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Stability of direct band gap under mechanical strains for monolayer MoS$_2$, MoSe$_2$, WS$_2$ and WSe$_2$

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Abstract

Single layer transition-metal dichalcogenides materials (MoS$_2$, MoSe$_2$, WS$_2$ and WSe$_2$) are investigated using the first-principles method with the emphasis on their responses to mechanical strains. All these materials display the direct band gap under a certain range of strains from compressive to tensile (stable range). We have found that this stable range is different for these materials. Through studying on their mechanical properties again using the first-principles approach, it is unveiled that this stable strain range is determined by the Young’s modulus. More analysis on strains induced electronic band gap properties have also been conducted.

Keywords: direct band gap; 2D materials; elastic properties; first principles.

1 Introduction

The transition-metal dichalcogenides (TMDs), such as MoS$_2$, MoSe$_2$, WS$_2$ and WSe$_2$ have attracted much attention owing to their superior electronic, optical, mechanical and catalytic properties [1-8]. It has been found that the monolayer TX$_2$ type ($T$, transition-metal atom; $X$, chalcogen atom) have several distinctive electronic and optical properties including a direct band gap (the band gap is in the range of 1.1 to 2.0 eV) at the $K$ point in the Brillouin zone [9-11], strong photoluminescence effects[12-15] and the possibility of full optical control of the valley and spin occupation[16-19]. These properties will significantly enhance the potential of using them in various applications such as pressure sensors[20], fast photodetection[21], and nanoelectromechanical systems (NEMS) devices[22].
Due to their important semiconducting properties, the band structures were extensively investigated in the past. A vast number of first-principles calculations\cite{11, 23-27} and experiments\cite{28} have been reported that under large mechanical strains, the band gap of the monolayer $TX_2$ changes from direct type to indirect type, more specifically the band gap has been narrowed, eventually leading to the CBM (conduction band minimum) plunge to below the Fermi level, which implicates that the material becomes exhibiting metal properties. Applying mechanical strains is one of the most promising ways to tune the band gap of the monolayer $TX_2$. However, a few theoretical groups reported the relationship between the Young’s modulus and the deformation region of direct band gap\cite{26, 29}. The study on the directional dependence of the mechanical strain has not been conducted so far\cite{11, 26}. Hence the critical issue is to conduct a thorough investigation to unveil the relation between their mechanical properties and electronic properties.

In this work, we simulate the Young’s modulus of monolayer $TX_2$ cells and investigate the band structures of hexagonal and orthorhombic monolayer $TX_2$ cells under a wide range of strain amplitudes in different directions using first-principles methods. This theoretical method has been used in many previous studies in TMDs\cite{23-27}. Our aim is to theoretically explore the relationship between the Young’s modulus and the width of the direct band gap region. Furthermore, the directional properties of strains acting on the orthorhombic monolayer $TX_2$ cells will also be explored. The effect of Young’s modulus and various directions of strains on the monolayer $TX_2$ will be detailed in the following sections of this paper. The phonon spectral properties of these four materials have been studied in reference [11], and our focus was not on studying this property. We also neglect other properties which could be extracted from DFT, as it was simulated, for examples, in references [30, 31] for similar zigzag sheet monolayers with high Young’s modulus and strong mechanical anisotropy.

2 Computational method

We start modelling the monolayer $TX_2$. By creating two types of crystal cells using the Atomistix ToolKit (ATK)\cite{32} simulation tools, the lattice constants and key bond lengths/angles calculated are listed in Table 1. Based on the lattice constant of Table 1, the initial model of monolayer $TX_2$ are established. Shown in the Figure 1, the monolayer $TX_2$ can be viewed as composed of two-dimensional (2D) $X$-$T$-$X$ sheets stacked on top of one
Table 1. Lattice parameters of monolayer $TX_2$

<table>
<thead>
<tr>
<th>Material</th>
<th>$a$(Å)</th>
<th>$d$(Å)</th>
<th>$\theta$(Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MoS$_2$</td>
<td>3.160</td>
<td>2.440</td>
<td>80.165</td>
</tr>
<tr>
<td>MoSe$_2$</td>
<td>3.288</td>
<td>2.524</td>
<td>82.477</td>
</tr>
<tr>
<td>WS$_2$</td>
<td>3.160</td>
<td>2.450</td>
<td>80.452</td>
</tr>
<tr>
<td>WSe$_2$</td>
<td>3.290</td>
<td>2.531</td>
<td>82.749</td>
</tr>
</tbody>
</table>

Figure 1. Schematic graphs of the 2D monolayer $TX_2$ type cells. The $x$ and $y$ lattice vectors are shown. The hexagonal (green) and orthorhombic (red) unite cells are indicated.

