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Decentralized Adaptive Control of a Class of Hidden Leader-follower Nonlinearly Parameterized Coupled Multi-agent Systems

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***Abstract*— In this paper, decentralized adaptive control is investigated for a class of discrete-time nonlinear hidden leader-follower multi-agent systems. Different from the conventional leader-follower multi-agent system, among all the agents, there exists a hidden leader that knows the desired reference trajectory, while the followers do not know the desired reference signal or are not aware of which agent is a leader. The history information of each agent has an influence on its neighbors. The dynamics of each agent is described by the nonlinear discrete-time auto-regressive model with unknown parameters. In order to deal with the uncertainties and nonlinearity, a normalized gradient algorithm is applied to estimate the unknown parameters. Based on the certainty equivalence principle in adaptive control theory, a decentralized adaptive controller is designed using neighborhood history information with the aid of Lyapunov techniques. Under the decentralized adaptive controller, rigorous mathematical proofs are provided to show that the output of the hidden leader tracks the desired reference trajectory and the average value of absolute error between each agent's outputs and the corresponding desired signals converges to zero as time goes on. The closed-loop system eventually achieves strong synchronization in sense of mean in the presence of strong couplings. In the end, the simulation results show the validity of this scheme.**

I. Introduction

In recent years, with the development of artificial intelligence, distributed control and distributed computer networks, analysis and control design of complex systems especially multi-agent systems (MASs) have gained more interests from the researchers in the control community[1], [2], [3], [4], [5], [6], [7], [8]. In general, control strategies developed for the multi-agent system (MAS) can be divided into two architectures: centralized and decentralized. In the centralized control, a whole group of agents is controlled using a powerful station; while in the decentralized control, the controller should be designed in a decentralized or distributed manner, and global information may not be available for all agents. Moreover, each agent should implement its own control law using only locally available information.

Since MAS is very susceptible to internal uncertainties and external disturbance, to deal with these uncertainties, adaptive control is the key technique. The least squares algorithm and the gradient algorithm are two important approaches for estimating the unknown parameters in the adaptive control system due to their easy-to-use recursive nature. Hence, studies to these algorithms or related algorithms, e.g. the least squares algorithm[9], [10], the extended least squares algorithm[11], the nonlinear least squares algorithm[12], the distributed gradient algorithm[13], are still active in the research community. Reviewing the history of adaptive control theory, adaptive control of linear systems has been a main stream for several decades since the closed-loop systems with adaptive controller are usually highly nonlinear despite of the linearity of the original plant. For instance, the closed-loop stability of Åström-Wittenmark self-tuning regulator had been a long-term open problem until the work of Chen and Guo [14], [15]. Besides, up to now, adaptive control of nonlinear systems has also drawn much attention in the research community of adaptive control in the past decades. However, most results on adaptive control are still concentrating on dealing with various uncertainties in one single system, and hence, centralized control is still the main considered strategy. In fact, because of nonlinear dynamics of MAS and interactions among agents, exploring on decentralized adaptive control of MAS is of great significance and difficulties. So far, there are relatively few results found in [16], [17], [18], etc.

Among MASs, a popular pattern is the leader-follower MAS. It has been extensively used to represent systems in many practical applications such as large scale robotic systems[19], large population stochastic multi-agent systems[20], [21], formation control of wheeled mobile robots and multi-agent network[22], [23], evacuation of large crowd in emergency[24], etc. In the leader-follower MAS, the leader is usually independent of its followers, but its behavior affects its followers' behaviors. Therefore, in order to achieve the control objective of all the agents, we only need to control the leader, which transfers the control of the global system to that of each agent.

Motivated by the works mentioned above, in this paper, we studied the decentralized adaptive control of the leader-follower MAS, where agents have self-governed but limited capability of sensing, decision-making and communicating, leading to the decentralized control for the whole system. In other words, control of each agent can only depend on its own and its neighborhood history information. Due to the interactions among agents and complexity of performance indices, it brings intrinsic difficulties and challenges. Only limited results can be found in [25], [26], etc.

It is worth mentioning that in a leader-follower multi-agent system, the conventional leader agent is assumed to be known to and can be sensed by all other agents. In [22], the leader is known to all other agents and independent of other agents, and the state of the leader is available to a portion of followers; if the state of each agent is measurable, under the authors' proposed distributed state feedback controllers, the state of each agent exponentially converges to the state of a leader. In [27], the behavior of the leader is independent of the followers and the state of the leader is a known constant; each follower receives information from its neighbors and the leader; the authors design a measurement-based distributed protocol such that as time goes on, each follower's state will finally converge to the leader's state.

In this contribution, the leader-follower problem considered is different from [22], [27] in terms that the leader agent is hidden and coupled with other agents. Namely, among all the agents, there exists a mysterious leader agent and all the followers are not aware of which agent is a leader, and the leader affects its neighbors' behaviors. Only the hidden leader agent knows the desired reference trajectory, while the followers do not know the desired reference signal or who is a leader agent. Consequently, it is more challenging to achieve global synchronization via decentralized adaptive control. Under an adjacency matrix of the direct graph, the output of the hidden leader tracks the desired reference trajectory successfully, and the whole system eventually achieves strong synchronization in sense of mean in the presence of the strong couplings. Only a few ideas and results can be found in [26].

Different from [26], in this paper, each agent is assumed to be a nonlinearly parameterized system, in other words, each agent is assumed to be nonlinear not only with output dynamics but also with unknown parameters. While, in [26], each agent is assumed to be linear with unknown parameters and nonlinear with output dynamics, which is characterized by a nonlinear function. Obviously, to explore the decentralized adaptive control of nonlinearly parameterized MAS is much more difficult than to study that of linearly parameterized MAS. Due to such subtle and intrinsic fundamental difference in the dynamics of the system, the only given directly update law for the estimated parameters in [26] may not work or even fail in this paper. To cope with such challenges, a new normalized gradient algorithm is put forward to estimate the unknown nonlinearized parameters in detail.

Most systems mentioned above, e.g. It is worth noting that most existing studies on adaptive control, distributed or decentralized control, multi-agent systems focus more on continuous-time or linear plants, which may bring convenience in technical analysis with mathematical tools like Lyapunov theory. However, considering the popularity of nonlinear plants especially nonlinearly-parametrized plants and the widely-used modern approach of digital control, discrete-time nonlinearly parameterized MASs are worthy of in-depth investigation, despite that there are few researches found in the literature because of the intrinsic difficulties and challenges involved in this area. To this end, this paper tries to make a new attempt to pioneer the development of decentralized adaptive control for nonlinearly parametrized plants and focuses on the leader-follower architecture with one hidden leader, with the following contributions highlighted below:

1) The decentralized adaptive control has been studied for a class of hidden leader-follower nonlinearly parameterized coupled MASs. In this MAS, each agent is a nonlinearly parameterized system with strongly couplings with other agents through its neighbors' information. In fact, the hidden leader considered can propagate its influence to all agents indirectly through its own limited neighbors rather than directly command all other agents to follow its desired reference signal.
2) A normalized gradient algorithm is adopted by each agent to estimate the nonlinearized parameters so as to provide online identification of each local plant, based on only available local information from its neighbors, by minimizing the weighted combination of output tracking errors and parameter estimation errors, where a Taylor approximation at the latest parameter estimate is used to overcome the difficulty of the unknown true parameters involved.
3) Based on the certainty equivalence principle, the controller of the hidden leader agent is designed by using its dynamic history information and the desired reference trajectory, and the adaptive control law of each following agent is designed by using its dynamics and its own neighbors' history information. for decentralized adaptive control of nonlinearly parameterized coupled MAS with the aid of Lyapunov technique and the
4) Under the proposed decentralized adaptive control law, it can be shown that the output of the hidden leader tracks the bounded desired reference trajectory, and the average value of absolute error between each agent's outputs and the corresponding desired signals converges to zero as time goes on under some mild conditions on the nonlinear functions involved and strong connectivity of the directed graph of the MAS. At last, the whole system is shown to achieve strong synchronization in sense of mean in the presence of the strong couplings.

