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A governing equation of the six-wave fiber optical parametric amplifier (FOPA) for the power and phase difference evolution of the six interacting waves is deduced. To optimize the gain of the six-wave FOPA, a multivariate stochastic optimization algorithm, i.e., the Genetic algorithm (GA), is applied. The effect of pump depletion on the gain characteristic of the six-wave FOPA is emphasized and the effect of the fiber length, the wavelength and power of two pumps on bandwidth, flatness and magnitude of gain spectrum has also been studied. A broader and flatter six-wave FOPA gain is obtained by adopting optimum design parameters, which theoretically provides a uniform gain of 65 dB with 0.3dB uniformity over an 110nm bandwidth for the six-wave FOPA. © 2017 Optical Society of America

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1. INTRODUCTION

Fiber optical parametric amplifiers (FOPAs) are attracting more and more research interests, for their many applications in various areas. Examples include high-gain amplification [1], wavelength conversion [2], regeneration [3], all-optical sampling [4], wide-bandwidth amplification [5], noise suppression [6], etc. [7-10]. Especially in the practical application of the fiber optical communication system, the amplifiers require the broader and flatter gain characteristics, and these are precisely the advantages of the FOPA.

So far, the gain properties of FOPA have been intensely researched and many techniques have been done to increase the gain of the FOPA [11-13], or improve the bandwidth-utilization efficiency [14-19]. Examples include inserting the dispersion compensating fibers to compensate the high nonlinear fibers (HNLFs) dispersion [11], adopting the cascade of phase-shifted fiber Bragg grating between two HNLFs [12], using a suitable length of standard single-mode fiber inserted between two HNLFs to reach phase matching [13], and so on. In addition, the genetic algorithm (GA) was used to optimize the FOPA [18, 19]. Compared with the one-pump architecture, the two-pump FOPA can offer a flatter and broader gain spectrum, which makes the two-pump FOPA more attractive for optical communication applications. In practice, when the signal wavelength is not far away from the pump wavelength and the signal power is amplified to relatively high, some new wavelength components would be generated due to cascaded four wave mixing (FWM) [20], consequently leading to the occurrence of six-wave mixing (SWM) for two-pump FOPA [6, 21], which consists of two pump waves, one signal wave and three idler waves.

For the high frequent appearance of the six-wave FOPA in practice, recently, some studies of the six-wave FOPA have been done [6, 15, 17, 21], however, the gain property did not have good quality compared with the four-wave FOPA, because the more waves interaction could not lead to the better gain spectrum. In this paper, we focus on optimizing the gain characteristics of the six-wave FOPA by using the GA under the condition of the pump depletion, since in order to achieve the higher signal gain, the pump depletion should be considered [19]. Consequently the flatter and broader gain of six-wave FOPA is obtained.

The paper is organized as follows. In Section 2, the six-wave model of FOPA is described and the governing equations for the power and phase difference evolution of the six interacting waves inside the two-pump FOPA are deduced. The results and discussion are analyzed in Section 3 and the conclusion is given in Section 4.

2. THEORY AND MODEL OF THE SIX-WAVE FOPA

The six-wave model is started from propagating in the direction along the fiber with continuous-wave pump.
Frequency assignments of the six-wave model are given in Refs. [6, 17, 21], which are shown in Fig. 1, where \( \omega_1 \) and \( \omega_2 \) denote the angular frequencies of pump1 and pump2, respectively. The center frequency is defined as \( \omega_0 = (\omega_1 + \omega_2) / 2 \) and the signal is injected at angular frequency \( \omega_2 \) close to the pump1. Due to the non-degenerate FWM among the pump1, pump2 and the signal waves, the first idler, at frequency \( \omega_5 \), is generated on the left side of pump2, with angular frequency satisfying \( \omega_5 + \omega_6 = \omega_2 + \omega_3 \). Since the signal is close to the pump1, also in the condition when the signal power is relatively high, some new wavelength components would be generated due to cascaded FWM [20], thus the second idler, at frequency \( \omega_4 \), is generated on the left side of pump1 owing to the degenerate FWM of the pump1 and the signal waves at the same time. This idler is symmetric to the signal with respect to the pump1 so that getting \( 2\omega_0 = \omega_1 + \omega_2 \). Another similar process occurs between the pump2 and the idler1 resulting in the third idler, with angular frequency \( \omega_3 \), where the frequencies satisfy \( 2\omega_3 = \omega_4 + \omega_5 \) [6, 17, 21].

![Fig. 1](image-url)  
**Fig. 1.** Frequency assignments of six interacting waves in the 2-P FOPAs based on the six-wave model.

