A Causal Relationship from R&D Subsidies to UK Growth

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We show that a DSGE model in which subsidies to private sector R&D stimulate economic growth, following the predictions of semi-endogenous growth theory, can account for the joint behaviour of UK output and total factor productivity for 1981-2010. R&D subsidies are measured as government-funded R&D performed by the private sector as a proportion of total private sector R&D. We estimate and test the performance of the model using Indirect Inference, and also investigate the robustness of the results using a Monte Carlo exercise. The implication of our findings are that sharp cuts in R&D subsidies tend to have highly persistent growth effects in the UK.

1. INTRODUCTION

Since Schultz (1953) and Griliches (1958) an influential literature has linked R&D activity to economic growth, and the R&D growth channel is now taken as given by many. For instance, Warda (2005) states simply that “Innovation is the engine of growth in a knowledge economy, and Research and Development (R&D) is the key ingredient of the innovation process,” going on to say that “Government has a major supporting role in this area by providing a favourable business environment, including appropriate and competitive incentive programs for R&D.” (p.2) However, while plainly innovation must cause productivity growth by definition, the empirical link between subsidies, corporate R&D and successful innovation remains less certain at the macroeconomic level, due principally to the difficulty of establishing causality.

This paper therefore investigates a structural model which embeds that key growth hypothesis. The research question is whether direct government R&D subsidies have incentivised the private sector to conduct R&D, and so enhanced innovation and productivity growth in the UK over the sample (1981-2010). The model is tested and estimated using Indirect Inference methods (Le et al., 2012, 2016).

The power of the Indirect Inference test has been shown to be strong in a variety of model contexts, using numerical methods - see for instance Le et al. (2016). We conduct a further Monte Carlo study for the particular model used here which reinforces that conclusion. The strong power of the test to reject models that are misspecified underlines the robustness of our findings. For this reason, the present paper is a useful complement to existing empirical work on the macroeconomic impact of direct R&D subsidies. By estimating the DSGE model and testing it in this way, the conclusions cannot be said to rely on an untested calibration.

A further value of the indirect estimation and testing approach taken here lies in the ability to specify a particular causal mechanism for growth in the DSGE model; hence there is no question surrounding the exogeneity of policy in the model. This approach therefore bypasses the difficulties associated with potential regressor endogeneity which are so hard to address conclusively in regression-based empirical work at the macroeconomic level, while also retaining the idea that hypotheses should be tested by classical econometric methods (an idea that receives less attention in the DSGE literature). To check the model’s identification, we apply the numerical identification test proposed in Le et al. (2017).
Another advantage is that we can look at a single country, the UK, without imposing homogeneity assumptions across a sample of countries which may actually differ in the relationship between R&D subsidies and growth. As a backdrop for the analysis we take an open economy model which has been shown elsewhere to account well for the UK macroeconomy’s behaviour (Meenagh et al., 2010), and add an unambiguous role for R&D subsidies which affect innovation incentives at the microfoundation level. The UK is a highly open economy and we judge that openness to be an important feature in an empirical analysis such as this.

Finally, we note continuing controversy over the importance of direct R&D subsidies to the private sector. R&D policy programmes represent a considerable outlay of public money, but whether they actually generate growth is debated. R&D expenditures and patent numbers are convenient measurables often used as proxies for innovation outputs in empirical studies, but how far they capture innovation is questionable. These proxies may be more closely correlated with non-innovative activities. Firms may patent as a signal to capital markets or to earn through licensing revenues, for instance. Increased R&D expenditures resulting from subsidies may also be channeled straight into researcher wage increases, since researcher supply is relatively inelastic (Goolsbee, 1998). The question of how past R&D subsidies have affected the UK aggregate is therefore of continuing relevance.

We find robust evidence in this paper of a positive impact of shocks to direct R&D subsidies on the path of Total Factor Productivity (TFP) and output. The estimated structural model is used to simulate the impact of a one-off, one percentage point shock to direct subsidies which dies out gradually; this generates a long-lasting growth episode in TFP. The episode translates into an increase in the average annual growth rate of output of 0.2 percentage points per annum for nearly two decades.

A review of some existing literature on R&D-driven growth is given in Section 2, focusing on the macroeconomic literature. Section 3 outlines the DSGE model including the growth process. Empirical work follows in Section 4, including an outline of the methodology and data, estimation results and a variance decomposition for the estimated DSGE model. We also report the results of our Monte Carlo exercise on the power of the testing method applied here, as well as simulation results for a controlled temporary R&D policy reform using the estimated model. Section 5 concludes.

2. LITERATURE

In the New Endogenous Growth theory, spillovers overcome diminishing returns to accumulable factors in the aggregate production function, generating sustained economic growth. They also undermine private incentives to innovate since the innovator cannot appropriate the full return from his investment (e.g. Aghion and Howitt, 1992; Romer, 1990). Supposing that a downward incentive effect dominates, the broad flavour of policy recommendations coming out of these models is that research activities should be subsidised directly - or indirectly through fiscal incentives - in order to bring private returns into line with the social rate, and that protection of intellectual property rights should be increased, enabling the innovator to appropriate a larger portion of the returns to his investment in spite of the non-rival nature of knowledge outputs. The underlying structure of the environment can also play a role, depending on the particular model; competition policy and the reduction of barriers to entry and other market frictions may increase the innovation rate (see discussion in Aghion et al. 2013).

Pure endogenous growth models in the style of Romer (1990) predict large long-run growth responses to changes in the scale of the economy’s R&D sector; but while R&D activity (in terms of labour inputs and investment) increased dramatically in the last century, long-run growth rates were largely stable. Since Jones (1995), a second generation of ‘semi-endogenous’ R&D-driven growth models has emerged which imply a weaker scale effect, allowing R&D and policies incentivising it to have important transitional effects on growth but not to determine the long-run. The choice of semi- versus fully endogenous growth mechanism can imply significantly different optimal R&D tax and subsidy policies; see Sener (2008) for discussion.
We discuss semi-endogenous growth modelling here, given our own empirical focus on the transitional growth effects of R&D policy.

A number of existing DSGE models explore the macroeconomic impacts of R&D policies by simulation, embedding a semi-endogenous R&D-driven growth mechanism and making additional modelling choices which offer various insights. For instance, policymakers may increase innovation through the R&D channel by subsidising human capital accumulation, exploiting complementarities between the two activities that arise through the use of highly skilled workers as an input to the R&D process. This complementarity is modelled in Papa-georgiou and Perez-Sebastian (2006) and explored in Varga and ’t Veld (2011) among others. Cozzi et al. (2017) take a Schumpeterian approach in which the technology frontier evolves semi-endogenously, combining creative destruction with price stickiness.

McMorrow and Roeger (2009) examine the impact of R&D policy on growth in a global DSGE model calibrated to the EU and to the US. They add the semi-endogenous growth mechanism in Jones (1995) to the European Commission’s QUEST III model (Ratto et al. 2009). They find that subsidies to R&D make only a modest contribution to productivity growth. Since the supply of high-skilled workers is constrained, much of the impact of R&D subsidies is absorbed by increases in researcher wages (cf. Goolsbee, 1998). Of course the overall impact is constrained by the semi-endogenous growth assumption. In the short run there is reallocation of high-skilled labour from the production sectors to the research sector, which has a dampening effect on output in the periods directly following the reform (this is the case in the model we propose as well).

An important issue in such models is calibration of the R&D externality parameter.\(^1\) This is generally either set based on the panel econometric literature or set indirectly by other parameter choices, themselves calibrated to results from econometric studies (e.g. Papa-georgiou and Perez-Sebastian, 2006). McMorrow and Roeger (2009) calibrate externalities to panel regression estimates from Botazzi and Peri (2007) and Coe and Helpman (1995). Bye et al. (2011) use a CGE model of a small open economy calibrated to Norway to simulate innovation policy reforms; while they calibrate the growth process from econometric results, they note that estimates are scarce for their purposes and they therefore rely heavily on sensitivity tests. It is of course evident that the simulated policy impacts produced from calibrated DSGE models depend strongly on calibration choices. The difficulties of interpretation posed by macro-level regressions of growth or productivity on policy variables are well known - causality is hard to establish and the scarcity of strong, exogenous instruments for potentially endogenous regressors leaves such regressions prone to bias.\(^2\) We therefore opt not to calibrate the growth process in our model from this literature, given that the magnitude of the parameter on R&D policy is pivotal for our conclusions.

There are rare exceptions to this calibration strategy. Cozzi et al. (2017) estimate structural parameters for a New Keynesian creative destruction model of the US using Bayesian methods. The structure of their model differs from ours, but we note their relatively high estimate of the intertemporal knowledge spillover parameter. This implies that shocks affecting R&D intensity will have long-lasting macroeconomic effects. They also find a high persistence for exogenous R&D policy shocks, consistent with our own results below. We prefer a frequentist estimation strategy here since our reading of the empirical literature does not suggest an appropriate prior for parameters governing the R&D subsidy impact in our UK model. The approach taken here also allows us to evaluate the model’s performance together with the estimated parameter, using the Indirect Inference test. Formal econometric evaluation of DSGE models

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1. Where international spillovers are included there is both a domestic R&D externality parameter and an international externality to be calibrated.
2. Macro-regression studies are often defended on the grounds that “they help us update our priors about the impact of certain types of policies” (Rodrik, 2012, p. 141) and that “even simple or partial correlations can restrict the range of possible causal statements that can be made” (Wacziarg, 2002, p. 909), but when models are not identified it is not clear that this is defensible. Rodrik (2012) points out a failure of the aggregate growth regression literature to address policy endogeneity, which is surprising given the attention paid to the issue in microeconometric studies.
is now receiving increasing attention in the literature; see Giacomini (2013).

Before presenting the UK model in the next section, we highlight some of our modelling choices in the context of the DSGE literature discussed here - one notable difference being that we abstract from knowledge spillovers in the growth process. Growth occurs in this model due to the representative agent’s decision to spend time ‘innovating’; the resulting innovation is excludably donated to the firm, of which the agent is sole shareholder. The assumption simplifies the model considerably while allowing the important testable policy implication to emerge, that R&D subsidies stimulate productivity growth. The broader DSGE literature accommodates increasing theoretical complexity which is insightful; our aim is to strip back this complexity for the time being and see whether we find robust empirical evidence for a simple DSGE model in which R&D subsidies cause TFP behaviour. This is a nontrivial question, since there is a strong possibility that the causation works in the opposite direction, or that the effect is simply negligible and that an exogenous growth model is more appropriate. If support is found for the simple mechanism we propose here, we can proceed to model the microfoundations with more complexity.

3. MODEL

We adapt the open economy Real Business Cycle model in Meenagh et al. (2010), adding an endogenous growth process based on Meenagh et al. (2007). It is a two-country model with a single industry; one broad type of consumption good is traded internationally, but the product of the home goods sector is differentiated from that of the foreign country. Consumers demand both home goods and imported goods. The home country is calibrated to the UK economy and the foreign country represents the rest of the world; its size therefore allows us to treat foreign prices and consumption demand as exogenous. International markets are cleared by the real exchange rate.