For simplicity, the following notations will be used throughout this paper: $\|\cdot\|$ denotes Euclidean norm of a matrix; $\|\cdot\|_p$ denotes p-norm of a matrix; $\lambda_{\max}(\cdot)$ $(\lambda_{\min}(\cdot))$ denotes maximum (minimum) eigenvalue of a matrix; $R^{m\times n}$ denotes the set of all $m\times n$ dimensional real matrices; $\mathrm{tr}(\cdot)$ denotes the trace of a square matrix.

The remainder of this paper is organized as follows. Firstly, in Section II, some relevant basic definitions and lemmas are presented with preliminary introduction to some relevant concepts of algebraic graph theory and hidden leader-follower problem as well as the model structure and technical assumptions adopted. Then, Section III describes the proposed normalized gradient algorithm and illustrate why it can be used to identify the nonlinearly-parameterized plant in detail. Consequently, in Section IV, based on the certainty equivalence principle, the decentralized adaptive controller is designed where each agent tries to make full information of its available local information. In Section V, the main theorems of this paper are presented with rigorous technical proofs , which are rather involving with certain techniques such as order estimation, inequalities, and series analysis. Moreover, to illustrate the applicability of the proposed decentralized adaptive controller, a concrete MAS is given in Section VI, and the simulation results demonstrated the consistence with our theoretical results. Finally, Section VII concludes this paper by summarizing the work done and future work to be done.

II. Problem Formulation

A. Preliminaries

Definition 2.1: [27] A square matrix $A_{m\times m}(a_{ij}\geq 0)$ is a sub-stochastic matrix if there exists at least one row i such that $\sum\limits_{j=1}^{m} a_{ij}<1$ and other rows i such that $\sum\limits_{j=1}^{m} a_{ij}=1$.

Let $a(k)$ and $b(k)$ be two discrete-time scalar or vector sequences defined for all $k\in N^+$, where N^+ is the set of all positive integers. Definitions 2.2 — 2.4 refer [28].

Definition 2.2: $b(k)$ is large order of $a(k)$, denoted by $a(k) = O(b(k))$, if there are $m_1 > 0, m_2 > 0$ and $k_0 > 0$ satisfying $\|a(k)\| \leq m_1 \max_{k \leq k'} \|b(k)\| + m_2, \forall k' > k_0$. It is clear to see that $a(k) = O(1)$ implies $a(k)$ is a bounded sequence.

Definition 2.3: $b(k)$ is small order of $a(k)$, denoted by $a(k) = o(b(k))$, if there is a discrete-time function $\alpha(k')$ satisfying $\lim_{k' \to \infty} \alpha(k') \to 0$ and $k_0 > 0$ such that $\|a(k)\| \leq \alpha(k') \max_{k \leq k'} \|b(k)\|, \forall k' > k_0$. It is easy to see that $a(k) = o(1)$ implies $a(k)$ is a sequence converging to zero.

Definition 2.4: $a(k)$ and $b(k)$ are of equivalent order, denoted by $a(k) \sim b(k)$, if $a(k) = O(b(k))$ and $a(k) = O(b(k))$. It is obvious that this equivalence relation is reflexive, symmetric and transitive, thus, symbol $\sim$ represents an equivalence relationship.

It is straightforward to verify the following properties.

$$O(a(k)) + O(b(k)) = O(a(k) + b(k)) \tag{II.1}$$

$$O(a(k))O(b(k)) = O(a(k)b(k)) \tag{II.2}$$

$$O(a(k)) + o(a(k)) = O(a(k)) \tag{II.3}$$

$$o(a(k))o(b(k)) = o(a(k)b(k)) \tag{II.4}$$

$$o(1)O(b(k)) = o(b(k)) \tag{II.5}$$

$$O(1)o(b(k)) = o(b(k)) \tag{II.6}$$

Lemma 2.1: [29] Consider the following iterative system

$$Y(k+1) = A(k)Y(k) + B(k) \tag{II.7}$$

where $\|B(k)\| = O(1)$, and $A(k) \to A$ as $k \to \infty$. Assume ρ is the spectral radius of A, i.e., $\rho = \max |\lambda(A)|$ and $\rho < 1$, then we can get the order estimation

$$Y(k+1) = O(1) \tag{II.8}$$

Lemma 2.2: If matrix A is nonnegative and irreducible, then $\rho(A) < \|A\|_\infty$, where $\rho(A)$ stands for the spectral radius of a matrix A.

B. Algebraic Graph Theory

Under an MAS study, each agent maybe coupled to other agents through its neighbors' available information. Let the communicated topology be represented by a directed graph from algebraic graph theory. A directed graph $\mathcal{G} = (\mathcal{V}, \varepsilon, \mathcal{A})$ with a set of N agents $\mathcal{V} = \{1, 2, \cdots, N\}$, and $\varepsilon = \mathcal{V} \times \mathcal{V}$ is a set of M ordered edges of the form (i, j), representing that agent j has access to the information of agent i. At the time, we call agent i is agent $j's$ neighbor. Each agent has only limited communication capability with access only to its individual neighborhood information. The set of all neighbors of agent i is denoted by $\mathcal{N}_i$. Matrix $\mathcal{A}(a_{ij} = 0, 1) \in R^{N \times N}$ is an adjacency matrix, whose entries $a_{ii} = 0$, $a_{ij} = 1$ if $(i, j) \in \varepsilon$, and $a_{ij} = 0$ if $(i, j) \notin \varepsilon$. It follows directly from the definition of element a_{ij} that $\mathcal{A}$ may be a non-symmetric matrix, and that $\mathrm{tr}(\mathcal{A}) = 0$. If agent i is agent $j's$ neighbor, we would call agent j is an indegree of agent i. The weighted in-degree matrix is defined as a diagonal matrix $D = \mathrm{diag}\{d_1, d_2, \cdots, d_N\}$ with $d_i = \sum_{j=1}^{N} a_{ij}, i = 1, \cdots, N$.

Definition 2.5: An adjacency matrix $\mathcal{A}(a_{ij} = 0, 1)$ is a strongly connected matrix if there exists a path that follows the direction of the edges of the directed graph such that agent i and agent j are connected.

C. Hidden leader-follower problem

Let us consider an MAS consisting of N dynamic agents. The control objective is to synthesize a controller of the hidden leader agent using its history output and the desired reference trajectory. And a local control input for each follower using its own and its neighborhood history information. At last, the average value of each agent's tracking error between this agent's output and its corresponding desired output converges to zero. It is worth mentioning that a desired reference trajectory $y^*(k)$ is only available to a hidden leader agent and unknown to other agents and all the followers are not aware of which agent is one leader agent.

Define the tracking error between the output $y_i(k)$ of agent i at time k and the corresponding reference trajectory $y^*(k)$ as

$$e_i(k) = y_i(k) - y^*(k) \tag{II.9}$$

For agent i, its goal is to design a local controller $u_i(k)$ at time k based on its own and its available neighborhood history information or the desired reference signal, so that the average tracking error converges to zero as $k \to \infty$.

Mathematically speaking,

$$\lim_{k \to \infty} \frac{1}{k} \sum_{k'=1}^{k} |e_i(k')| = 0 \tag{II.10}$$

Define the difference between the output $y_i(k')$ of agent i at time k' and the output $y_j(k')$ of agent j at time k' as

$$e_{ij}(k') = y_i(k') - y_j(k'), i \neq j \tag{II.11}$$

Definition 2.6: [29] If the errors $e_{ij}(k)$ satisfy

$$\frac{1}{k} \sum_{k'=1}^{k} |e_{ij}(k')| \to 0, k \to \infty, \tag{II.12}$$

then we say that this system achieves strong synchronization in sense of mean.