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We have limited ourselves to situation in which all the waves are launched into the fiber with the same linear state of polarization (SOP) and remain in that state along the entire fiber [17]. When the fiber losses are neglected, and under the assumption that the six-waves are sufficiently separated in frequency, the 6-coupled equations governing the complex field amplitudes evolution of the two pumps (waves 1 and 2), one signal (wave 3), and three generated idlers (waves 4, 5 and 6) along the fiber can be written as [6, 17, 21].

\[
\frac{dA_i}{dz} = i\beta_i(\{A_1^* A_2^* A_1 A_2\}) A_i + 2 A_i A_j A_l A_k \exp(i\Delta\beta_{ijkl} z)
+ 2 A_i A_{ij} A_{kl} \exp(i\Delta\beta_{ijkl} z)
+ 2 A_i A_{il} A_{jk} \exp(i\Delta\beta_{ijkl} z)
\]

(8)

\[
\frac{dA_i}{dz} = i\beta_i(\{A_1^* A_2^* A_1 A_2\}) A_i
+ 2 A_i A_{ij} A_{kl} \exp(i\Delta\beta_{ijkl} z)
+ 2 A_i A_{ij} A_{kl} \exp(i\Delta\beta_{ijkl} z)
+ 2 A_i A_{ij} A_{kl} \exp(i\Delta\beta_{ijkl} z)
\]

(9)

\[
\frac{dA_i}{dz} = i\beta_i(\{A_1^* A_2^* A_1 A_2\}) A_i
\]

(10)

Finally, the Eqs. (1)-(6) can be deduced and simplified as:

\[
\frac{dP_i}{dz} = -i\gamma(\{P_1 P_2 P_3 P_4\}) P_i + \gamma(\{P_1 P_2 P_3 P_4\}) P_i
\]

(11)

\[
\frac{dP_i}{dz} = -i\gamma(\{P_1 P_2 P_3 P_4\}) P_i + \gamma(\{P_1 P_2 P_3 P_4\}) P_i
\]

(12)

\[
\frac{dP_i}{dz} = -i\gamma(\{P_1 P_2 P_3 P_4\}) P_i + \gamma(\{P_1 P_2 P_3 P_4\}) P_i
\]

(13)

\[
\frac{dP_i}{dz} = -i\gamma(\{P_1 P_2 P_3 P_4\}) P_i + \gamma(\{P_1 P_2 P_3 P_4\}) P_i
\]

(14)
Numerical solutions of the six coupled wave equations were solved with the fourth-order Runge-Kutta algorithm based on the amplitude and power evolution. As will be shown below, the equations we deduced are correct and appropriate for the theoretical and numerical analysis for the six-wave FOPA. Furthermore, the GA is a kind of multivariate stochastic optimization algorithm based on natural selection and genetic evolution [22]. It has been proved to be an effective advanced optimization algorithm, which has been applied to the FOPA based on four wave mixing [18, 19]. In this work, by using the GA, we consider optimizing the peak gain, bandwidth and gain flatness, where the fiber length, the wavelength and power of two pumps are chosen as the operating variables, while the other parameters, such as the nonlinearity coefficient and the second-order and forth-order dispersion, are usually defined for a given HNLF [17, 21].

3. RESULTS AND DISCUSSIONS

For the six-wave FOPA, the parameters of the optical fiber are determined according to the determination of optical fiber. To optimize the gain characteristic of the six-wave FOPA, we find the optimal combination by adjusting the wavelength of two pumps, the power of two pumps and the length of the optical fiber. By adopting the genetic algorithm we obtained a peak gain of 65 dB and gain bandwidth of about 110 nm (1475 nm-1585 nm) with flatness of less than 0.3 dB. The HNLF parameters set for simulation are same as those in Refs. [5, 17], which are \( \lambda_s = 1556 \) nm, \( \gamma = 20 \text{W}^{-1}\text{km}^{-1} \), \( \beta_2 = 0.49 \times 10^{-21} \text{s}^3/\text{m} \), and \( \beta_4 = -5.8 \times 10^{-4} \text{s}^4/\text{m} \).

![Fig. 2](image-url) (a) The power transformation between the signal (solid red curve) and pump light (dotted blue curve) in the FOPA based on six-wave FOPA model, (b) The signal gain as a function of the fiber length based on six-wave FOPA model.

To show a first sight of the gain characteristics of the six-wave FOPA with pump depletion, we present the pump power, the signal power and the signal gain as the function of the fiber length in Fig. 2, where the powers of two pumps are both 2.5 W, the pump wave is 1460.9 nm and 1595.5 nm respectively, the signal wave is 1550 nm and the fiber length is 200 m. As seen from the dotted blue curve in the Fig. 2 (a), due to the six-wave mixing interaction, the power of the two pumps reaches the minimum 1.163 W at the fiber length of 135 m, and leads to the effective energy transformation with the signal power being the maximum of 1.347 W as shown from the solid red curve in the Fig. 2 (a). And with the increase of the fiber length, the pumps power turn to increase with the signal power reduction, as can be seen in Fig. 2 (a), which will lead to a lower signal gain in the output of the HNLF. Also, it can be seen from the Fig. 2 (b) that the signal reach the highest gain of 61.3 dB at the fiber length of 135 m. On the other hand, it can be observed that the pump depletion could not be ignored when the greatest signal gain are demanded.

