to\infty}\varepsilon(t) = 0$.

Defining that

$$z(t) = s(t) - w_0 K_d\varepsilon_2(t) + K_p e(0) + K_d\hat{\dot{e}}(0) + K_d e(0) \tag{19}$$

where $z(t) = [z_1(t), \cdots, z_6(t)]^\top \in \mathbb{R}^{6\times 1}$.

Substituting $\hat{\dot{e}} = \hat{\dot{\eta}} - \dot{\eta}_r = \dot{e} - w_0\varepsilon_2$ into (A.27), we have

$$K_p e(t) + K_i\int_0^t e(\tau)d\tau + K_d\dot{e}(t) + K_d e(0) = z(t) \tag{20}$$

Because K_p, K_i and K_d are positive definite diagonal matrices, we have

$$z_i(t) = K_{pi}e_i(t) + K_{ii}\int_0^t e_i(\tau)d\tau + K_{di}\dot{e}_i(t) + K_{di}e_i(0) \tag{21}$$

where $z_i(t)$ is the ith element of $z(t)$, and $i = 1, \ldots, 6$.

Then, take Laplace transformation of (21), we have

$$\frac{e_i(p)}{z_i(p)} = \frac{p}{K_{di}p^2 + K_{pi}p + K_{ii}} \tag{22}$$

where p is the Laplace transformation operator, $e_i(p)$ and $z_i(p)$ are the Laplace transformations of $e_i(t)$ and $z_i(t)$, respectively.

Using the final value theorem, we have

$$e(\infty) = \lim_{p\to 0}\frac{p^2 z_i(p)}{K_{di}p^2 + K_{pi}p + K_{ii}} \tag{23}$$

Since the initial error $e(0)$ and $\hat{\dot{e}}(0)$ are bounded, and $\varepsilon_2(t)$ is bounded, from (19), $z_i(t)$ is bounded. $z_i(t)$ can converge to $K_p e(0) + K_d\hat{\dot{e}}(0) + K_d e(0)$ as time goes to infinity. Then, we have $|z_i(t)| \leq z_{i\max} < \infty$. The Laplace transformation of $z_i(t)$ satisfies

$$\begin{aligned}|z_i(p)| &= \left|\int_0^\infty e^{-p\tau}z_i(\tau)d\tau\right| \leq \int_0^\infty |e^{-p\tau}z_i(\tau)|d\tau \\ &\leq z_{i\max}\int_0^\infty |e^{-p\tau}|dt \leq \frac{z_{i\max}}{p}\end{aligned} \tag{24}$$

Then, we have

$$\lim_{p\to 0}|p^2 z_i(p)| = 0 \tag{25}$$

Hence, it can be induced from (25) that $\lim_{p\to 0}p^2 z_i(p) = 0$, and then we have

$$e_i(\infty) = \lim_{p\to 0}\frac{p^2 z_i(p)}{K_{di}p^2 + K_{pi}p + K_{ii}} = 0 \tag{26}$$

The system given by (21) and (22) is stable if the parameters K_{di}, K_{pi} and K_{ii} are chosen as positive constants to satisfy Hurwitz stability criterion. According to (23) and (26), we have $\lim_{t\to\infty}e(t) = 0$. This completes the proof. ■

REFERENCES

[1] R. Cui, L. Chen, C. Yang, and M. Chen, "Extended state observer-based integral sliding mode control for an underwater robot with unknown disturbances and uncertain nonlinearities," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 8, pp. 6785–6795, 2017.