Definition 2.7: [29] If the errors $e_{ij}(k)$ satisfy

$$\frac{1}{k} \sum_{k'=1}^{k} e_{ij}(k') \to 0, k \to \infty, \tag{II.13}$$

then we say that this system achieves weak synchronization in sense of mean.

D. Problem Statement

To study decentralized adaptive control problem for MAS with the time-invariant parameters. Consider an MAS consisting of N agents, and dynamic model of agent i is given by

$$y_i(k+1) = f_i(\theta_i, y_i(k), \varphi_i(k)) + u_i(k) \tag{II.14}$$

where $y_i(k) \in R$ is the output at time k of agent i. The unknown time-invarying parameter $\theta_i \in R^{p_i \times 1}$ is to be identified for agent i. Agent i needs to design a local control input $u_i(k) \in R$ at time k with the available information. If agent i is neighbor of m agents, then $\varphi_i(k)$ is a vector of the outputs from m neighbor agents at time k. Here, f_i is internal structure known nonlinear function, and $f_i(\theta_i, y_i(k), \varphi_i(k))$ is first-order continuously differentiable with respect to corresponding θ_i. Denote $\Phi_i(k) \triangleq \frac{\partial f_i(\theta, y_i(k), \varphi_i(k))}{\partial \theta^T}|_{\theta=\hat{\theta}_i(k)}$.

Before analyzing decentralized adaptive control for this MAS, we make some assumptions as follows.

A1: The directed graph of the MAS is strongly connected so that the adjacent matrix $\mathcal{A}$ is irreducible.

A2: The desired reference $y^*(k)$ for the MAS is a bounded sequence and satisfies $y^*(k+1) - y^*(k) = o(1)$.

A3: Without loss of generality, it is assumed that the first agent is a hidden leader who knows the desired reference $y^*(k)$, while other agents are not aware of the desired reference or which agent is the leader.

A4: Each $\Phi_i(\cdot)$ is Lipschitz function with Lipschitz coefficient L_i.

III. Normalized Gradient Algorithm

Due to the fact that each agent is assumed to be nonlinear with unknown parameters, to study the nonlinearly parameterized MAS is very difficulty. In this section, we adopt a normalized gradient algorithm to estimate the unknown parameters.

Consider a parametric estimator criterion as follow.

$$\begin{aligned} J_i(\theta_i) =& [y_i(k+1) - f_i(\theta_i, y_i(k), \varphi_i(k)) - u_i(k)]^2 + \\ & \mu_i \left\| \theta_i - \hat{\theta}_i(k) \right\|^2 \end{aligned} \tag{III.1}$$

where $\hat{\theta}_i(k)$ denotes the estimation of θ_i at time k, μ_i is weighted factor.

Calculating the Taylor series expansion at $\hat{\theta}_i(k)$ of function $f_i(\theta_i, y_i(k), \varphi_i(k))$.

$$\begin{aligned} f_i(\theta_i, y_i(k), \varphi_i(k)) \cong & f_i(\hat{\theta}_i(k), y_i(k), \varphi_i(k)) + \\ & \Phi_i(k)[\theta_i - \hat{\theta}_i(k)] \end{aligned} \tag{III.2}$$

where $\Phi_i(k) = \frac{\partial f_i(\theta, y_i(k), \varphi_i(k))}{\partial \theta^T}|_{\theta=\hat{\theta}_i(k)} \in R^{1\times p_i}$. Putting Eq. (III.2) into Eq. (III.1), it is easy to obtain that

$$\begin{aligned} J_i(\theta_i) \cong & [y_i(k+1) - f_i(\hat{\theta}_i(k), y_i(k), \varphi_i(k)) - \\ & \Phi_i(k)(\theta_i - \hat{\theta}_i(k)) - u_i(k)]^2 + \mu_i \left\| \theta_i - \hat{\theta}_i(k) \right\|^2 \end{aligned} \tag{III.3}$$

Using the gradient algorithm

$$\nabla J_i(\theta_i) = 0 \tag{III.4}$$

that is

$$\begin{aligned} & [y_i(k+1) - f_i(\hat{\theta}_i(k), y_i(k), \varphi_i(k)) - \Phi_i(k)(\theta_i - \hat{\theta}_i(k)) \\ & - u_i(k)]\Phi_i(k) - \mu_i(\theta_i - \hat{\theta}_i(k))^T = 0 \end{aligned} \tag{III.5}$$

It is difficult to get the update law of the estimated parameters

$$\begin{aligned} & \hat{\theta}_i(k+1) = \\ & \hat{\theta}_i(k) + \frac{\Phi_i^T(k)[y_i(k+1) - f_i(\hat{\theta}_i(k), y_i(k), \varphi_i(k)) - u_i(k)]}{\mu_i + \|\Phi_i(k)\|^2} \end{aligned} \tag{III.6}$$

Remark 3.1: Punishment factor μ_i plays an important role in the algorithm. Because the linear expansion of function $f_i(\theta_i, y_i(k), \varphi_i(k))$ at the point $\hat{\theta}_i(k)$ is approximate to the nonlinear model, we can choose the appropriate punishment factor μ_i to limit the range of $\theta_i(k+1) - \hat{\theta}_i(k)$. The numerator of Eq. (III.6) is positive if we take $\mu_i > 0$, so this algorithm has no singular case.

IV. Decentralized Adaptive Controller

According to Assumption $A3$, the first agent at time k knows the reference signal $y^*(k)$, by the certainty equivalence principle to track a desired reference trajectory $y^*(k)$, we have the controller

$$u_1(k) = -f_1(\hat{\theta}_1(k), y_1(k), \varphi_1(k)) + y^*(k+1) \tag{IV.1}$$

Since other agents are not aware of either the existence of the leader or the reference trajectory, and their available neighborhood information are the only external information available for them. At present, our objective is to design the controller such that the output of each agent $i (i \neq 1)$ tightly tracks the average value of the history outputs of the corresponding agent's neighbors. Based on the certainty equivalence principle, we consider the following adaptive controller.

$$u_i(k) = -f_i(\hat{\theta}_i(k), y_i(k), \varphi_i(k)) + z_i(k), i = 2, \cdots, N \tag{IV.2}$$

where $z_i(k)$ is the average value of the outputs of the i^{th} agent's neighbors, defined as

$$z_i(k) = \frac{1}{d_i} \sum_{l \in \mathcal{N}_i} y_l(k) \tag{IV.3}$$

where $\mathcal{N}_i$ represents the set of agent i neighborhood and $d_i = \sum_{j=1}^{N} a_{ij}$ defined before is the number of agents in $\mathcal{N}_i$, that is to say, d_i is the number of agent i's neighbors.

Remark 4.1: From Eq. (IV.1) and Eq. (IV.2), we can see that the desired reference signal is only available for the hidden leader and other agents are not ware of who is a leader agent nor the desired reference signal. The hidden leader's controller is designed using its history information and the desired signal. Each follower control law is designed using its dynamics and its own neighbors' history information.

Define the error signal between the output of agent 1 at time $(k+1)$ and the corresponding reference trajectory as

$$\tilde{y}_1(k+1) = y_1(k+1) - y^*(k+1) \tag{IV.4}$$

As for other agents, define the error signal between the output of agent i at time $(k+1)$ and the average value of the output at time k of the i^{th} agent's neighbors as

$$\tilde{y}_i(k+1) = y_i(k+1) - z_i(k), i \neq 1 \tag{IV.5}$$

Substituting Eq. (II.14) and Eq. (IV.1) into Eq. (IV.4), it yields

$$\tilde{y}_1(k+1) = f_1(\theta_1, y_1(k), \varphi_1(k)) - f_1(\hat{\theta}_1(k), y_1(k), \varphi_1(k)) \tag{IV.6}$$

Putting Eq. (III.2) into Eq. (IV.6), it is easy to get that

$$\tilde{y}_1(k+1) \cong -\Phi_1(k)\tilde{\theta}_1(k) \tag{IV.7}$$

where $\tilde{\theta}_1(k) = \hat{\theta}_1(k) - \theta_1$.