The model is a standard UK workhorse in terms of expected macroeconomic and open economy reactions. It is used as a testing vehicle in order to examine whether the productivity path is systematically affected by shocks to R&D subsidies in the UK - a relationship derived below from the model’s microfoundations. This model has the added advantage for the UK of capturing real exchange rate movements while abstracting from monetary policy, which underwent several regime changes in the UK during this period. Since the calibrated UK model has performed well in similar tests (Meenagh et al., 2010), the introduction of the R&D policy variable should test whether this policy hypothesis alone has caused the rejection.

3.1. Consumer Problem

The representative consumer chooses paths for consumption \( C_t \) and leisure \( x_t \) to maximise lifetime utility, \( U \):

\[
U = \max E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(C_t, x_t) \right]
\]

(1)

\( u(.) \) takes the following additively separable form.

\[
u(C_t, x_t) = \theta_0 \gamma_t C_t^{(1-\rho_1)} + (1 - \theta_0) \xi_t x_t^{(1-\rho_2)}
\]

(2)

\( \rho_1, \rho_2 > 0 \) are Arrow-Pratt coefficients of relative risk aversion for consumption and leisure; the inverse of \( \rho_1 \) (\( \rho_2 \)) is the intertemporal substitution elasticity between consumption (leisure) in two consecutive periods. \( \gamma_t \) and \( \xi_t \) are preference shocks, and \( 0 < \theta_0 < 1 \) is a preference weighting on consumption.

While the firm makes zero profits, the agent obtains the full benefit of productivity increases through resulting real wage increases.
The agent divides time among three activities: leisure, labour $N_t$ supplied to the firm for the real wage $w_t$, and an activity $z_t$ that is unpaid at $t$ but known to have important future returns. The time endowment is normalised at one:

$$N_t + x_t + z_t = 1$$  \hspace{1cm} (3)

This section outlines the agent’s choices of leisure versus non-leisure activity, consumption, domestic and foreign bonds ($b_{t+1}$, $b^ho_{t+1}$) and a bond issued by the firm to finance its capital investment ($\tilde{b}_{t+1}$), and new shares ($S^p_t$) purchased at the current price ($q_t$). Income at $t$ is from wages, maturing bonds and dividends ($d_t$) on shares purchased last period, with additional purchasing power from shareholdings, $q_t S^p_{t-1}$. The taxbill $T_t$ is defined further below. The only taxed choice variable in the model is $z_t$; all other taxes are treated as lump sum, adjusting to rule out wealth effects. Since the choice of $z_t$ is left aside until Section 3.4 on endogenous growth, the taxbill is not relevant at this stage of the problem. The agent’s real terms budget constraint is as follows.$^4$

$$C_t + b_{t+1} + Q_t b^ho_{t+1} + q_t S^p_t + \tilde{b}_{t+1} = w_t N_t - T_t + b_t (1 + r_{t-1}) + Q_t b^ho_t (1 + r_{t-1}) + (q_t + d_t) S^p_{t-1} + (1 + \tilde{r}_{t-1}) \tilde{b}_t$$  \hspace{1cm} (4)

$Q_t$ reflects the price of the foreign consumption bundle relative to the general price level at home defined as $Q_t = \frac{P_t^f}{P_t} \tilde{E}_t$. $\tilde{E}_t$ is the nominal exchange rate (domestic currency value of one unit of foreign currency). We assume $\tilde{E}_t \equiv 1$, so $Q_t$ is the import price relative to the domestic CPI. It therefore moves inversely to the real exchange rate, generally thought of as the price of exports relative to imports.$^5$ For fixed $\tilde{E}$, a rise in $Q_t$ implies a real depreciation of the domestic good on world markets and hence an increase in the competitiveness of domestic exports; this can be thought of as a real exchange rate depreciation.

The consumer maximises utility (equations 1 and 2) with respect to $C_t$, $x_t$, $b_{t+1}$, $b^ho_{t+1}$, $\tilde{b}_{t+1}$ and $S^p_t$, subject to equations 3 and 4. The first order conditions yield the Euler equation (5), the intratemporal condition (6),$^6$ real uncovered interest parity (7), and the share price formula (8). The first order conditions on $b_{t+1}$ and $b^ho_{t+1}$ combine to show that $\tilde{r}_t = r_t$, equating the real rate of return on the firm’s bond to the domestic real interest rate. Indeed, returns on all assets ($S^p_t$, $b_{t+1}$, $\tilde{b}_{t+1}$ and $b^ho_{t+1}$) are equal at the margin.

$$\frac{1}{(1 + r_t)^{\gamma_t} C_t^{-p_1}} = \beta E_t[\gamma_{t+1} C_{t+1}^{-p_1}]$$  \hspace{1cm} (5)

$$U_x |_{U=0} = \frac{(1 - \theta_0) \xi_t x_t^{-p_2}}{\theta_0 \gamma_t C_t^{-p_1}} = w_t$$  \hspace{1cm} (6)

$$x_t = \frac{q_{t+1} + d_{t+1}}{(1 + r_t)} = \sum_{i=1}^{\infty} \frac{d_{t+i}}{\prod_{j=0}^{i-1} (1 + r_{t+j})}$$  \hspace{1cm} (8)

The condition in equation 8 rests on the further assumption that $q_t$ does not grow faster than

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$^4$Price $P_t$ of the consumption bundle is numeraire

$^5$b$^ho_{t+1}$ is a real bond - it costs what a unit of the foreign consumption basket ($C^*_t$) would cost, i.e. $P^*_t$ (the foreign CPI). In domestic currency, this is $P^*_t \tilde{E}_t$. Assuming $P^*_t \approx P^f_t$ (i.e. exported goods from the home country have little impact on the larger foreign country) the unit cost of $b^ho_{t+1}$ is $Q_t$.

$^6$Later we show that the return on labour time, $w_t$, is equal at the margin to the return on $z_t$. 

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the interest rate, \( \lim_{t \to \infty} \frac{q_{t+j}}{\prod_{j=0}^{t}(1+r_{t+j})} = 0. \)

The domestic country has a perfectly competitive final goods sector, producing a version of the final good differentiated from the product of the (symmetric) foreign industry. The model features a multi-level utility structure (cf. Feenstra et al. 2014). Differentiated varieties of the final good yield utility to the agent via a CES sub-function: having discovered the agent’s choice of \( C_t \) for the level-one utility maximisation, we treat it as parametric and consider how that amount of consumption breaks down between domestic and foreign varieties, \( C^d_t \) and \( C^f_t \).

The level of \( C_t \) chosen above must satisfy the expenditure constraint,

\[
C_t = p^d_tC^d_t + Q_tC^f_t
\]

\( p^d_t \) is the domestic good price relative to \( P_t \). \( C^d_t \) and \( C^f_t \) are chosen to maximise \( \tilde{C}_t \) according to the following utility function (equation 10), subject to the constraint that \( \tilde{C}_t \leq C_t \).

\[
\tilde{C}_t = [\omega(C^d_t)^{-\rho} + (1 - \omega)\zeta_t(C^f_t)^{-\rho}]^{-\frac{1}{\rho}}
\]

At a maximum the constraint binds and consumption-equivalent utility, \( \tilde{C}_t \), equals the amount spent on consumption, \( C_t \). There is fixed preference bias towards domestic goods, \( 0 < \omega < 1 \).

Import demand is subject to a shock, \( \zeta_t \). The elasticity of substitution between domestic and foreign varieties is constant at \( \sigma = \frac{1}{1+\rho} \). First order conditions imply the relative demands for the import (equation 11) and the domestic good (equation 12).

\[
\frac{C^f_t}{C_t} = \left( \frac{1 - \omega)\zeta_t}{Q_t} \right)^{\sigma}
\]

(11)

\[
\frac{C^d_t}{C_t} = \left( \frac{\omega}{p^d_t} \right)^{\sigma}
\]

(12)

Given equation 11 above, the symmetric equation for foreign demand for domestic goods (exports) relative to general foreign consumption is

\[
(C^d_t)^* = C^*_t \left( (1 - \omega^F) \zeta^*_t \right)^{\sigma^F} (Q^*_t)^{-\sigma^F}
\]

(13)

* signifies a foreign variable; \( \omega^F \) and \( \sigma^F \) are foreign equivalents to \( \omega \) and \( \sigma \). \( Q^*_t \) is the foreign equivalent of \( Q_t \), the ratio of the import price to the CPI, and \( \ln Q^*_t \simeq \ln p^d_t - \ln Q_t \).

An expression for \( p^d_t \) as a function of \( Q_t \) follows from the maximised equation 10:

\[
1 = \omega^F (p^d_t)^{\rho^F} + [(1 - \omega)\zeta_t]^\rho Q^*_{t}^{\rho^F}
\]

(14)

A first order Taylor expansion around a point where \( p^d_t \simeq Q \simeq \zeta \simeq 1 \), with \( \sigma = 1 \), yields a loglinear approximation for this expression:

\[
\ln p^d_t = \hat{k} - \frac{1 - \omega}{\omega} \frac{1}{\rho} \ln \zeta_t - \frac{1}{\omega} \ln Q_t
\]

(15)

The export demand equation is then

\[
\ln (C^d_t)^* = \hat{c} + \ln C^*_t + \sigma^F \frac{1}{\omega} \ln Q_t + \varepsilon_{ex,t}
\]

(16)

where \( \hat{c} \) collects constants and \( \varepsilon_{ex,t} = \sigma^F [\ln \zeta_t + \frac{1 - \omega}{\omega} \frac{1}{\rho} \ln \zeta_t]. \)

\( Q^*_t = P^d_t \) - since \( Q_t = \frac{p^d_t}{P^d_t} \) and \( P_t \) is numeraire, \( Q_t = P^f_t \). If domestic export prices hardly influence the foreign CPI then \( P^*_t \simeq P^f_t \).
Assuming no capital controls, the real balance of payments constraint is satisfied so that the current account surplus and capital account deficit sum to zero.

\[
\Delta b^f_{t+1} = r^f b^f_t + \frac{p^d_t E_t}{Q_t} - IM_t
\]  

(17)

3.2. Firm Problem

The representative firm produces the final good via a Cobb-Douglas function with constant returns to scale, where \(A_t\) is total factor productivity:

\[
Y_t = A_t K_t^\alpha N_t^\beta
\]  

(18)

There are diminishing marginal returns to labour and capital. The firm also faces convex adjustment costs to capital, taking a quadratic form. The firm undertakes capital investment, raising funds to purchase new capital by issuing debt \((\hat{b}_{t+1})\) at \(t\), the cost of which is \(\hat{r}_t\) payable at \(t+1\). Bonds are issued one for one with units of capital demanded: \(\hat{b}_{t+1} = K_t\). The cost of capital covers not only the return demanded by debt-holders, but also capital depreciation \(\delta\) and adjustment costs, represented by \(\hat{\alpha}_t\). The profit function is:

\[
\pi_t = Y_t - \hat{b}_{t+1}(\hat{r}_t + \delta + \hat{\alpha}_t) - (\hat{w}_t + \chi_t) N_t
\]

\(\hat{w}_t\) is the unit cost of labour; \(\kappa_t\) and \(\chi_t\) are shocks to the net rental costs of capital and labour, capturing random movements in marginal tax rates, for instance in depreciation allowances or national insurance. The consumer’s first order conditions imply that \(\hat{r}_t = r_t\). Substituting for \(\hat{b}_{t+1} = K_t\) and \(\hat{r}_t = r_t\), the profit function is

\[
\pi_t = Y_t - K_t(r_t + \delta + \kappa_t) - \frac{1}{2} \zeta(\Delta K_t)^2 - (\hat{w}_t + \chi_t)N_t
\]

(19)

Here adjustment costs are explicit, having substituted \(\hat{b}_{t+1}\hat{\alpha}_t = K_t\hat{\alpha}_t = \frac{1}{2} \zeta(\Delta K_t)^2\). Parameter \(\zeta\) is constant.