![Fig. 3](image-url) (a) The FWM Signal gain (solid black curve) and un-optimized SWM (dotted blue curve) gain as functions of signal wavelength, (b) The signal gain with GA (solid red curve) and without GA (dotted blue curve) as functions of signal wavelength.

In the following, it is significant to analyze the gain bandwidth and flatness of the six-wave FOPA. Firstly, based on the optimized parameters used in Ref. [19] researched by us, the optimized gain property can be seen from the solid black curve in Fig. 3 (a), which have a good gain for the four wave FOPA, but unfortunately, the bad gain characteristics was gained by using the same previous parameters in the six-wave FOPA system, as shown in the dotted blue curve in Fig. 3 (a). Hence, we should further search for the parameters appropriate for the six-wave FOPA to optimize the gain.
using genetic algorithm to optimize the two pump wavelength with the fiber length of 135 m, it can be seen from the Fig. 3 (b) that the better gain flatness can be achieved with the optimized wavelength of 1460.9 nm and 1595.5 nm.

Next, it is meaningful to study the power of pumps on the influence of the six-wave FOPA, which played vital role on the gain bandwidth, amplitude and flatness. Under the same conditions and same parameters expect of the pump power used previously, the optimized pump power of 3.414 W are obtained as shown in Fig. 4. As seen from the Fig. 4, the optimized pump power can lead to a broader gain bandwidth; however, the gain flatness is not good. Note that the value of the phase matching affects the gain of FOPA. It also can be seen from the Eq. (10) that the relative phase difference in different fiber length is not only affected by the nonlinear coefficient, but also by the power of different waves and the linear phase mismatch. When the pump power is changed while the other parameters remain unchanged, the phase matching will become worse. Therefore, it is necessary to further modulate the other parameters.

Additionally, although there have been many techniques used for the efficient simulated Brillouin scattering (SBS) suppression until now, it is somewhat necessary to consider the SBS, which is needed to be suppressed for the FOPA to operate properly. Hence, we further optimize the gain characteristics of the six-wave FOPA with GA under the condition of the low pump power, based on the previous parameters, the power of pump was reduced to 2 W, and by optimizing the fiber length with GA, the fiber length of 156 m can lead to the better gain property, as shown if Fig. 6, the peak gain of 60 dB and the gain bandwidth of 123 nm ranging from 1468 nm to 1591 nm is achieved.

At last, the nonlinear coefficient of the HNLF plays a very important role in the FOPA, under the condition of low pump power, we further increase the nonlinear coefficient, by
adopting the parameters same as the Ref. [23], which are $\beta_1 = 1 \times 10^{33} \text{s}^{-1} / \text{m}$, $\beta_2 = -1 \times 10^{33} \text{s}^{-1} / \text{m}$, and $\gamma = 35 \text{W}^{-1} \text{km}^{-1}$, respectively. Seen from the Fig. 7, the gain bandwidth of 112 nm covering from 1475 nm to 1587 nm and the peak gain of 60 dB with 0.9 dB uniformity can be achieved when the pump power is 2 W and the fiber length is 88 m. Also can be seen from the solid black curve of the Fig. 7, the gain bandwidth of 123 nm can be achieved when the pump power of 1.5 W. That is to say, even in the condition of low pump power, we can also get the better gain property of six-wave FOPA with GA. In addition, the six-wave FOPA model cannot be ignored especially in the practical system, therefore, it is necessary to find ways to optimize the six-wave FOPA model.

4. CONCLUSION

In summary, we have deduced the governing equations for the power and phase difference evolution of the six interacting waves inside the two-pump FOPA, which provide consistent results with the amplitude evolution equations based on numerical analysis. In addition, we have provided a method to obtain the optimal combination of parameters in six-wave FOPA with GA. The effects of pump depletion, fiber length, pump wavelength and pump power on the gain characteristic of six-wave FOPA has been discussed, which is proved to be an effective way and a series of optimum results are gained. The numerical simulation shows that the GA is an appropriate method to this multi-objective and multi-variate problem for the six-wave FOPA model. We can obtain a flat gain optimization combination scheme readily with the given optical fiber parameters, e.g., a theoretically estimated flat gain of 65 dB with 0.3 dB uniformity over 110 nm bandwidth, which can be potentially used in the optical communication system.

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