The similar method to Eq. (IV.7), one has

$$\tilde{y}_i(k+1) \cong -\Phi_i(k)\tilde{\theta}_i(k), i \neq 1 \tag{IV.8}$$

where $\tilde{\theta}_i(k) = \hat{\theta}_i(k) - \theta_i$.

Define

$$Y(k) = [y_1(k), y_2(k), \cdots, y_N(k)]^T \tag{IV.9}$$

$$\tilde{Y}(k) = [\tilde{y}_1(k), \tilde{y}_2(k), \cdots, \tilde{y}_N(k)]^T \tag{IV.10}$$

$$H = [1, 0, \cdots, 0]^T \in R^{N\times 1} \tag{IV.11}$$

And denote

$$\Lambda \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{d_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{d_N} \end{bmatrix} \tag{IV.12}$$

then the product matrix

$$\Lambda\mathcal{A} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{d_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{d_N} \end{bmatrix} \begin{bmatrix} 0 & a_{12} & \cdots & a_{1N} \\ a_{21} & 0 & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \frac{1}{d_2}a_{21} & 0 & \cdots & \frac{1}{d_2}a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{d_N}a_{N1} & \frac{1}{d_N}a_{N2} & \cdots & 0 \end{bmatrix} \tag{IV.13}$$

where $\mathcal{A}$ is an adjacent matrix of MAS consisting of N agents, the dynamic system of agent i is defined in Eq. (II.14). And

$$\Lambda\mathcal{A}Y(k) = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \frac{1}{d_2}a_{21} & 0 & \cdots & \frac{1}{d_2}a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{d_N}a_{N1} & \frac{1}{d_N}a_{N2} & \cdots & 0 \end{bmatrix} \begin{bmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_N(k) \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ \frac{1}{d_2}\sum\limits_{l\in\mathcal{N}_2} y_l(k) \\ \vdots \\ \frac{1}{d_N}\sum\limits_{l\in\mathcal{N}_N} y_l(k) \end{bmatrix} \tag{IV.14}$$

From Eq. (IV.2) and Eq. (IV.14), it is obvious to obtain that

$$\begin{bmatrix} 0 \\ z_2(k) \\ \vdots \\ z_N(k) \end{bmatrix} = \Lambda\mathcal{A}Y(k) \tag{IV.15}$$

which together with Eq. (IV.4) and Eq. (IV.5), the closed-loop MAS can be written as

$$\begin{bmatrix} y_1(k+1) \\ y_2(k+1) \\ \vdots \\ y_N(k+1) \end{bmatrix} = \begin{bmatrix} 0 \\ z_2(k) \\ \vdots \\ z_N(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} y^*(k+1) + \begin{bmatrix} \tilde{y}_1(k+1) \\ \tilde{y}_2(k+1) \\ \vdots \\ \tilde{y}_N(k+1) \end{bmatrix} \tag{IV.16}$$

that is

$$Y(k+1) = \Lambda\mathcal{A}Y(k) + Hy^*(k+1) + \tilde{Y}(k+1) \tag{IV.17}$$

V. Analysis of Control Performance

Up to now, we have obtained the update law for the estimated parameters and designed the decentralized adaptive control. The control performances for the MAS are analyzed in this section.

Theorem 5.1: Under Assumptions $A1 - A4$, the closed-loop MAS consisting of N open loop systems in Eq. (II.14), parameter estimates update law in Eq. (III.6), decentralized adaptive control law defined in Eq. (IV.1) and Eq. (IV.2), the control objective given by Eq. (IV.13) is achieved.

To make mathematical analysis, the proofs of this main results are divided into two steps. In the first step, we show that $\tilde{y}_i(k) \to 0$, which implies $y_1(k) - y^*(k) \to 0$, that is, output of the hidden leader at time k tracks the reference trajectory $y^*(k)$.

In the second step, although $y_i(k) - y^*(k) \to 0$ cannot be expected, the control objective $\lim\limits_{k\to\infty} \frac{1}{k}\sum\limits_{k'=1}^{k} |e_i(k')| = 0$ is achieved.

Proof: **Step 1**: Consider a Lyapunov candidate

$$V_i(k) = \left\|\tilde{\theta}_i(k)\right\|^2 \tag{V.1}$$

The difference of Lyapunov function is

$$\begin{aligned} &\triangle V_i(k) = V_i(k) - V_i(k-1) \\ &= \left\|\tilde{\theta}_i(k)\right\|^2 - \left\|\tilde{\theta}_i(k-1)\right\|^2 \\ &= \left\|\tilde{\theta}_i(k) - \tilde{\theta}_i(k-1)\right\|^2 + 2\tilde{\theta}_i(k-1)[\tilde{\theta}_i(k) - \tilde{\theta}_i(k-1)] \end{aligned} \tag{V.2}$$

Because of $\tilde{\theta}_i(k) = \hat{\theta}_i(k) - \theta_i$, obviously,

$$\tilde{\theta}_i(k) - \tilde{\theta}_i(k-1) = \hat{\theta}_i(k) - \hat{\theta}_i(k-1) \tag{V.3}$$

Putting Eq. (V.3) into Eq. (V.2), one has

$$\begin{aligned} &\triangle V_i(k) = \\ &\left\|\hat{\theta}_i(k) - \hat{\theta}_i(k-1)\right\|^2 + 2\tilde{\theta}_i(k-1)[\hat{\theta}_i(k) - \hat{\theta}_i(k-1)] \end{aligned} \tag{V.4}$$

By the update law Eq. (III.6), the difference of the Lyapunov function can be written as

$$\begin{aligned} &\triangle V_i(k) = \\ &= \frac{\|\Phi_i(k-1)\|^2 \tilde{y}_i^2(k)}{[\mu_i + \|\Phi_i(k-1)\|^2]^2} + 2\tilde{\theta}_i^T(k-1)\frac{\Phi_i^T(k-1)\tilde{y}_i(k)}{\mu_i + \|\Phi_i(k-1)\|^2} \end{aligned} \tag{V.5}$$

Substituting Eq. (IV.7) and Eq. (IV.8) into the right side of the above equation, it is easy to get

$$\triangle V_i(k) = -\frac{\tilde{y}_i^2(k)}{\mu_i + \|\Phi_i(k-1)\|^2} \leq 0 \tag{V.6}$$

From the above equation, it is easy to see the difference of Lyapunov function is nonpositive, so Lyapunov function is bounded, which implies $\left\|\tilde{\theta}_i(k)\right\|$ is bounded. Thus, $\hat{\theta}_i(k)$ is bounded.

Taking summation on both sides of Eq. (V.6), after some simple manipulations, it is easy to see that

$$\sum_{k=1}^{\infty} \frac{\tilde{y}_i^2(k)}{\mu_i + \|\Phi_i(k-1)\|^2} \le V_i(0) \tag{V.7}$$

By one property of the positive term series, it is easy to know that Eq. (V.7) implies

$$\lim_{k\to\infty} \frac{\tilde{y}_i^2(k)}{\mu_i + \|\Phi_i(k-1)\|^2} = 0 \tag{V.8}$$

or

$$\tilde{y}_i(k) = \alpha_i(k)[\mu_i + \|\Phi_i(k-1)\|^2]^{\frac{1}{2}} \tag{V.9}$$

where $\alpha_i(k) \in L^2[0,\infty)$.