The firm maximises expected profits subject to these constraints, choosing capital \((K_t)\) and labour \((N_t)\), taking prices \(r_t\) and \(\hat{w}_t\) as given. Assume free entry into the sector and a large number of firms operating under perfect competition. The Lagrangian for the problem is \(L_0\):

\[
L_0 = E_0 \sum_{t=0}^{\infty} d^t E_t \left\{ Y_t - K_t(r_t + \delta + \kappa_t) - \frac{1}{2} \zeta(\Delta K_t)^2 - (\hat{w}_t + \chi_t)N_t \right\}
\]

(20)

d is the firm’s discount factor. The optimality condition for \(K_t\) equates the marginal product of capital (net of adjustment costs and depreciation) to its price, plus cost shock.

\[
(1 - \alpha) \frac{Y_t}{K_t} - \delta - \zeta \Delta K_t + d \zeta E_t(\Delta K_{t+1}) = r_t + \kappa_t
\]

(21)

This gives a non-linear difference equation in capital.

\[
K_t = \frac{1}{1 + d} K_{t-1} + \frac{d}{1 + d} E_t K_{t+1} + \frac{1 - \alpha}{\zeta(1 + d)} \frac{Y_t}{K_t} - \frac{1}{\zeta(1 + d)} (r_t + \delta) - \frac{1}{\zeta(1 + d)} \kappa_t
\]

(22)

Given capital demand, the firm’s investment, \(I_t\), follows via the capital accumulation identity.

\[
K_t = I_t + (1 - \delta)K_{t-1}
\]

(23)

\(^8\text{the adjustment cost attached to}\ \hat{b}_{t+1} \text{ is:} \ \hat{b}_{t+1}\hat{\alpha}_t = \hat{b}_{t+1} \frac{1}{2} \zeta \left( \hat{b}_{t+1} + \frac{\hat{b}_{t+1}^2}{\hat{b}_{t+1}} - 2\hat{b}_t \right) = \frac{1}{2} \zeta(\Delta \hat{b}_{t+1})^2\)
The optimal labour choice equates the marginal product to its real price, $\tilde{w}_t$, plus the cost shock $\chi_t$. This gives the firm’s labour demand condition.

$$N_t = \alpha \frac{Y_t}{\tilde{w}_t + \chi_t} \quad (24)$$

Note that differentiated goods at the international level introduce a wedge between the consumer real wage, $w_t$, and the real unit cost of labour for the firm, $\tilde{w}_t$. The wedge is

$$p^d_t = \frac{w_t}{\tilde{w}_t} \quad (25)$$

implying, via 15, the relationship in equation 26.

$$\ln w_t = \hat{k} + \ln \tilde{w}_t - \frac{1 - \omega}{\omega} \ln Q_t - \frac{1 - \omega}{\omega} \frac{1}{\rho} \ln \chi_t \quad (26)$$

### 3.3. Government

The government spends on the consumption good ($G_t$) subject to its budget constraint.

$$G_t + b_t(1 + r_{t-1}) = T_t + b_{t+1} \quad (27)$$

Spending is assumed to be non-productive and made up strictly of welfare transfers. As well as raising tax revenues $T_t$ the government issues bonds maturing one period ahead. Each period the government raises tax revenues to cover spending and the current bill for debt interest, so that $T_t = G_t + r_{t-1}b_t$ and $b_t = b_{t+1}$. Therefore the level government debt is fixed in the model and the government is fully solvent. Revenue $T_t$ is made up as follows.

$$T_t = \Phi_t - s_t z_t \quad (28)$$

$s_t$ is a proportional subsidy rate on time spent in innovative activity $z_t$. $\Phi_t$, a lumpsum tax capturing the revenue effects of all other tax instruments, responds to changes in $s_t z_t$ to keep tax revenue neutral in the government budget constraint. Government spending is modeled as an exogenous trend stationary AR(1) process.

$$\ln G_t = g_o + g_1 t + \rho_g \ln G_{t-1} + \eta_{g,t} \quad (29)$$

where $|\rho_g| < 1$ and $\eta_{g,t}$ is a white noise innovation.

### 3.4. Productivity Growth

Assume that productivity growth is a linear function of time spent in some innovation-enhancing activity $z_t$.

$$\frac{A_{t+1}}{A_t} = a_0 + a_1 z_t + u_t \quad (30)$$

where $a_1 > 0$. $z_t$ is the systematic channel through which policy incentives, $s_t$, drive growth.\(^9\)

The characterisation of $z_t$ depends on the data used for $s_t$. Here it is assumed to be the investment of time in R&D. By manipulating the first order condition, $z_t$ can be bypassed altogether in the model; productivity growth ultimately depends on the subsidy $s_t$ alone (equation 32).

\(^9\)The real cost of labour for the domestic firm is the nominal wage $W_t$ relative to the unit value of the domestic good, $P^d_t$, while the real consumer wage is $W_t$ relative to the general price $P_t$ of the mixed consumption bundle.\(^10\)

\(^10\)All other factors that might systematically affect growth - such as human capital - are therefore in the error term.
The model is conceptually similar to Lucas (1988, 1990) where growth depends on time spent in human capital accumulation. Once accumulated, human capital enhances labour efficiency and increases earnings, but in the short term the return to labour (for a given level of human capital) is foregone to raise the human capital stock. This implies a trade-off; a unit of non-leisure time can be allocated as an input to the human capital production function or as an input to goods production. The endogenous growth process below is adapted from Meenagh et al. (2007) to a decentralised framework.

The consumer maximises utility in equations 1 and 2 with respect to \( z_t \), subject to budget and time constraints and the taxbill (equations 3, 4 and 28)). We assume that for all \( t \), the consumer’s shareholdings are equivalent to a single share:\(^{11}\) \( S^0_t = \hat{S} = 1 \). The value per share given in equation 8 is then the value of the firm. Dividend income \( d_t \) received by shareholder is everything leftover from revenue after labour and capital input costs are paid, i.e. profits. The rational agent expects \( z_t \) to raise her own consumption possibilities through her role as the firm’s sole shareholder. She knows that, given equation 30, a marginal change in \( z_t \) permanently raises productivity from \( t + 1 \). This higher productivity is fully excludable and donated to the atomistic firm she owns; higher productivity is anticipated to raise household income via firm profits paid out as dividends. The choice is assumed not to affect economy-wide aggregates; all prices are taken as parametric (note that the productivity increase is not expected to increase the consumer real wage here, though it does so in general equilibrium - cf. Boldrin and Levine, 2002 and 2008).\(^{12}\)

Substituting into the first order condition for \( z_t \) using equation 30 and rearranging for \( \frac{A_{t+1}}{A_t} \) yields (after some approximation)

\[
\frac{A_{t+1}}{A_t} = a_t \left( 1 - \frac{\beta \tau - \tau}{1 - (1 - \tau_t)^2} \right)
\]

The full derivation is given in the Appendix. We define \( \frac{z_t}{w_t} \equiv s'_t \), to refocus the driver variable as the ratio of subsidies to real wages (the opportunity cost of spending time outside regular labour). \( s'_t \) is a unit free measure with the dimensions of a rate as opposed to \( s_t \) which, like the wage, is an amount of money payable per unit of time. Therefore \( s'_t \) is easier to take to the data. A first order Taylor expansion of the righthand side of equation 31 around a point where \( s'_t = s' \) gives a linear relationship between \( \frac{A_{t+1}}{A_t} \) and \( s'_t \) of the form

\[
d \ln A_{t+1} = b_0 + b_1 s'_t + \varepsilon_{A,t} \tag{32}
\]

where \( b_1 = a_t \left( 1 - \frac{\beta \tau - \tau}{1 - (1 - \tau_t)^2} \right) \).\(^{13}\) Note that this relationship came out of the first order condition for \( z_t \). The household chooses \( z_t \) taking all other sources of productivity growth as exogenous; other factors outside the model (like human capital) therefore affect the error term in the productivity time series. Equation 32 drives the behaviour of the model in simulations.

There are of course notable aspects of the R&D growth channel that we abstract from here. We do not deal with catch-up or distance from the global technological frontier; nor do we explicitly include spillovers in the micro-foundations. There is no suggestion that growth is in reality as simple as this model suggests. We look simply at whether the approximations made here are empirically justifiable.

Substituting into 31 using 30 reveals a relationship between \( z_t \) and \( s'_t \). Define \( \frac{\partial z_t}{\partial s'_t} \equiv c_1 \), and assume this to be a constant. This parameter enters the simulation explicitly in the producer

\(^{11}\)This assumption allows the substitution to be made in the budget constraint that \( q_t S^0_t = (q_t + d_t) S^0_{t-1} = -d_t \).

\(^{12}\)Given the time endowment \( 1 = N_t + x_t + z_t \), the agent has indifference relations between \( z_t \) and \( x_t \), between \( x_t \) and \( N_t \), and \( z_t \) and \( N_t \). The intratemporal condition in 6 gives the margin between \( x_t \) and \( N_t \); here we focus on the decision margin between \( z_t \) and \( N_t \), so the margin between \( x_t \) and \( x_t \) is implied. Therefore the substitution \( N_t = 1 - x_t - z_t \) can be made in the budget constraint.

\(^{13}\)Other terms in the expansion are treated as part of the error term.
labour cost equation:

\[ \ln w_t = \text{const}_4 + \rho_2 \ln N_t + \rho_1 \ln C_t + \left[ \frac{1 - \omega}{\omega} \right]^{\sigma} \ln Q_t - \rho_2 c_1 s'_t + e_{w,t} \]  

(33)

where

\[ e_{w,t} = -\ln \gamma_t + \ln \xi_t + \frac{1}{\rho} \left[ \frac{1 - \omega}{\omega} \right]^{\sigma} \ln \zeta_t \]  

(34)

i.e. the unit labour cost shock is a combination of preference shocks to consumption and leisure and to import demand. This equation is derived from the intratemporal condition (equation 6) which governs labour supply choices (for the step-by-step derivation, see Appendix). Since \( s'_t \) represents an incentive to R&D, \( c_1 > 0 \) and hence \( \frac{d \ln w_t}{ds'_t} > 0 \) and equally \( \frac{d \ln N_t}{ds'_t} < 0 \), since equation 33 is simply the labour supply condition rearranged; so the worker's response to a higher subsidy rate on \( z_t \) is to reduce time spent in ordinary employment.

3.5. Closing the model

Goods market clearing is required to close the model. In volume terms, the supply of the domestic good is equated to the demand for consumption (net of imports), investment, government consumption and exports.

\[ Y_t = C_t + I_t + G_t + EX_t - \text{IM}_t \]  

(35)

All asset markets also clear.