According to Assumption $A4$, the Lipschitz condition of $\Phi_i(\cdot)$, it is clear to obtain the order estimation

$$\Phi_i(k) = O(y_i(k) + \varphi_i(k)) \tag{V.10}$$

thus, it is simple to see

$$(\mu_i + \|\Phi_i(k-1)\|^2)^{\frac{1}{2}} = \mu_i + O(y_i(k) + \varphi_i(k-1)) \tag{V.11}$$

By Eq. (II.5) and Eq. (V.9), we can obtain that

$$\tilde{y}_i(k) = o(1) + o(y_i(k) + \varphi_i(k-1)) \tag{V.12}$$

Because of $o(\varphi_i(k-1)) \sim \sum_{l\in\mathcal{N}_i} o(y_l(k-1))$, Eq. (V.12) can be written as

$$\begin{bmatrix} \tilde{y}_1(k) \\ \tilde{y}_2(k) \\ \vdots \\ \tilde{y}_N(k) \end{bmatrix} \sim \begin{bmatrix} o(1) & o(a_{12}) & \cdots o(a_{1N}) \\ o(a_{21}) & o(1) & \cdots o(a_{2N}) \\ \vdots & \vdots & \ddots \vdots \\ o(a_{N1}) & o(a_{N2}) & \cdots o(1) \end{bmatrix} \begin{bmatrix} y_1(k-1) \\ y_2(k-1) \\ \vdots \\ y_N(k-1) \end{bmatrix} + \tag{V.13}$$

$$\begin{bmatrix} o(1) \\ o(1) \\ \vdots \\ o(1) \end{bmatrix} \tag{V.14}$$

$$= \begin{bmatrix} o(1) & 0 & \cdots 0 \\ 0 & o(1) & \cdots 0 \\ \vdots & \vdots & \ddots \vdots \\ 0 & 0 & \cdots o(1) \end{bmatrix} \begin{bmatrix} 1 & a_{12} & \cdots a_{1N} \\ a_{21} & 1 & \cdots a_{2N} \\ \vdots & \vdots & \ddots \vdots \\ a_{N1} & a_{N2} & \cdots 1 \end{bmatrix} \begin{bmatrix} y_1(k-1) \\ y_2(k-1) \\ \vdots \\ y_N(k-1) \end{bmatrix} + \tag{V.15}$$

$$\begin{bmatrix} o(1) \\ o(1) \\ \vdots \\ o(1) \end{bmatrix} \tag{V.16}$$

that is

$$\tilde{Y}(k) \sim \text{diag}[o(1), o(1), \cdots, o(1)](\mathcal{A} + I)Y(k-1) + [o(1), o(1), \cdots, o(1)]^T \tag{V.17}$$

Putting Eq. (V.17) into Eq. (IV.17), it is easy to have

$$Y(k+1) = \{\Lambda\mathcal{A} + \text{diag}[o(1), o(1), \cdots, o(1)](\mathcal{A} + I)\}Y(k) + [y^*(k+1) + o(1), o(1), \cdots, o(1)]^T \tag{V.18}$$

It is easy to know that

$$\{\Lambda\mathcal{A} + \text{diag}[o(1), o(1), \cdots, o(1)](\mathcal{A} + I)\}Y(k) \to \Lambda\mathcal{A}Y(k) \tag{V.19}$$

as $k \to \infty$, and according to Assumption A2,

$$(y^*(k+1) + o(1), o(1), \cdots, o(1))^T = O(1) \tag{V.20}$$

thus

$$Y(k+1) = \Lambda\mathcal{A}Y(k) + O(1) \tag{V.21}$$

From Eq. (IV.13) and Definition 2.1, it is easy to get the product matrix $\Lambda\mathcal{A}$ is a sub-stochastic matrix; according to Assumption $A1$, it is obvious to see the product matrix $\Lambda\mathcal{A}$ is irreducible. Thus, by Lemma 2.2, $\rho(\Lambda\mathcal{A}) < \|\Lambda\mathcal{A}\|_\infty = 1$, which together with Lemma 2.1, it is clear to get that

$$Y(k+1) = O(1) \tag{V.22}$$

Combining Eq. (V.17) and Eq. (V.22), it yields that

$$\tilde{Y}(k) = (o(1), o(1), \cdots, o(1))^T \tag{V.23}$$

that is

$$\begin{bmatrix} \tilde{y}_1(k) \\ \tilde{y}_2(k) \\ \vdots \\ \tilde{y}_N(k) \end{bmatrix} = \begin{bmatrix} o(1) \\ o(1) \\ \vdots \\ o(1) \end{bmatrix} \tag{V.24}$$

or

$$\begin{bmatrix} y_1(k) - y^*(k) \\ y_2(k) - z_2(k-1) \\ \vdots \\ y_n(k) - z_N(k-1) \end{bmatrix} = \begin{bmatrix} o(1) \\ o(1) \\ \vdots \\ o(1) \end{bmatrix} \tag{V.25}$$

From Eq. (V.25), it is easy to see that

$$\tilde{y}_1(k) = y_1(k) - y^*(k) \to 0 \tag{V.26}$$

and

$$\tilde{y}_i(k) = y_i(k) - z_i(k-1) \to 0, i \ne 1 \tag{V.27}$$

Remark 5.1: From Eq. (V.26) we can see the output of the hidden leader tracks the desired reference signal. From Eq. (V.27) we can find that each follower follows the average value of its own neighborhood history outputs.

Step 2: Define the error between each agent's output $y_i(k)$ and the hidden leader's output $y_1(k)$ as follows.

$$\begin{bmatrix} e_{11}(k) \\ e_{21}(k) \\ \vdots \\ e_{N1}(k) \end{bmatrix} \triangleq \begin{bmatrix} y_1(k) - y_1(k) \\ y_2(k) - y_1(k) \\ \vdots \\ y_N(k) - y_1(k) \end{bmatrix} \tag{V.28}$$

that is

$$\begin{aligned} E(k) &= Y(k) - [1, 1, \cdots, 1]^T y_1(k) \\ &= [e_{11}(k), e_{21}(k), \cdots, e_{N1}(k)]^T \end{aligned} \tag{V.29}$$

or

$$e_{11}(k) = y_1(k) - y_1(k) = 0 \tag{V.30}$$

$$e_{i1}(k) = y_i(k) - y_1(k), i \ne 1 \tag{V.31}$$

By Eq. (IV.16), we have

$$y_i(k+1) = z_i(k) + \tilde{y}_i(k+1), i \ne 1 \tag{V.32}$$

Combining Eq. (V.32) and Eq. (V.31), thus

$$e_{i1}(k+1) = z_i(k) - y_1(k+1) + \tilde{y}_i(k+1), i \neq 1 \quad \text{(V.33)}$$

which together with Eq. (IV.15) yields

$$\begin{aligned} E(k+1) =& \Lambda\mathcal{A}Y(k) - [0,1,\cdots,1]^T y_1(k+1) + \\ & \mathrm{diag}(0,1,\cdots,1)\tilde{Y}(k+1) \end{aligned} \quad \text{(V.34)}$$

After some simple calculations, one has

$$\begin{aligned} E(k+1) =& \Lambda\mathcal{A}Y(k) - [0,1,\cdots,1]^T y_1(k) + \\ & [0,1,\cdots,1]^T y_1(k) - [0,1,\cdots,1]^T y_1(k+1) + \\ & \mathrm{diag}(0,1,\cdots,1)\tilde{Y}(k+1) \\ =& \Lambda\mathcal{A}Y(k) - [0,1,\cdots,1]^T y_1(k) + \\ & [0,1,\cdots,1]^T (y_1(k) - y_1(k+1)) + \\ & \mathrm{diag}(0,1,\cdots,1)\tilde{Y}(k+1) \end{aligned} \quad \text{(V.35)}$$

Noting that

$$\begin{bmatrix} 0 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = \Lambda\mathcal{A} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \quad \text{(V.36)}$$

thus Eq. (V.35) can be written as

$$\begin{aligned} E(k+1) =& \Lambda\mathcal{A}Y(k) - \Lambda\mathcal{A}[1,1,\cdots,1]^T y_1(k) + \\ & [0,1,\cdots,1]^T (y_1(k) - y_1(k+1)) + \\ & \mathrm{diag}(0,1,\cdots,1)\tilde{Y}(k+1) \end{aligned} \quad \text{(V.37)}$$