A transversality condition is also required to ensure a balanced growth equilibrium is reached for this open economy in which trade deficits (surpluses) cannot be run forever via borrowing from (lending) abroad. This rules out a growth path financed by insolvent borrowing rather than growing fundamentals. The transversality condition imposes the restriction on the balance of payments identity that in the long run the change in net foreign assets (the capital account) must be zero. At some notional terminal date \( T \) when the real exchange rate is constant, the cost of servicing the current level of debt must be met by an equivalent trade surplus.

\[ r^f_T b^f_T = - \left( \frac{p^d_T E X_T}{Q_T} - \text{IM}_T \right) \]  

(36)

This is the only transversality condition in the model, and the numerical solution path is forced to be consistent with the constraints it places on the rational expectations. In practice it is a constraint on household borrowing since government solvency is ensured already by other means, and firms do not borrow from abroad.

When solving the model, the balance of payments constraint is scaled by output so that the terminal condition imposes that the ratio of debt to gdp must be constant in the long run, \( \Delta b^f_{t+1} = 0 \) as \( t \to \infty \), where \( b^f_{t+1} = \frac{b^f_{t+1}}{r^f_{t+1}} \). This implies that the growth rate of debt equals the growth rate of real gdp \( (g_Y) \).

The model is loglinearised before solution and simulation; the full loglinearised model is listed in Appendix B.

3.6. Exogenous variables

Stationary exogenous variables in the system are shocks to the real interest rate (Euler equation), labour demand, the real wage, capital demand, export demand and import demand. These are not directly observable but are implied by the difference between the data and the model predicted values (given some set of structural parameters). Those differences \( e_{i,t} \) are
then treated as trend stationary AR(1) processes as follows:

$$e_{i,t} = a_i + b_i t + \rho_i e_{i,t-1} + \eta_{i,t}$$  (37)

where $\eta_{i,t}$ is an i.i.d mean zero innovation term, and $i$ identifies the endogenous variable to which the residual belongs. We model foreign consumption demand, government consumption, foreign interest rates and the policy variable $s_t$ similarly.

The AR(1) coefficients $\rho_i$ are estimated. To find the model’s structural residuals where expectations enter, expectational variables are estimated using a robust instrumental variable technique due to Wickens (1982) and McCallum (1976); they are the one step ahead predictions from an estimated VECM. Where $a_i \neq 0$ and $b_i \neq 0$, the linearly detrended residual $\hat{e}_i$ is used, where

$$\hat{e}_{i,t} = r_i \hat{e}_{i,t-1} + \eta_{i,t}$$  (38)
$$\hat{e}_{t,i} = e_{i,t} - \hat{a}_i - \hat{b}_i t$$  (39)

The innovations $\eta_{i,t}$ are approximated by the fitted residuals from estimation of equation 38, $\hat{\eta}_{i,t}$. The Solow residual $\ln A_t$ is modelled as a unit root process with drift driven by a stationary AR(1) shock and by exogenous variable $s_t$, based on equation 32.

$$\ln A_t = d + \ln A_{t-1} + b_1 s_t - 1 + e_{A,t}$$  (40)
$$e_{A,t} = \rho_A e_{A,t-1} + \eta_{A,t}$$  (41)

Deterministic trends are removed from the exogenous variables since they enter the model’s balanced growth path. We focus here on how the economy deviates from that steady state in response to shocks - in particular, stationary shocks to R&D subsidies. Such shocks will have a permanent shift effect on the path of TFP via its unit root. Due to their persistence they also generate long-lasting transitional TFP growth episodes above long-run trend.

The model is solved using a projection method (cf. Fair and Taylor, 1983), which checks that the one period ahead expectations are consistent with the model’s own predictions and that the expectations satisfy the model’s terminal conditions at the end of the simulation window. These conditions force the endogenous variables’ simulated paths to converge to the model’s long run equilibrium at the ‘terminal’ date. Since the model is not stationarised, the long run levels at which the endogenous variables reach balanced growth depend on the behaviour of the non-stationary driving variables as they have evolved stochastically over the simulation period.

4. EMPIRICAL WORK

4.1. Indirect Inference Methods

The model in the preceding section is tested and estimated using the Indirect Inference method following Le et al. (2011). The model evaluation approach is similar to traditional Real Business Cycle moment-matching, with the addition of a formal statistical test for the closeness of those moments. The samples generated from the bootstrapped model and the observed data are described atheoretically by an auxiliary model, used as a basis for the comparison. The full methodology is given in Le et al. (2016). We describe it briefly here.

Using calibrated parameter set $(\theta)$, $J$ bootstrap simulations are generated from the DSGE model. Having added back the effects of deterministic trends removed from shocks, an auxiliary model is estimated for all $J$ pseudo-samples. The estimated auxiliary model coefficient vectors $a_j$ ($j = 1, ..., J$) yield the variance-covariance matrix $\Omega$ of the DSGE model’s implied distribution for these coefficients. Hence the small-sample distribution for the Wald statistic $W_S(\theta)$ is obtained:

$$W_S(\theta) = (a_j - \bar{a}_j(\theta))^T W(\theta) (a_j - \bar{a}_j(\theta))$$  (42)
\( \bar{a}_j(\theta) \) is the arithmetic mean of the \( J \) estimated vectors and \( W(\theta) = \Omega(\theta)^{-1} \) is the inverse of the estimated variance-covariance matrix.\(^{14}\) The test statistic, \( WS^*(\theta) \), is

\[
WS^*(\theta) = (\hat{\alpha} - \bar{a}_j(\theta))^\prime W(\theta) (\hat{\alpha} - \bar{a}_j(\theta))
\]

a function of the distance between \( \bar{a}_j(\theta) \) and \( \hat{\alpha} \), where \( \hat{\alpha} \) is the coefficient vector estimated from the UK data. Inference proceeds by comparing the percentile of the Wald distribution at which the test statistic falls with the chosen size of the test; for a 5% significance level, a percentile above 95% signifies rejection. We can present the same information as a t-statistic (Mahalanobis distance\(^{15}\)) or as a p-value.

For estimation, a 'simulated annealing' algorithm performs the indirect inference Wald test for points inside a bounded parameter space. We look for a parameter set for this model such that the restrictions it imposes, including the hypothesised causal relationship from R&D subsidies to TFP, do not lead it to be rejected as a data generating process. This is discussed further below.

### 4.2. Data

#### 4.2.1. UK Macroeconomic Data

We use unfiltered data from 1981 to 2010. This is due to problems inherent in data filtering (see e.g. Hamilton, 2016). Particularly in this case where we are interested in relatively long-lasting growth episodes in response to shocks propagated through non-stationary TFP, the risk of mistaking that response for a change in underlying trend and filtering some of it out is high if an HP filter is used. The auxiliary model is therefore a Vector Error Correction Model since we have non-stationary data; this is discussed further in section 4.3 below. The macroeconomic data for the UK is plotted in Figure 1. Data sources are listed in Appendix C.

#### 4.2.2. Data on R&D Subsidies

Equation 32 is repeated here for convenience:

\[
D \ln A_t = b_0 + b_1 s_t' + e_{A,t}
\]

The hypothesis is that \( b_1 > 0 \), i.e. that \( s_t' \) encourages the growth driving activity \( z_t \) defined here as R&D. Since \( z_t \) itself is not included in simulations of the model, the choice of data for \( s_t \) identifies the growth channel. Data is available post-1981 for R&D. The policy variable used is the ratio of business-performed R&D expenditure (BERD) financed directly by government, to the total level of BERD (all sources of funding).\(^{16}\) This is referred to below as the subsidy rate. Aggregate data on BERD from 1981 is annual with missing values at 1982 and 1984. Each missing value has been interpolated as the arithmetic average of the two contiguous values. Robustness checks have been conducted around the interpolation of these missing values and are reported below. The ratio obtained at annual frequency is interpolated to a quarterly frequency using a constant average match interpolation. The ratio is plotted in Figure 2 for the constant average and quadratic average interpolations. The detrended subsidy variable is modeled as a persistent but stationary AR(1) process (see exogenous variables section above).

Since the R&D subsidy variable does not include fiscal incentives to R&D, which have increased in the UK since 2000, it is only a partial proxy for the policy incentives to R&D. However, fiscal incentives as measured by the OECD B-Index may affect R&D and productivity growth differently to direct subsidies (e.g. Foreman-Peck, 2013), so it is not immediately clear that we should combine them into a single index. Likewise, no indicator of policy surrounding

\(^{14}\)The high power of this method relative to Likelihood based tests is due in part to the use of the restricted covariance matrix in constructing the Wald statistic, as opposed to the unrestricted COV matrix.

\(^{15}\)Since the Wald is a chi-squared, the square root is asymptotically a normal variable.

\(^{16}\)Source: OECD (2014).
FIG. 1 Key quarterly UK data for 1980-2010.

FIG. 2 Business R&D Subsidy Variable. Ratio of Government Funded BERD to Total BERD. Constant and average match interpolation. Source, OECD
intellectual property rights has been discovered spanning a long enough time frame for this investigation; in any case within the UK taken singly such an indicator would show little time series variation and be uninformative. We could resort to patent counts to proxy innovation policy, but a) these are an outcome and may not be a good proxy for policy and b) they respond in a way that may have nothing to do with productivity (there is a large literature on the appropriateness of patents as a measure of innovation, see e.g. van Pottelsberghe, 2011). For these reasons, the subsidy variable employed here is preferred.

4.3. Auxiliary Model

The full solution to the structural model gives the endogenous variables as a function of the structural parameters and current and past exogenous variables, which can be represented as a cointegrated VECM rearranged as a VARX(1) – see Appendix D. The general form is

\[ y_t = [I - K]y_{t-1} + K\Pi x_{t-1} + n + \phi t + q_t \]  \hspace{1cm} (43)

The error \( q_t \) contains suppressed lagged difference regressors, while \( t \) captures the deterministic trend in \( x_t \) (the balanced growth behaviour of the exogenous variables) affecting both the endogenous and exogenous variables. \( x_{t-1} \) contains unit root variables, present to control for the impact of past shocks on the long run path of both \( x \) and \( y \). This VARX(1) approximation to the reduced form of the model is the unrestricted auxiliary model used to assess the closeness of model-simulated samples to the observed data.

Since the focus is on the transitional growth of output and TFP and whether our assumptions about the causal role of R&D policy are correct, we use a ‘directed’ Wald (Le et al. 2011) that focuses on the endogenous variables of central interest. Endogenous variables in the auxiliary VARX(1) are therefore output and TFP, while exogenous lagged variables are the subsidy variable and net foreign assets, \( b_{t-1}^f \). The latter captures the stochastic trend in the model through its unit root.

The test is whether the model replicates the features not just of output and productivity taken singly, but the joint behaviour of those variables conditional on the behaviour of any non-stationary predetermined variables and of the policy variable. Although this VARX(1) is a severe approximation of the model’s solution, the power of the test remains strong; the small sample properties of Indirect Inferences are discussed for a variety of models in Le et al. (2016). Since Monte Carlo studies can be model-dependent, we also investigate the power of the test in this particular context in Section 4.6 below.

The vector \( a_j \) used to construct the Wald distribution (eq. 42) includes OLS estimates of coefficients on the lagged endogenous and exogenous variables, as well as the variances of the fitted auxiliary model errors; the same coefficients make up vector \( \hat{\alpha} \) estimated on the observed data. The VARX errors are also tested for stationarity. The trend term in the VARX(1) captures the deterministic trend in the data and simulations. Since the focus of the study is on the stochastic trend resulting from the shocks, the deterministic trend is not part of the Wald test on which the model’s performance is evaluated.