According to Assumption $A2$, one has

$$\begin{aligned} & y_1(k) - y_1(k+1) \\ & = y_1(k) - y^*(k) - y_1(k+1) + y^*(k+1) + y^*(k) - \\ & y^*(k+1) = o(1) \end{aligned} \quad \text{(V.38)}$$

which together with Eq. (V.23) and Eq. (V.37), it is clear to get that

$$\begin{aligned} E(k+1) =& \Lambda\mathcal{A}\{Y(k) - [1,1,\cdots,1]^T y_1(k)\} + \\ & [o(1), o(1), \cdots, o(1)]^T \end{aligned} \quad \text{(V.39)}$$

By Eq. (V.29), Eq. (V.39) can be written in the following form

$$E(k+1) = \Lambda\mathcal{A}E(k) + [o(1), o(1), \cdots, o(1)]^T \quad \text{(V.40)}$$

The spectral radius ρ of the product matrix $\Lambda\mathcal{A}$ is less than 1. And we know that there exists a matrix norm $\|\cdot\|_p$ such that

$$\|E(k+1)\|_p \leq \rho \|E(k)\|_p + o(1) \quad \text{(V.41)}$$

After some manipulations, it is easy to obtain that

$$\sum_{k'=1}^{k+1} \|E(k')\|_p \leq \rho \sum_{k'=1}^{k} \|E(k')\|_p + o(k) + \|E(0)\|_p \quad \text{(V.42)}$$

Define

$$S(k) = \sum_{k'=1}^{k} \|E(k')\|_p \quad \text{(V.43)}$$

Putting Eq. (V.43) into Eq. (V.42), Eq. (V.42) can be written as

$$S(k+1) \leq \rho S(k) + o(k) + C \quad \text{(V.44)}$$

where $C = \|E(0)\|_p$. From Eq. (V.44), it is easy to see that

$$S(2) \leq \rho S(1) + \alpha(1) + C \quad \text{(V.45)}$$

$$\begin{aligned} S(3) &\leq \rho S(2) + \alpha(2) + C \\ &\leq \rho(\rho S(1) + \alpha(1) + C) + \alpha(2) + C \\ &\leq \rho^2 S(1) + (\rho\alpha(1) + \alpha(2)) + (\rho+1)C \end{aligned} \quad \text{(V.46)}$$

Using the same method to Eq. (V.46), it is difficult to get that

$$\begin{aligned} S(k) \leq& \rho^{k-1}S(1) + (\rho^{k-2}\alpha(1) + \rho^{k-3}\alpha(2) + \cdots + \\ & \alpha(k-1)) + (\rho^{k-2} + \rho^{k-1} + \cdots + 1)C \\ =& \rho^{k-1}S(1) + \sum_{k=0}^{k-2} \rho^k \alpha(k-k'-1) + \frac{(1-\rho^{k-1})C}{1-\rho} \end{aligned} \quad \text{(V.47)}$$

where $\alpha(k) \in L^2[0,\infty)$, $\alpha(k) = o(k)$ is guaranteed as $k \to \infty$. Using the Schwartz's inequality, the second term on the right-hand side of the above equation can be estimated by

$$\sum_{k'=0}^{k-2} \rho^k \alpha(k-k'-1) \leq (\sum_{k=0}^{k-2} \rho^{2k'})^{\frac{1}{2}} (\sum_{k'=0}^{k-2} \alpha^2(k-k'-1))^{\frac{1}{2}} \quad \text{(V.48)}$$

Putting Eq. (V.48) into Eq. (V.47), one has

$$\begin{aligned} S(k) \leq& \rho^{k-1}S(1) + (\sum_{k=0}^{k-2} \rho^{2k'})^{\frac{1}{2}} (\sum_{k'=0}^{k-2} \alpha^2(k-k'-1))^{\frac{1}{2}} + \\ & \frac{(1-\rho^{k-1})C}{1-\rho} \end{aligned} \quad \text{(V.49)}$$

Taking the limit on both sides of the above equation, it is easy to get that

$$\begin{aligned} \lim_{k\to\infty} S(k) \leq& o(1) + \frac{1}{(1-\rho^2)^{\frac{1}{2}}} \lim_{k\to\infty} (\sum_{k'=0}^{k-2} \alpha^2(k-k'-1))^{\frac{1}{2}} \\ & + \frac{C}{1-\rho} \end{aligned} \quad \text{(V.50)}$$

Noting that $\alpha(k) \in L^2[0,\infty)$, $\alpha(k) = o(k)$ for $k \to \infty$, thus

$$S(k) = o(1) + o(k) + \frac{C}{1-\rho} \quad \text{(V.51)}$$

in other words,

$$\frac{S(k)}{k} = o(1) \quad \text{(V.52)}$$

which together with Eq. (V.43) and Definition 2.3, it is easy to obtain that for $k' \to \infty$,

$$\frac{1}{k} \sum_{k'=1}^{k} \|E(k')\|_p \to 0 \quad \text{(V.53)}$$

According to the equivalence among norms, we have

$$\frac{1}{k} \sum_{k'=1}^{k} \|E(k')\|_2 \to 0 \quad \text{(V.54)}$$

By Eq. (V.29), it is obvious to see that for $k \to \infty$,

$$\frac{1}{k}\sum_{k'=1}^{k}|e_{i1}(k')| \to 0 \tag{V.55}$$

Combining Eq. (V.30) and Eq. (V.31), it yields

$$\begin{aligned} e_{i1}(k) &= y_i(k) - y_1(k) \\ &= y_i(k) - y^*(k) + y^*(k) - y_1(k) \\ &= y_i(k) - y^*(k) - \tilde{y}_1(k) \\ &= e_i(k) - \tilde{y}_1(k) \end{aligned} \tag{V.56}$$

which leads to

$$e_i(k) = e_{i1}(k) + \tilde{y}_1(k) \tag{V.57}$$

$$\begin{aligned} \frac{1}{k}\sum_{k'=1}^{k}|e_i(k')| &= \frac{1}{k}\sum_{k'=1}^{k}|e_{i1}(k') + \tilde{y}_1(k')| \\ &\leq \frac{1}{k}\sum_{k'=1}^{k}|e_{i1}(k')| + \frac{1}{k}\sum_{k'=1}^{k}|\tilde{y}_1(k')| \end{aligned} \tag{V.58}$$

Noting that Eq. (V.26) and Eq. (V.55), then for $k' \to \infty$,

$$\frac{1}{k}\sum_{k'=1}^{k}|e_i(k')| \to 0 \tag{V.59}$$

■

Theorem 5.2: Under the conditions of Theorem 5.1, then the closed-loop system achieves strong synchronization in sense of mean, i.e.,

$$\frac{1}{k}\sum_{k'=1}^{k}|e_{ij}(k')| \to 0 \tag{V.60}$$

Proof: From Eq. (II.11), we know that the error between the output of agent i and the output of agent j, that is

$$\begin{aligned} e_{ij}(k') &= y_i(k') - y_j(k') \\ &= y_i(k') - y^*(k) - (y_j(k') - y^*(k)) \\ &= e_i(k') - e_j(k') \end{aligned} \tag{V.61}$$

Obviously,

$$|e_{ij}(k')| \leq |e_i(k')| + |e_j(k')| \tag{V.62}$$

After some simple manipulations and by Eq. (V.59), one has

$$\begin{aligned} &\lim_{k\to\infty}\frac{1}{k}\sum_{k'=1}^{k}|e_{ij}(k')| \leq \lim_{k\to\infty}\frac{1}{k}\sum_{k'=1}^{k}(|e_i(k')| + |e_j(k')|) \\ &= \lim_{k\to\infty}\frac{1}{k}\sum_{k'=1}^{k}|e_i(k')| + \lim_{k\to\infty}\frac{1}{k}\sum_{k'=1}^{k}|e_j(k')| \\ &= 0 \end{aligned} \tag{V.63}$$

The Eq. (V.60) is completed. Thus, by Definition 2.6, this closed-loop system achieves strong synchronization in sense of mean. ■

Remark 5.2: Up to now, only the hidden leader knows the desired reference trajectory, while other agents are not aware of the desired reference trajectory and who is the leader agent. Eventually the whole system achieves strong synchronization in sense of mean.