4.4. Indirect Inference Testing and Estimation Results

We first test a baseline calibration of the model, using parameter values from Meenagh et al (2010). We then estimate a number of the structural parameters of the model using Indirect Inference and report those estimates as well as the Wald test statistic for the model with that set of parameters. The structural parameters we chose to estimate are listed in Table 2. They are generally preference-related parameters, as well as the policy-growth parameter, for which no strong priors exist. Due to the attention paid in the literature to adjustment inertia in the response of R&D to policy determinants (Guellec and van Pottelsbergh, 2000; Westmore, 2013; Di Comite et al. 2015), we also test and estimate the model with a 4 quarter lag in
the subsidy rate, whereas the baseline model assumes a 1 quarter lag. Estimated structural parameters are therefore reported for two models.

1. SUBS Model 1, where the productivity process is

\[ \ln A_t = \ln A_{t-1} + b_0 + b_1 s_{t-1} + e_{A,t} \]

2. SUBS Model 2, where productivity is

\[ \ln A_t = \ln A_{t-1} + b_0 + b_1 s_{t-4} + e_{A,t} \]

Some structural model coefficients are kept fixed throughout the empirical work here, at values taken from Meenagh et al. (2010); see Table 1. Long run ratios featuring in the loglinearised model for \( \frac{M}{Y} \), \( \frac{X}{Y} \), \( \frac{Y}{C} \) and \( \frac{G}{C} \) are calibrated to UK post-war averages. \( \frac{X}{C} \) and \( \frac{M}{C} \) are then set to be consistent with those values.

The baseline calibration is given in Table 2, column 3. The implied AR(1) coefficients for the stationary exogenous variables are given in column 3 of Table 3. Analysis of impulse response functions show real business cycle behaviour consistent with Meenagh et al. (2010); impulse responses for a one-oﬀ policy shock are likewise as expected - see section below on temporary policy reform for further discussion.

The macroeconometric literature does not oﬀer a strong prior for \( b_1 \), the impact of the subsidy shock on next period’s TFP, in terms of sign or magnitude. Estimates for the impact of R&D on TFP and of direct subsidies on TFP or output growth vary across different regression models and estimators, for different samples and for different measures of R&D or of the policy environment. The same holds for \( c_1 \); compare e.g. Falk (2006) to Westmore (2013). Lacking a compelling rationale for calibrating this model from the existing literature, starting values chosen for these are 0.1 and 0.06 respectively, and we search around these values in the estimation procedure.\(^{17}\) A preliminary to the estimation is to set bounds on the parameter space; these are set at 30\% either side of the baseline calibration. If the parameter starting value is inappropriate, the estimation process will move towards one of the initial bounds, indicating that the bounds should be shifted.

The addition of the policy-driven TFP process leads the model to be rejected by the II test with this structural calibration (Table 2, column 3). This is true when the relevant policy shock in the TFP process is assumed to be the 1-quarter lagged subsidy and also for the 4-quarter

\(^{17}\) A small starting value for \( c_1 \) is preferred since the labour supply effects induced by policy change should plausibly be small.
lagged subsidy; in each case the test statistic falls in the 100th percentile of the bootstrapped Wald distribution.

However, when the model is estimated by Indirect Inference a structural parameter set is found such that the model is not rejected by the test. For the parameters listed in Table 2, column 4, the test statistic falls in the 77th percentile of the distribution, signifying a comfortable non-rejection. Some coefficients have moved some way from their starting values. Indeed, when the model is assessed on output and productivity alone, the Wald-minimising coefficients are very close to the set found for the L.Minford and Meenagh (2017) model in which productivity is driven by labour market regulation and top marginal income tax rates. Only $b_1$ and $c_1$ are different in absolute magnitude, as we might expect given the different policy variable driving the model here. This implies that the same structural model can accommodate various policy drivers of TFP.

The model also passed when the subsidy variable is assumed to have a lagged impact of one year, though it is a borderline non-rejection with a Wald percentile of 94.48 and this is obtained given a different structural calibration (Table 2, col. 5). This is a weaker result and suggests that assuming a 4-quarter lagged impact for the subsidy is excessive.

4.5. Variance Decomposition

A variance decomposition for key variables in the model with this coefficient set is reported in Table 4. To obtain this we bootstrap the model and calculate the variance of the simulated endogenous variables generated by each of the eleven shocks, taken one at a time. For each column, the cell values indicate the proportion of the total model variance for that endogenous variable generated by each exogenous variable; columns of Table 4 sum to unity. See Appendix E for the full variance decomposition (all endogenous variables); here we pick out output and TFP due to their relevance for the growth question, as well as labour supply (impacted by the subsidy) and key open economy variables (the real interest rate, real exchange rate and net foreign assets).

The penultimate column in Table 4 shows that the R&D subsidy shock accounts for 62.8% of the total variance in TFP in the estimated model, more than the independent shock to TFP (note that shocks to subsidies and shocks to the AR(1) productivity error term $e_{A,t}$ are bootstrapped independently). The estimated value of $b_1$ is clearly large enough to distinguish this model clearly from an exogenous productivity growth model. The identified subsidy shock generates considerable variability across all endogenous variables.
### TABLE 2
Structural Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Starting calibration</th>
<th>Estimates I</th>
<th>Estimates II (4-quarter lag)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRRA coefficient ($C_t$)</td>
<td>$\rho_1$</td>
<td>1.0</td>
<td>0.9712</td>
</tr>
<tr>
<td>CRRA coefficient ($x_t$)</td>
<td>$\rho_2$</td>
<td>1.2</td>
<td>1.5198</td>
</tr>
<tr>
<td>Preference weight on $C_t$</td>
<td>$\theta_0$</td>
<td>0.5</td>
<td>0.5267</td>
</tr>
<tr>
<td>Home bias in consumption</td>
<td>$\omega$</td>
<td>0.7</td>
<td>0.5431</td>
</tr>
<tr>
<td>Foreign equivalent of $\omega$</td>
<td>$\omega^F$</td>
<td>0.7</td>
<td>0.8819</td>
</tr>
<tr>
<td>Import demand elasticity</td>
<td>$\sigma$</td>
<td>1.0</td>
<td>0.7676</td>
</tr>
<tr>
<td>Elasticity of substitution ($C_t^{d*}, C_t^{f*}$)</td>
<td>$\sigma^F$</td>
<td>0.7</td>
<td>0.8522</td>
</tr>
<tr>
<td>Capital equation coefficients $^{18}$</td>
<td>$\zeta_1, \zeta_2, \zeta_3, \zeta_4$</td>
<td>0.51, 0.47, 0.02, 0.25</td>
<td>0.63, 0.35, 0.02, 0.24</td>
</tr>
<tr>
<td>Capital equation coefficients $^{18}$</td>
<td>$c_1$</td>
<td>0.06</td>
<td>0.0632</td>
</tr>
<tr>
<td>Capital equation coefficients $^{18}$</td>
<td>$b_1$</td>
<td>0.1</td>
<td>0.0901</td>
</tr>
<tr>
<td>Wald percentile</td>
<td>100</td>
<td>77.04</td>
<td>94.48</td>
</tr>
</tbody>
</table>

18. $\frac{\partial R_t}{\partial c_t}$ and $\frac{\partial R_t}{\partial d_t}$ refer to the partial derivatives of the capital and income equations with respect to the capital and income variables, respectively.
### TABLE 3
AR coefficients for stationary exogenous variables

<table>
<thead>
<tr>
<th>Exogenous variable</th>
<th>AR coefficient</th>
<th>Starting calibration</th>
<th>Estimated Model 1</th>
<th>Estimated Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shock to real interest rate</td>
<td>$\rho_r$</td>
<td>0.860</td>
<td>0.858</td>
<td>0.839</td>
</tr>
<tr>
<td>Shock to $TFP$</td>
<td>$\rho_A$</td>
<td>0.589</td>
<td>0.577</td>
<td>0.588</td>
</tr>
<tr>
<td>Shock to labour demand</td>
<td>$\rho_N$</td>
<td>0.897</td>
<td>0.897</td>
<td>0.913</td>
</tr>
<tr>
<td>Shock to capital demand</td>
<td>$\rho_K$</td>
<td>0.765</td>
<td>0.951</td>
<td>0.950</td>
</tr>
<tr>
<td>Shock to real wage</td>
<td>$\rho_{\tilde{r}}$</td>
<td>0.879</td>
<td>0.837</td>
<td>0.943</td>
</tr>
<tr>
<td>Shock to export demand</td>
<td>$\rho_X$</td>
<td>0.939</td>
<td>0.939</td>
<td>0.938</td>
</tr>
<tr>
<td>Shock to import demand</td>
<td>$\rho_M$</td>
<td>0.848</td>
<td>0.832</td>
<td>0.854</td>
</tr>
<tr>
<td>Shock to R&amp;D subsidy</td>
<td>$\rho_S$</td>
<td>0.974</td>
<td>0.974</td>
<td>0.971</td>
</tr>
<tr>
<td>Shock to foreign consumption demand</td>
<td>$\rho_{CF}$</td>
<td>0.939</td>
<td>0.939</td>
<td>0.953</td>
</tr>
<tr>
<td>Shock to foreign real interest rate</td>
<td>$\rho_{r,F}$</td>
<td>0.851</td>
<td>0.851</td>
<td>0.837</td>
</tr>
<tr>
<td>Shock to government consumption</td>
<td>$\rho_G$</td>
<td>0.972</td>
<td>0.972</td>
<td>0.951</td>
</tr>
<tr>
<td>Shock to real interest rate, r</td>
<td>0.169</td>
<td>0.002</td>
<td>0.009</td>
<td>0.012</td>
</tr>
<tr>
<td>Shock to TFP</td>
<td>0.231</td>
<td>0.350</td>
<td>0.228</td>
<td>0.300</td>
</tr>
<tr>
<td>Shock to labour demand</td>
<td>0.031</td>
<td>0.002</td>
<td>0.015</td>
<td>0.001</td>
</tr>
<tr>
<td>Shock to capital demand</td>
<td>0.160</td>
<td>0.025</td>
<td>0.045</td>
<td>0.014</td>
</tr>
<tr>
<td>Shock to real wage</td>
<td>0.122</td>
<td>0.020</td>
<td>0.162</td>
<td>0.006</td>
</tr>
<tr>
<td>Shock to export demand</td>
<td>0.034</td>
<td>0.010</td>
<td>0.103</td>
<td>0.145</td>
</tr>
<tr>
<td>Shock to import demand</td>
<td>0.028</td>
<td>0.001</td>
<td>0.014</td>
<td>0.013</td>
</tr>
<tr>
<td>R&amp;D subsidy shock</td>
<td>0.142</td>
<td>0.589</td>
<td>0.406</td>
<td>0.470</td>
</tr>
<tr>
<td>Foreign consumption shock</td>
<td>0.005</td>
<td>0.002</td>
<td>0.016</td>
<td>0.036</td>
</tr>
<tr>
<td>$r^F$ shock</td>
<td>0.071</td>
<td>0.000</td>
<td>0.001</td>
<td>0.004</td>
</tr>
<tr>
<td>Gov consumption shock</td>
<td>0.005</td>
<td>0.000</td>
<td>0.002</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**TABLE 4**

Variance Decomposition for key endogenous variables based on estimated structural parameters, Subsidy Model 1. NFA is Net Foreign Assets. Q is the inverse of the real exchange rate.
4.6. Power Exercise

The small sample properties of Indirect Inference have been investigated elsewhere (see Le et al. 2016 for references). However, since Monte Carlo results can be difficult to generalise from one context to another, we check the power of the Indirect Inference test for our particular setup. To do this, we introduce falseness into the structural parameters, $\theta$, moving them away from predefined true values by a certain percentage (in either a positive or negative direction). Using a bootstrapped Wald distribution based on the misspecified model, we see whether the Indirect Inference test as implemented above will correctly reject this model given a sample from the true (correctly specified) model. Finding the rate at which the test statistic falls in the 95th-100th percentile range of the distribution, for a particular degree of falseness, we get a sense of how reliable the procedure is. Rejection rates are given in Table 5. The results of the exercise indicate that the testing method applied in the study is powerful. Coefficients just 1.5% away from their true values will result in a certain rejection.19

4.7. Policy Reform and Growth Episode

A temporary shock to the detrended R&D subsidy has the effect in the model of increasing the level of TFP permanently and also generates a long-lasting TFP growth episode, with knock-on effects on the rest of the economy. Impulse responses to a one-off, 1 percentage point increase in $s_0$ are shown in Figure 3. The simulation is based on the estimated structural parameter set found above. After 70 quarters the loglevel of output is 2 percentage points higher than its no shock state (note, balanced growth has been removed here). The average annual growth increase over the 17.5 year episode is therefore 0.11 percentage points per annum.

How confident can we be in these results? The power exercise above demonstrates that the Indirect Inference test is robust against misspecification in the model’s structural parameters. There is the further issue of identification. Work checking the identification of rational expectations DSGE models finds that they generally are overidentified (the notable exception is models featuring sunspots); see Le et al. 2017. It is a priori likely that the model we use here is identified since models of this type routinely pass identification tests, but in particular we would like to show that the reduced form of this model could not be confused with a model in which R&D subsidies respond endogenously to TFP. To check identification we apply the numerical identification test developed by Le et al (2017) to this model.20 This is work currently in progress.

4.7.1. Robustness checks and sensitivity to assumptions

Robustness checks showed the results for Subsidy Model 1 to be invariant to the interpolation technique (quadratic versus constant) and to the way in which missing values were

---

19 The above power function holds when all the parameters are falsified together to the same degree. We would most of all like to know here whether the addition here of the R&D subsidy is appropriate. This policy affects the model via parameters $b_1$ and $c_1$. We therefore conduct a power test when these coefficients alone are misspecified, holding all other coefficients to their true values. This is work in progress.

20 The idea is to check whether any other structural model could generate the model’s reduced form by creating a large data number of data samples of large size from the model, and testing whether any alternative possible model is rejected by these at the same 5% confidence level as the true model itself.
FIG. 3 Impulse Responses for a 1 pc point increase in R&D subsidies; 70 quarters.

supplied for years 1982 and 1984. Further robustness checks in progress are whether the test’s conclusions for Model 1 are unaffected when the sample is cut in 2008 to exclude the full financial crisis period.

Another point of interest is whether the detrending process for the R&D subsidy variable affects the conclusions. Though direct subsidies have been falling steadily since 1980s, we see from the late 1990s that the trend slows (Appendix F). We therefore investigate what happens when a nonlinear deterministic time trend is removed from the subsidy variable. This work is currently underway.

5. CONCLUSION

In this paper, a DSGE model of the UK has been tested and estimated using Indirect Inference when productivity is driven systematically by direct subsidies to private sector R&D, for the period 1981 – 2010.

After Indirect Inference estimation, the model is comfortably not rejected by the Indirect Inference Wald test based on an auxiliary vector error correction model. Our test focuses on whether the model can explain output and productivity as endogenous variables.

The estimated impact of current direct subsidies to private R&D on total factor productivity growth one-quarter ahead is 0.09, signifying that in this sample a 1 percentage point increase in the detrended ratio of government funded BERD to total BERD raises productivity by 0.09 percent over the quarter, with permanent effects on the level. Given the estimated structural model, we conduct a simulated policy reform experiment. A one-off one percentage point increase in direct subsidies dying out gradually generates a transitional growth episode in TFP lasting nearly two decades. This translates into an increase in the average annual growth rate of output of 0.2 percentage points over those decades.

The power exercise we conduct for the method in Section 4.6 lends significant robustness to these conclusions about the role of R&D subsidies. Our Monte Carlo study finds that the introduction of 1.5% misspecification in the model’s structural parameters leads to a 100%

---

21 Missing values were calculated as i) the average of two contiguous values, ii) equal to previous value, iii) equal to following value. The Wald test result was similar for all three.
rejection rate. We also apply the numerical model identification test proposed in Le et al. (2016) - work in progress.

This study provides additional information on the impact of policy on transitional growth in the UK since the 1980s. Since the policy variable consists of government-funded formal R&D activity, there is little doubt that this policy works on growth through the channel of formal R&D undertaken by firms. Taken together with the results from L.Minford and Meenagh (2017), the conclusion is that government policy has had an impact on the UK productivity experience, both through direct subsidies to R&D, and more indirectly by reforming the policy environment in which firms (both new and established) make decisions.

Finally, the study is a first step in a wider research agenda on the role of R&D policy in UK growth. This model has abstracted heavily from the processes surrounding the R&D investment decision and the way that direct subsidies enter it in practice. A different and more elaborate model of the R&D channel could give more insight into exactly how direct subsidies are working to drive TFP at the level of microfoundations, but this must be for future research. In this study we provide evidence of the positive direction of the subsidy impact and the extent of that effect on the macroeconomy, findings which can inform future work.

REFERENCES

APPENDIX A: MODEL DERIVATIONS, CONT.

A.1. First order condition for \( z(t) \)

The first order condition for \( z_t \) is:

\[
\frac{dL}{dz_t} = 0 = -\beta^{t} \lambda_t w_t + \beta^{t} \lambda_t s_t + E_t \sum_{i=1}^{\infty} \beta^{t+i} \lambda_{t+i} \frac{d z_{t+i}}{dz_t}
\]  

(44)

At the \((N_t, z_t)\) margin, the optimal choice of \( z_t \) trades off the impacts of a small increase \( dz_t \) on labour earnings (lower in period \( t \) due to reduced employment time), subsidy payments (higher at \( t \)
in proportion to the increase in \( z_t \), and expected dividend income. With substitution from 30, the first order condition can be rearranged as follows:

\[
\beta^t \gamma_t C_t^{\rho_t} w_t = \frac{a_1}{a_0 + a_1 z_t + u_t} E_t \sum_{i=1}^{\infty} \beta^{t+i} \gamma_{t+i} C_{t+i}^{\rho_i} Y_{t+i} + \beta^t \lambda_t s_t
\]  

(46)

On the left hand side is the return on the marginal unit of \( N_t \), the real consumer wage; on the right is the present discounted value of the expected increase in the dividend stream as a result of a marginal increase in \( z_t \), plus time \( t \) subsidy incentives attached to R&D activity. Substituting again from 30 for \( z_t \) yields

\[
\frac{A_{t+1}}{A_t} = a_1 \frac{E_t \sum_{i=1}^{\infty} \beta^{t+i} \gamma_{t+i} C_{t+i}^{\rho_i} Y_{t+i}}{\gamma_t C_t^{\rho_t} (w_t - s_t)}
\]  

(47)

Modeling the preference shock to consumption, \( \gamma_t \), as an AR(1) stationary process such that \( \gamma_t = \rho_\gamma \gamma_{t-1} + \eta_{t-\gamma} \), setting \( \rho_\gamma \approx 1 \), we approximate \( \frac{\gamma}{Y} \) as a random walk, so \( E_t \frac{\gamma_{t+i}}{C_{t+i}} = \frac{\gamma}{C_t} \) for all \( i > 0 \). The expression becomes

\[
\frac{A_{t+1}}{A_t} = a_1 \frac{\frac{\gamma}{C_t}}{\frac{w_t}{C_t}(1 - s'_t)}
\]  

(48)

where \( \frac{\gamma}{z} \equiv s'_t \). A first order Taylor expansion of the right hand side of equation 31 around a point where \( s' = s' \) gives a linear relationship between \( \frac{A_{t+1}}{A_t} \) and \( s'_t \) of the form

\[
d \ln A_{t+1} = b_0 + b_1 s'_t + \varepsilon_{A,t}
\]  

(49)

where \( b_1 = a_1 \frac{\frac{\gamma}{C_t}}{\frac{w_t}{C_t}(1 - s'_t)} \). Other terms in the expansion are treated as part of the error term.

A.2. Deriving the labour supply response to subsidies

Taking the total derivative of the time endowment in 3 gives \( dx_t = -dN_t - dz_t \), and hence

\[
\frac{dx_t}{x_t} = d \ln x_t \approx -d \ln N_t - \frac{dz_t}{N} = -d \ln N_t - 2dz_t
\]  

(50a)

\[
\frac{dA_{t+i}}{A_{t+i-1}} = \frac{A_{t+i}}{A_{t+i-1}} a_1. \quad \text{Hence for } i \geq 1,
\]

\[
\frac{d}{dz_t} \frac{A_{t+i}}{A_{t+i-1}} = \frac{d}{dz_t} \frac{A_{t+i}}{dA_{t+i-1}} \frac{dA_{t+i-1}}{A_{t+i-2}} \ldots \frac{dA_{t+2}}{dA_{t+1}} \frac{dA_{t+1}}{dz_t} = \frac{A_{t+i}}{A_{t+i-1}} a_1
\]  

(45)

so \( \frac{dA_{t+i}}{dz_t} = \frac{A_{t+i}}{A_{t+i-1}} A_{t+i+1} A_{t+i} a_1 \). It may be objected that \( dz_t \) will enhance output directly through its effect on productivity (holding inputs fixed), and will also induce the firm to hire more capital in order to exploit its higher marginal product (similarly for labour). I assume that the effect of \( dz_t \) on the future dividend \( dz_{t+1} = \pi_{t+1} \) is simply its direct effect through higher TFP, on the basis that any effects on the firm’s input demands are second order and can be ignored. Therefore the expected change in the dividend stream is based on forecasts for choice variables (see on other first order conditions) that are assumed independent of the agent’s own activities in context of price forecasts; she anticipates only the effect of \( z_t \) on the level of output that can be produced with given inputs from \( t + 1 \) onwards.

23 The non-policy cost of generating new productivity via \( z_t \) is assumed to be zero. The model abstracts from a fixed or sunk cost of innovating. Moreover, time in \( z_t \) leads in a certain fashion to higher productivity, except in so far as the relationship is subject to a random shock.