Remark 5.3: From Definition 2.6 and 2.7, it is easy to know this system achieves weak synchronization in sense of mean, too.

VI. SIMULATION RESULTS

To illustrate the output of the hidden leader tracks the desired reference trajectory, the whole system achieves strong synchronization in sense of mean. An MAS with nonlinearly parameterized couplings is considered. It is assumed that there are five agents, among which, each agent's output affects the outputs of the corresponding neighbors. The structure of each agent is given as follows.

$$y_i(k+1) = f_i(\theta_i, \varphi_i(k)) + u_i(k) \tag{VI.1}$$

where

$$\begin{aligned} &\theta_1 = 1 \\ &\theta_2 = 2 \\ &\theta_3 = 3 \\ &\theta_4 = 4 \\ &\theta_5 = 5 \\ &f_1(k) = \theta_1 y_1(k) + \sin(\theta_1 y_4(k)) \\ &f_2(k) = y_2(k) + \theta_2 e^{-|y_3(k)|} + \cos(\theta_2 y_5(k)) \\ &f_3(k) = \theta_3 y_3(k) + \cos(\theta_3 y_1(k)) + e^{-y_4(k)} \\ &f_4(k) = \theta_4 y_4(k) + e^{-y_4(k)} + \cos(y_2(k)) \\ &f_5(k) = \theta_5 y_5(k) + \sin(y_3(k)) + \cos(y_2(k)) \end{aligned} \tag{VI.2}$$

Obviously, from the above-mentioned dynamic model, we know the adjacency matrix

$$\mathcal{A} = \begin{bmatrix} 0\,0\,0\,1\,0 \\ 0\,0\,1\,0\,1 \\ 1\,0\,0\,1\,0 \\ 0\,1\,0\,0\,0 \\ 0\,1\,1\,0\,0 \end{bmatrix} \tag{VI.3}$$

Firstly, by Definition 2.5, this multi-agent system is strongly connected, that is to say, this plant satisfies Assumption A1. Secondly, here, we take the desired reference trajectory as $y^*(1) = 21, y^*(k+1) = 20 + \frac{(-1)^{k+1}}{k}$, and thus this signal satisfies Assumption A2. Thirdly, the first agent, which is the hidden agent, knows that $y^*(1) = 21, y^*(k+1) = 20 + \frac{(-1)^{k+1}}{k}$, while other agents are not aware of $y^*(1) = 21, y^*(k+1) = 20 + \frac{(-1)^{k+1}}{k}$ or who is a leader agent, and thus, Assumption A3 holds. Lastly, it is easy to check that Assumption A4 holds.

The initial outputs are set as $[1, 1, 1, 1, 1]^T$. The initial parameter estimates are set as $[0, 0, 0, 0, 0]^T$. We estimate the parameters using the update law defined by Eq. (III.6) with $\mu_1 = 0.7, \mu_2 = 0.6, \mu_3 = 0.5, \mu_4 = 0.4, \mu_5 = 0.3$.

The parameter tracking errors of all five agents are shown in Fig. 1 - Fig. 5. As we can see, for each agent, the parameter estimation tends to converge toward the true parameter value as steps increase.

From Fig.6, we can find that the laws defined by Eq. (IV.1) and Eq. (IV.2) are bounded. As we can see, for each input control eventually is stable, that is because $y^*(k+1)$ converges a fixed value 20 and the error between parameter estimate and the corresponding true parameter value is almost surely zero.

Fig.7 shows that the first agent's output, which is the hidden leader's output, tracks the desired trajectory faster than the followers' outputs because the hidden leader tracks the reference directly. The third and fourth agents are connected with the first agent, but they do not know the first agent is the leader agent. The other followers are not connected with the first agent directly, they are connected by their own neighbors. But whether or no, the closed-loop system achieves strong synchronization in sense of mean in the presence of strong couplings.

To sum up, although the desired trajectory is only available for the hidden leader, and the followers do not know any information about the leadership of the leader agent or the desired signal, the local adaptive control law of each agent can achieve the objective through the nonlinear couplings among neighboring agents.

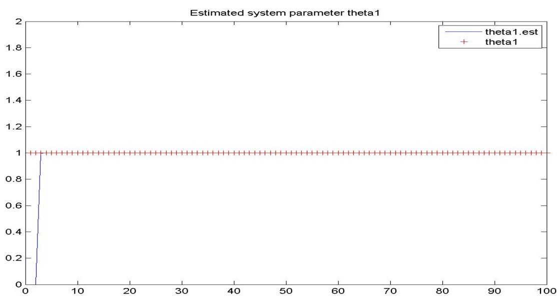

Fig. 1. Parameter θ_1's true value and its estimation

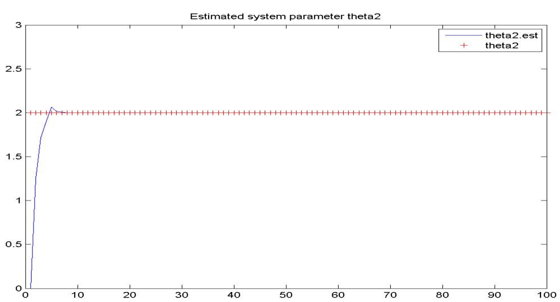

Fig. 2. Parameter θ_2's true value and its estimation

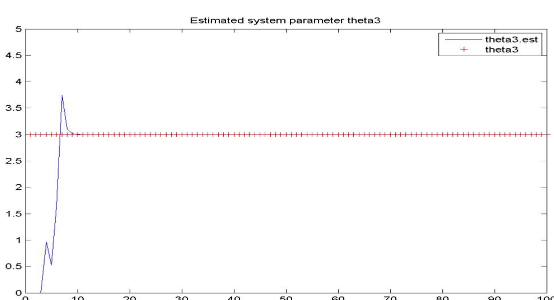

Fig. 3. Parameter θ_3's true value and its estimation

VII. Conclusions

In this paper, we have investigated the decentralized adaptive control for a class of hidden leader-follower nonlinearly parameterized coupled MASs. The dynamics of each agent is a nonlinearly parameterized system with strongly coupled terms through its neighbors' information. Among all the agents, there is a hidden leader, which knows the desired signal. The followers are not aware of which agent is a leader