24 Although in balanced growth \( C_t \) is constant, in the presence of shocks the ratio will move in an unpredictable way (see Meenagh et al. 2007 for discussion). At any given point in the sample, the model is not in balanced growth, though it tends to it in the future if no further shocks are expected.
Substituting into the loglinearised intratemporal condition for \( \ln w_t \) from 26 and using 50a, we obtain
\[
\frac{1}{\rho_2} \left[ k + d \ln \tilde{w}_t - \frac{1}{\rho} \left[ \frac{1 \cdot \omega}{\omega} \right] \sigma \ln \xi_t - \left[ \frac{1 \cdot \omega}{\omega} \right] \sigma \ln Q_t \right] = \ln N_t - 2c_1ds_t + \ln C_t + \frac{1}{\rho_2} d \ln \xi_t + \frac{1}{\rho_2} d \ln \gamma_t - \frac{\rho_1}{\rho_2} d \ln C_t + \frac{1}{\rho_2} d \ln \xi_t
\]  
(50b)

Integrating this and rearranging for the log of the real unit cost of labour to the firm, \( \ln \tilde{w}_t \), gives
\[
\ln \tilde{w}_t = \text{const} + \rho_2 \ln N_t + \rho_1 \ln C_t + \frac{1}{\rho} \left[ \frac{1 \cdot \omega}{\omega} \right] \sigma \ln Q_t - \rho_2 c_1s_t + \epsilon_{w,t}
\]  
(51)

where
\[
\epsilon_{w,t} = -\ln \gamma_t + \ln \xi_t + \frac{1}{\rho} \left[ \frac{1 \cdot \omega}{\omega} \right] \sigma \ln \xi_t
\]  
(52)

Substituting into equation (31) from (30) and rearranging for \( z_t \), then taking the derivative with respect to \( s_t \), we find \( c_1 = \frac{\partial z_t}{\partial s_t} \); we could potentially calibrate \( c_1 \) from this, taking appropriate values for righthand side variables. However there is flexibility around what values might be appropriate in practice.

**APPENDIX B: THE LINEARISED SYSTEM**

The linearised system of optimality conditions and constraints solved numerically is given below. Each equation is normalised on one of the endogenous variables (constants are suppressed in the errors). Variables are in natural logs except where already expressed in percentages. For clarity, \( \ln(C_t^*) \) and \( \ln C_t^f \) are denoted \( \ln EX_t \) and \( \ln IM_t \),

\[
\begin{align*}
r_t &= \rho_1 (E_t \ln C_{t+1} - \ln C_t) + e_{r,t} & \quad \text{(53)} \\
\ln Y_t &= \alpha \ln N_t + (1 - \alpha) \ln K_t + \ln A_t & \quad \text{(54)} \\
\ln N_t &= \ln Y_t - \tilde{w}_t + e_{n,t} & \quad \text{(55)} \\
\ln K_t &= \zeta_1 \ln K_{t-1} + \zeta_2 \ln K_{t-1} + \zeta_3 \ln Y_t - \zeta_4 r_t + e_{k,t} & \quad \text{(56)} \\
\ln C_t &= \frac{\bar{Y}}{C} \ln Y_t - \frac{EX}{C} \ln EX_t + \frac{IM}{C} \ln IM_t = \frac{\bar{K}}{C} \ln K_t + (1 - \delta - \gamma_k) \frac{\bar{K}}{C} \ln K_{t-1} - \frac{\bar{C}}{C} \ln G_t & \quad \text{(57)} \\
\ln \tilde{w}_t &= \rho_2 \ln N_t + \rho_1 \ln C_t + \frac{1}{\rho} \left[ \frac{1 \cdot \omega}{\omega} \right] \sigma \ln Q_t + \rho_2 c_1s_t + e_{w,h,t} & \quad \text{(58)} \\
\ln w_t &= \ln \tilde{w}_t - \frac{1}{\rho} \left[ \frac{1 \cdot \omega}{\omega} \right] \sigma \ln Q_t + \ln C_t^* + \epsilon_{w,t} & \quad \text{(59)} \\
\ln EX_t &= \ln C_t^* + \sigma F \frac{1}{\omega} \ln Q_t + e_{X,t} & \quad \text{(60)} \\
\ln IM_t &= \ln C_t - \sigma \ln Q_t + e_{M,t} & \quad \text{(61)} \\
\ln Q_t &= E_t \ln Q_{t+1} + r_t - r_t & \quad \text{(62)} \\
\Delta h_t^{f+1} &= \frac{\hat{h}_t^f}{1 + g} + \frac{\tilde{h}_t^f}{1 + g} + \left( \frac{1}{1 + g} \right) \left( \frac{EX}{\bar{Y}} \ln EX_t - \frac{EX}{\bar{Y}} \frac{1}{\ln \bar{IM}_t} \ln IM_t \right) & \quad \text{(63)} \\
\ln A_t &= \ln A_t - b_3 s_{t-1} + e_{A,t} & \quad \text{(64)} \\
\ln C_t^* &= \rho_{C^*} \ln C_{t-1} + \eta_{C^*} & \quad \text{(65)} \\
\ln G_t &= \rho_{G} \ln G_{t-1} + \eta_{G,t} & \quad \text{(66)} \\
r_t^f &= \rho_{rf} r_{t-1}^f + \eta_{rf,t} & \quad \text{(67)} \\
s_t^f &= \rho s_{t-1} + \eta_{s,t} & \quad \text{(68)}
\end{align*}
\]

Three of these equations hold as identities (market clearing, real uncovered interest parity and the balance of payments), and the consumer wage shock is also set to zero (it has common elements with
the shock to $\bar{w}_t$, see equation 26). The last four equations describe exogenous variables. Shocks $\epsilon_{i,t}$ are ARIMA(1,0,0) processes. $\eta_{it}$ are stationary zero-mean residuals.

APPENDIX C: DATA DEFINITIONS AND SOURCES

Most UK data are sourced from the UK Office of National Statistics (ONS); others from International Monetary Fund (IMF), Bank of England (BoE), UK Revenue and Customs (HMRC) and Organisation for Economic Cooperation and Development (OECD). Labour Market Indicators are taken from the Fraser Institute Economic Freedom Project, which sources them from the World Economic Forum’s Global Competitiveness Report (GCR) and the World Bank (WB). All data seasonally adjusted and in constant prices unless specified otherwise.

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\footnote{25}{Where equations are not straightforwardly linear in logs, they are linearised around sample mean values, denoted by overbar. The balance of payments constraint is scaled by output and its linearisation therefore includes the parameter $g$, the assumed balanced growth rate of output. An additional assumption applied in the linearisation of the balance of payments is that $\bar{k} = \frac{1}{\bar{w}} \frac{1}{p} \ln \bar{\xi_t} = 0$ in eq. 15, allowing the approximation: $\ln p_t^d - \ln Q_t = -\left(\frac{1-w}{\bar{w}} + 1\right) \ln Q_t = -\frac{1}{2} \ln Q_t.$}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Variable</th>
<th>Definition and Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Y)</td>
<td>Output</td>
<td>Gross Domestic Product; constant prices.</td>
<td>ONS</td>
</tr>
<tr>
<td>(N)</td>
<td>Labour</td>
<td>Ratio of total employment to 16+ working population(^1)</td>
<td>ONS</td>
</tr>
<tr>
<td>(K)</td>
<td>Capital Stock</td>
<td>Calculated from investment data ((I)) using Eqn.23 (na)</td>
<td>(na)</td>
</tr>
<tr>
<td>(I)</td>
<td>Investment</td>
<td>Gross fixed capital formation + changes in inventories</td>
<td>ONS</td>
</tr>
<tr>
<td>(C)</td>
<td>Consumption</td>
<td>Household final consumption expenditure by households</td>
<td>ONS</td>
</tr>
<tr>
<td>(A)</td>
<td>Total Factor Productivity</td>
<td>Calculated as the Solow Residual in Eqn. 18 (na)</td>
<td>(na)</td>
</tr>
<tr>
<td>(G)</td>
<td>Government Consumption</td>
<td>General government, final consumption expenditure</td>
<td>ONS</td>
</tr>
<tr>
<td>(IM)</td>
<td>Imports (also (C^F_t))</td>
<td>UK imports of goods and services</td>
<td>ONS</td>
</tr>
<tr>
<td>(EX)</td>
<td>Exports (also (C^{dr}_t))</td>
<td>UK exports of goods and services</td>
<td>ONS</td>
</tr>
<tr>
<td>(Q)</td>
<td>Terms of Trade</td>
<td>Calculated from (\frac{E}{P^F}) (na)</td>
<td>(na)</td>
</tr>
<tr>
<td>(E)</td>
<td>Exchange Rate</td>
<td>Inverse of Sterling effective exchange rate</td>
<td>ONS</td>
</tr>
<tr>
<td>(P_F)</td>
<td>Foreign Price Level</td>
<td>Weighted av. of CPI in US (0.6), Germany (0.19) &amp; Japan (0.21)</td>
<td>IMF</td>
</tr>
<tr>
<td>(P)</td>
<td>Domestic General Price Level</td>
<td>Ratio, nominal to real consumption</td>
<td>ONS</td>
</tr>
<tr>
<td>(b_F)</td>
<td>Net Foreign Assets</td>
<td>Ratio of nominal net foreign assets (NFA) to nominal GDP (^2)</td>
<td>ONS</td>
</tr>
<tr>
<td>(w)</td>
<td>Consumer Real Wage</td>
<td>Average Earnings Index (^3) divided by (P_t)</td>
<td>ONS</td>
</tr>
<tr>
<td>(\bar{w})</td>
<td>Unit cost of labour</td>
<td>Average Earnings Index (^4) divided by GDP deflator</td>
<td>ONS</td>
</tr>
<tr>
<td>(r)</td>
<td>Real Interest Rate, Domestic</td>
<td>Nominal interest rate minus one period ahead inflation.</td>
<td>(na)</td>
</tr>
<tr>
<td>(R)</td>
<td>Nominal Interest Rate, Domestic</td>
<td>UK 3 month treasury bill yield</td>
<td>BoE</td>
</tr>
<tr>
<td>(r_F)</td>
<td>Real Interest Rate, Foreign</td>
<td>(R_F) minus one-period ahead inflation ((\text{year-on-year change in } P_F))</td>
<td>(na)</td>
</tr>
<tr>
<td>(R_F)</td>
<td>Nominal Interest Rate, Foreign</td>
<td>Weighted av., 3-month discount rates, US, Germany &amp; Japan (^4)</td>
<td>IMF</td>
</tr>
<tr>
<td>(C^F)</td>
<td>Foreign Consumption Demand</td>
<td>World exports in goods and services</td>
<td>IMF</td>
</tr>
<tr>
<td>(s)</td>
<td>R&amp;D Subsidy, Direct</td>
<td>BERD funded by government as proportion of total BERD</td>
<td>OECD</td>
</tr>
</tbody>
</table>

**TABLE 6**

Data Description
APPENDIX D: AUXILIARY MODEL

The full linearised structural model, comprising a \( p \times 1 \) vector of endogenous variables \( y_t \), a \( r \times 1 \) vector of expected future endogenous variables \( E_t y_{t+1} \), a \( q \times 1 \) vector of non-stationary variables \( x_t \) and a vector of i.i.d. errors \( e_t \), can be written in the general form

\[
A(L)y_t = BE_t y_{t+1} + C(L)x_t + D(L)e_t
\]  

(69)

\[
\Delta x_t = a(L)\Delta x_{t-1} + d + b(L)z_{t-1} + c(L)e_t
\]  

(70)

\( x_t \) is a vector of unit root processes, elements of which may have a systematic dependency on the lag of \( z_t \), itself a stationary exogenous variable (this variable is subsumed into the shock below). \( e_t \) is an i.i.d., zero mean error vector. All polynomials in the lag operator have roots outside the unit circle. Since \( y_t \) is linearly dependent on \( x_t \) it is also non-stationary. The general solution to this system is of the form

\[
y_t = G(L)y_{t-1} + H(L)x_t + f + M(L)e_t + N(L)e_t
\]  

(71)

where \( f \) is a vector of constants. Under the null hypothesis of the model, the equilibrium solution for the endogenous variables is the set of cointegrating relationships (where \( \Pi \) is \( p \times p \)):\n
\[
y_t = \Pi x_t + g
\]  

(72)

(73)

though in the short run \( y_t \) is also a function of deviations from this equilibrium (the error correction term \( \eta_t \)):

\[
y_t - (\Pi x_t + g) = \eta_t
\]  

(74)

In the long run, the level of the endogenous variables is a function of the level of the unit root variables, which are in turn functions of all past shocks.

\[
\bar{y}_t = \Pi \bar{x}_t + g
\]  

(75)

\[
\bar{x}_t = [1-a(1)]^{-1}[d + c(1)\xi_t]
\]  

(76)

\[
\xi_t = \Sigma_{s=0}^{t-1} \epsilon_{t-s}
\]  

(77)

Hence the long-run behaviour of \( x_t \) can be decomposed into a deterministic trend part \( x_t^D = [1-a(1)]^{-1}d \) and a stochastic part \( x_t^S = [1-a(1)]^{-1}c(1)\xi_t \), and the long run behaviour of the endogenous variables is dependent on both parts. Hence the endogenous variables consist of this trend and of deviations from it; one could therefore write the solution as this trend plus a VARMA in deviations from it. An alternative formulation is as a cointegrated VECM with a mixed moving average error term

\[
\Delta y_t = -[I - G(1)](y_{t-1} - \Pi x_{t-1}) + P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + \omega_t
\]  

(78)

\[
\omega_t = M(L)e_t + N(L)e_t
\]  

(79)

which can be approximated as

\[
\Delta y_t = -K[y_{t-1} - \Pi x_{t-1}] + R(L)\Delta y_{t-1} + S(L)\Delta x_t + h + \zeta_t
\]  

(80)

or equivalently, since \( \bar{y}_{t-1} - \Pi \bar{x}_{t-1} = g = 0 \),

\[
\Delta y_t = -K[(y_{t-1} - \bar{y}_{t-1}) - \Pi(x_{t-1} - \bar{x}_{t-1})] + R(L)\Delta y_{t-1} + S(L)\Delta x_t + m + \zeta_t
\]  

(81)

considering \( \zeta_t \) to be i.i.d. with zero mean. Rewriting equation 80 as a levels VARX(1) we get

\[
y_t = [I - K]y_{t-1} + K\Pi x_{t-1} + n + \phi t + \eta_t
\]  

(82)

where the error \( \eta_t \) now contains the suppressed lagged difference regressors, and the time trend is

\footnote{In fact the matrix \( \Pi \) is found when we solve for the terminal conditions on the model, which constrain the expectations to be consistent with the structural model’s long run equilibrium.}
included to pick up the deterministic trend in $\tilde{x}_t$ which affects both the endogenous and exogenous variables. $x_{t-1}$ contains unit root variables which must be present to control for the impact of past shocks on the long run path of both $x$ and $y$. This VARX(1) approximation to the reduced form of the model is the basis for the unrestricted auxiliary model used throughout the estimation.

APPENDIX E: VARIANCE DECOMPOSITION (ALL ENDOGENOUS VARIABLES)
<table>
<thead>
<tr>
<th></th>
<th>r</th>
<th>Y</th>
<th>N</th>
<th>K</th>
<th>C</th>
<th>w</th>
<th>(\tilde{w})</th>
<th>X</th>
<th>M</th>
<th>Q</th>
<th>(b^T)</th>
<th>A</th>
<th>d(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shock to (r)</td>
<td>0.16915</td>
<td>0.00197</td>
<td>0.00946</td>
<td>0.01392</td>
<td>0.04238</td>
<td>0.03864</td>
<td>0.00063</td>
<td>0.01232</td>
<td>0.06000</td>
<td>0.01156</td>
<td>0.03056</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Shock to TFP</td>
<td>0.23125</td>
<td>0.34965</td>
<td>0.22750</td>
<td>0.21159</td>
<td>0.23919</td>
<td>0.11268</td>
<td>0.35403</td>
<td>0.31997</td>
<td>0.14627</td>
<td>0.30031</td>
<td>0.01221</td>
<td>0.37125</td>
<td>0.90164</td>
</tr>
<tr>
<td>Shock to (N)</td>
<td>0.03092</td>
<td>0.00198</td>
<td>0.01529</td>
<td>0.00104</td>
<td>0.00167</td>
<td>0.02777</td>
<td>0.01385</td>
<td>0.00073</td>
<td>0.00066</td>
<td>0.00069</td>
<td>0.00074</td>
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<td>0</td>
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<tr>
<td>Shock to (K)</td>
<td>0.16036</td>
<td>0.02454</td>
<td>0.04456</td>
<td>0.41922</td>
<td>0.02423</td>
<td>0.01442</td>
<td>0.01927</td>
<td>0.01438</td>
<td>0.00731</td>
<td>0.01350</td>
<td>0.01233</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Shock to (w)</td>
<td>0.12176</td>
<td>0.01992</td>
<td>0.16159</td>
<td>0.01146</td>
<td>0.01462</td>
<td>0.03621</td>
<td>0.00125</td>
<td>0.00653</td>
<td>0.00058</td>
<td>0.00613</td>
<td>0.00049</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Shock to (X)</td>
<td>0.03448</td>
<td>0.01006</td>
<td>0.10313</td>
<td>0.00026</td>
<td>0.16463</td>
<td>0.44235</td>
<td>0.00103</td>
<td>0.16006</td>
<td>0.37763</td>
<td>0.14660</td>
<td>0.65353</td>
<td>0</td>
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<tr>
<td>Shock to (M)</td>
<td>0.02802</td>
<td>0.00123</td>
<td>0.01439</td>
<td>0.00062</td>
<td>0.02004</td>
<td>0.03956</td>
<td>0.00022</td>
<td>0.01404</td>
<td>0.09832</td>
<td>0.01318</td>
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<tr>
<td>Subsidy shock</td>
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<td>0.58885</td>
<td>0.40561</td>
<td>0.33892</td>
<td>0.41954</td>
<td>0.15387</td>
<td>0.60936</td>
<td>0.50078</td>
<td>0.17308</td>
<td>0.47001</td>
<td>0.09621</td>
<td>0.62875</td>
<td>0.09836</td>
</tr>
<tr>
<td>(C^r) shock</td>
<td>0.00527</td>
<td>0.00159</td>
<td>0.01643</td>
<td>0.00066</td>
<td>0.06412</td>
<td>0.11844</td>
<td>0.00017</td>
<td>0.02681</td>
<td>0.12097</td>
<td>0.03590</td>
<td>0.13144</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(r^f) shock</td>
<td>0.07095</td>
<td>0.00009</td>
<td>0.00053</td>
<td>0.00285</td>
<td>0.00928</td>
<td>0.01594</td>
<td>0.00016</td>
<td>0.00423</td>
<td>0.01577</td>
<td>0.00397</td>
<td>0.00392</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(G) shock</td>
<td>0.00539</td>
<td>0.00012</td>
<td>0.00150</td>
<td>0.00006</td>
<td>0.00030</td>
<td>0.00014</td>
<td>0.00002</td>
<td>0.00013</td>
<td>0.00001</td>
<td>0.00013</td>
<td>0.00031</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

TABLE 7
Variance Decomposition, Subsidy Model 1
APPENDIX F: R&D POLICY TRENDS ACROSS OECD COUNTRIES

Three significant policy shifts have emerged since the 1980s across OECD countries. The proportion of Business Enterprise R&D (BERD) financed by government fell sharply, then levelled off from the late 1990s; there has been a strong movement towards indirect government support for R&D in the form of tax credits; and there has been interest in increased patent protection, though for the UK this is less significant.

The early 1980s saw a strong downward trend in direct government funding for private sector R&D expenditure, as supply side policy gained popularity across the OECD and many industries were privatised. In 1981 the UK level was high and far above the OECD average; the percentage subsequently plunged across the area (see Figure 4). However there has been a notable levelling out since the late 1990s. This halting of the downward trend seems to have been a response to the R&D-focused innovation literature, at least in part.

Another increasingly important aspect of the R&D policy mix is the trend towards fiscal incentives, adopted by the majority of OECD countries over the last decade. Tax credits constitute indirect government financial support; they reduce the marginal cost of R&D investment for the firm and do not require government to screen the projects that firms choose to undertake. Figure 5 shows the OECD ‘tax subsidy rate’, constructed as one minus the B-Index (Warda, 2001). The B-Index measures the income required before tax in order to break even on one unit of R&D spending (in USD); it is lower when the tax credit is higher. Thus the subsidy rate proxies the generosity of the tax schedule towards business R&D. A number lower than zero for the subsidy rate indicates that the tax code penalises R&D, while a number greater than zero indicates tax preference towards R&D.

For the UK this measure includes the large company R&D expenditure credit introduced in 2002, but it excludes the more generous SME R&D tax credit introduced in 2000, as well as accelerated depreciation schemes. The SME tax credit is also excluded from the aggregate measures of BERD funded directly by government, so this element of government policy is generally missing altogether from cross-country comparisons. Given the differences in design of tax credit schemes, which may be volume-based (as in the UK) or incremental (applying only to increases in expenditure above some threshold) and comprise various additional allowances, the comparability of the tax subsidy rate across countries may be limited. Note that the SME and the large company R&D tax credit became more generous in the UK as of 2013, when the complementary Patent Box scheme was also introduced, a credit on profits arising from patents (HM Treasury, 2010a). The patent box alone was predicted to cost £1.1bn per year in the June 2010 Budget (HM Treasury, 2010b, Table 2.4, footnote 3); these are expensive policies, at least in the short run.

For a recent survey of the empirical literature on the effectiveness of R&D tax credits, much of it looking at the firm or industry level, see Lokshin and Mohnen (2010) and HMRC (2010). The UK R&D tax credit is very costly and evidence for its effectiveness on aggregate productivity growth is of great interest. See e.g. Rassenfosse and van Pottelsberge (2009) and Hall (2013) for evidence on the policy determinants of patenting and productivity growth. However, since the UK R&D tax credit has not spanned even half the sample period analysed in this study, we do not investigate this aspect of the R&D policy mix.

Likewise, though policy surrounding patenting activity continues to receive much attention, this is beyond our scope. We discuss it here for the sake of completeness. A major OECD policy trend in response to the innovation literature is the general increase in patent protection; see Figure 6. According to the Ginarte-Park Index (Park, 2008) the UK, Belgium and the US had the highest levels of patent protection amongst this group of OECD countries in 1985. Between 1985 and 2008 the UK level of patent protection increased, but many others increased by more and overtook; however, the difference in the level between the 2008 leader (the US) and the UK is very small. Given the lack of variation in this index over the time period, we would not expect the addition of this dimension to a composite R&D policy indicator to add much information.\(^\text{27}\)

\(^{27}\)Intellectual property rights protection is classified as a framework policy in the literature (e.g. Westmore, 2013), and other framework policies sometimes looked at in the context of R&D incentives include labour market and product market regulation.
FIG. 4 % BERD Financed by Government (Source, OECD)

FIG. 5 Indirect Subsidy Rates (1-B-Index), UK and OECD. Source, OECD.
FIG. 6 Ginarte-Park Index of Patent Protection. Figure Reproduced from Westmore (2013, p. 13, Figure 6).

Source: Park (2008)