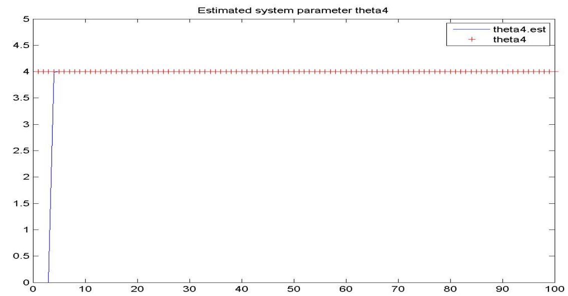

Fig. 4. Parameter θ_4's true value and its estimation

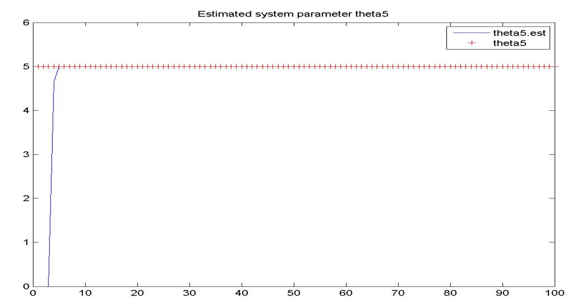

Fig. 5. Parameter θ_5's true value and its estimation

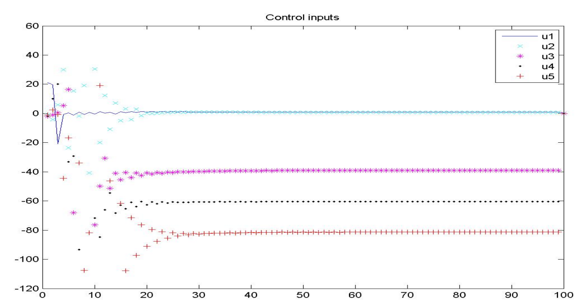

Fig. 6. Adaptive decentralized control inputs

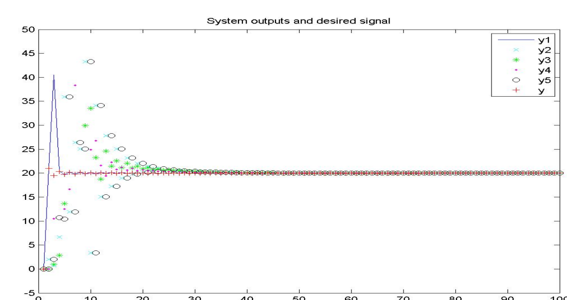

Fig. 7. Each agent's outputs and desired signals

or the desired reference signal. For each agent, a parameter update law is proposed to estimate unknown parameter using the normalized gradient algorithm. Based on the certainty equivalence principle, for the leader agent, the controller is designed by its history information and the desired reference signal. Adaptive control law of each follower is designed using its dynamics and its own neighbors' history information. Under the decentralized control, the whole system achieves strong synchronization in sense of mean in the presence of strong couplings.

Since few researchers studied the decentralized adaptive control for leader-follower nonlinearly parameterized coupled MASs with hidden leader and strong uncertain couplings, as a preliminary work towards understanding such challenging problems, we only investigated an MAS of this type without external disturbance. Many unsolved problems need to be explored in the future, and predictably, these problems are very difficult in both theoretical analysis and strict mathematical proof, hence there is a long way to go in this area, which needs in-depth applications of mathematical skills from stochastic analysis.

REFERENCES

[1] X. K. Wang, Z. W. Zeng, and Y. R. Cong. Multi-agent distributed coordination control: Developments and directions via graph viewpoint. *Neurocomputing*, 199(26):204 – 218, 2016.

[2] C. Y. Wang and Z. T. Ding. H_∞ consensus control of multi-agent systems with input delay and directed topology. *Control Theory and Applications*, 10(26):617 – 624, 2016.

[3] Q. Song, J. Cao, and W. W Yu. Second-order leader-following consensus of nonlinear multi-agent systems via pinning control. *Systems & Control Letters*, 59:553 – 562, 2010.

[4] W. Ni, X. L. Wang, and C. Xiong. Consensus controllability, observability and robust design for leader-following linear multi-agent systems. *Automatica*, 49:2199–2205, 2013.

[5] H. B. Ma, Y. N. Lv, C. G. Yang, and M. Y. Fu. Decentralized adaptive filtering for multi-agent systems with uncertain couplings. *Acta Automatica Sinica*, 1(1):94 – 105, 2014.

[6] C. L. Liu and F. Liu. Delayed-compensation algorithm for second-order leader-following consensus seeking under communication delay. *Entropy*, 17(6):3752 – 3765, 2015.

[7] Q. Y. Liu, Z. D. Wang, X. He, and D. H. Zhou. Event-based H_∞ consensus control of multi-agent systems with relative output feedback: The finite-horizon case. *IEEE Transactions on Automatic Control*, 60(9):2253 – 2258, 2015.

[8] C. G. Yang, H. B. Ma, and M. Y. Fu. Adaptive predictive control of periodic NARMA systems using nearest-neighbor compensation. *IET Control Theory and Applications*, 7:1 – 16, 2013.

[9] R. Nadakuditi and J. C. Preisig. A channel subspace post-filtering approach to adaptive least-squares estimation. *IEEE Transactions on Signal Processing*, 52(7), July 2004.

[10] C. Y. Li and J. Lam. Stabilization of discrete-time nonlinear uncertain systems by feedback based on LS algorithm. *SIAM J. Control and Optimization*, 51:1128–1151, 2013.

[11] Y. Toshio. Design of an adaptive fuzzy sliding mode control for uncertain discrete-time nonlinear systems based on noisy measurements. *International Journal of Syetems Science*, 47:617–630, 2016.

[12] C. Y. Li and M. Z. Q. Chen. Simulaneous identification and stabilization of nonlinearly parameterized discrete-time systems by nonlinear least squares algorithm. *IEEE Transaction on Automatica Control*, PP(99):1 – 13, 2015.

[13] P. Yi, Y. G. Hong, and F. Liu. Distributed gradient algorithm for constrained optimization with application to load sharing in power systems? *Systems & Control Letters*, 83:45 – 52, 2015.

[14] L. Guo and H. F. Chen. The Astrorm-Wittenmark self-tuning regulator revisited and ELS-based adaptive trackers. *IEEE Transaction on Automatic Control*, 36(7):802 – 812, 1991.

[15] L. Guo. Convergence and logarithm laws of self-tuning regulators. *Automatica*, 31:435 – 450, 1995.

[16] S. J. Liu, J. F. Zhang, and Z. P. Jiang. Decentralized adaptive output-feedback stabilization for large-scale stochastic nonlinear systems. *Automatica*, 43(2):238–251, 2007.

[17] C. Wen and J. Zhou. Decentralized adaptive stabilization in the presence of unknown backlash-like hysteresis. *Automatica*, 43(3):426–440, 2007.

[18] Q. Zhang and J. F. Zhang. Adaptive tracking-type games for coupled large population ARMAX systems. In *2010 8th IEEE International Conference on Control and Automation*, June 2010.

[19] C. Belta and V. Kumar. Trajectory design for formations of robots by kinetic energy shaping. In *IEEE International Conference on Robotics and Automation*, volume 3, pages 2593–2598, May 2002.

[20] T. Li and J. F. Zhang. Asymptotically optimal decentralized control for large population stochastic multiagent systems. *IEEE Transactions on Automatic Control*, 53(7):1643–1659, 2008.

[21] M. Nourian, P. E. Caines, R. P. Malhamé, and M. Y. Huang. Mean field lqg control in leader-follower stochastic multi-agent systems: Likelihood ratio based adaptation. *IEEE Transactions on Automatic Control*, 57(11):2801 – 2816, 2012.

[22] W. J. Dong and V. Djapic. Leader-following control of multiple non-holonomic systems over directed communication graphs. *Internation Journal of Systems Science*, 47(8):1877 – 1890, 2016.

[23] X. Y. Luo, N. N. Han, and X. P. Guan. Leader-following consensus protocols for formation control of multi-agent network. *Journal of Systems Engineering and Electronics*, 22:991 – 997, 2011.

[24] D. Helbing, I. Farkas, and T. Vicsek. Simulating dynamic features of escape panic. *Nature*, 47(6803):487–490, 2000.

[25] H. J. Chu, L. X. Gao, and W. D. Zhang. Distributed adaptive containment control of heterogeneous linear multi-agent systems: an output regulation approach. *IET Control Theory and Applications*, 10:95 – 102, 2016.

[26] S. Z. Sam, C. G. Yang, Y. N. Li, and T. H. Lee. Decentralized adaptive control of a class of discrete-time multi-agent systems for hidden leader following problem. In *International Conference on Intelligent Robots and Systems*, pages 5065 – 5070. IEEE, October 2009.

[27] C. Q. Ma, T. Li, and J. F. Zhang. Further results on limitations to the capability of feedback. *Journal of Systems Science and Complexity*, 23:35 – 49, 2010.

[28] L. J. Chen and K. S. Narendra. Nonlinear adaptive control using neural networks and multiple models. *Automatica*, 37:1245 – 1255, 2001.

[29] H. B. Ma. Decentralized adaptive synchronization of a stochastic discrete-time multi-agent dynamic model. *SIAM Journal of Control and Optimization*, 48(2):859 – 880, 2009.