The band structures of the monolayer $TX_2$ type cells are calculated by the density functional theory (DFT) using the ATK. In our DFT calculation, we use a numerical Linear Combination of Atomic Orbitals (LCAO) basis sets, where the method was described in [33]. We use the generalized gradient approximation (GGA) with the parametrization of Perdew-
Burke–Ernzerhof (PBE)[34], mesh cut-off energy of 7.5 Hartree and a 9×9×9 k-point grid is used for the relaxation calculations. A large vacuum spacing of at least 20 Å is added along the z-direction to avoid the interactions that arise while calculating the periodic boundary conditions. In order to optimize the geometries, we have utilized the Limited-memory Broyden Fletcher Goldfarb Shanno (LBFGS) algorithm[35] with the maximum stress tolerance value of 0.01 eV/Å³. The structure is fully relaxed until the force on each atom becomes smaller than 0.01 eV/Å. Show in the Figure 2, for the hexagonal cell, in order to maintain the crystal symmetry, the biaxial symmetrical strains ε have to be applied in both the x/y plane. However, for the orthorhombic cell, we can apply biaxial asymmetric strains along the armchair (ε_a) and zigzag (ε_z) directions within the x/y plane. The deformation is simulated by setting the lattice parameter to a fixed larger value and relaxing the atomic positions. The amplitude of deformation is defined as: ε = (a − a₀)/a₀, where a₀ and a are the lattice parameters of the unstrained and strained cells, respectively.

![Figure 2](image)

Figure 2. The applied strains directions are indicated by white arrows. (a) Symmetrical biaxial strains on the hexagonal cell. (b) Asymmetrical biaxial strains on the orthorhombic cell.

Using the DFT-GGA calculations, we show that the band gap (E_g) of the monolayer MoS₂, MoSe₂, WS₂ and WSe₂ are 1.76 eV, 1.43 eV, 1.95 eV and 1.62 eV, which are very close to recent DFT calculations[11, 13, 36, 37], but slightly smaller than that measured using complementary techniques of optical absorption, photoluminescence and photoconductivity of monolayer TX₂[11, 15]. Although the PBE exchange correlation functional in DFT underestimates the band gap, the GLLB-Sc and hybrid functional lead to better agreement with experiments. This trend is not generalized, and more often depends on the material and...
For example, our calculated PBE band gap for monolayer MoS$_2$ underestimates the experimental band gap of 1.80 eV by just 0.04 eV[9]. However, Kuc et al and Ataca et al showed that PBE$_0$ and HSE$_06$ hybrid functionals overestimate this band gap by approximately 1 eV and 0.45 eV, respectively[38, 39]. Moreover, Ataca et al also reported a higher value of the band gap by use G$_0$W$_0$ (2.78 eV) and GW$_0$ (2.50 eV)[38]. Analyzing the aforementioned data, we believe that our choice of the PBE exchange correlation functional for monolayer MoS$_2$, MoSe$_2$, WS$_2$ and WSe$_2$ is appropriate.

3 Simulation results and discussion

We calculated band structures for various amplitudes of biaxial symmetric compressive and tensile strains on the monolayer TX$_2$ hexagonal cell, as show in Figure 3 and Figure 4, respectively. The bold black lines are the band when no strain is applied. At equilibrium, the monolayer TX$_2$ are direct band gap semiconductors with the conduction band minimum (CBM) and valence band maximum (VBM) at $K$ point. Under strains (Figure 3 and Figure 4), the conduction band is decreasing and the valence band is increasing as the value of strains increase. The band structures have also changed from a direct band gap to an indirect band gap. The electronic states near the CBM and VBM contributed mainly from $d_{xz}, d_{x^2-y^2}$ and $d_{xy}$ orbitals of the $T$ atom (Mo and W) and the $p$ orbitals of the $X$ atom (S and Se)[11]. The strong coupling between $p$ orbitals of the $X$ atom and $d_{xz} + d_{yz}$ orbitals of the $T$ atom leads to a large splitting between their bonding and antibonding states, which influence the strain-induced band structures.
Figure 3. Band structures of monolayer MoS$_2$ (a), MoSe$_2$ (b), WS$_2$ (c) and WSe$_2$ (d) corresponding to 0%, 2%, 4%, 6%, 8%, 10% symmetrical biaxial compressive strains, respectively. The shift trend of the CBM and VBM are indicated by orange arrows.

Figure 4. Band structures of monolayer MoS$_2$ (a), MoSe$_2$ (b), WS$_2$ (c) and WSe$_2$ (d) corresponding to 0%, 2%, 4%, 6%, 8%, 10% symmetrical biaxial tensile strains, respectively. The shift trend of the CBM and VBM are indicated by orange arrows.

In equilibrium, the monolayer $TX_2$ are direct band gap semiconductors with the CBM and VBM at the $K$ point. Figure 5 shows the CBM evolution with the applied symmetrical biaxial strains for monolayer $TX_2$ hexagonal cells into three regimes. (1) In the central regime, monolayer $TX_2$ retains direct band gap. The deformation ranges ($\Delta \varepsilon$) in which the materials remain in the direct band gap region ($R_d$) of the CBM-strain curves are 0.04, 0.08, 0.02 and 0.06 for monolayer MoS$_2$, MoSe$_2$, WS$_2$ and WSe$_2$, respectively. It is deduced that $R_{d-WS_2}<R_{d-MoS_2}<R_{d-WSe_2}<R_{d-MoSe_2}$. (2) Exceeding gray dashed lines, the tensile and compressive strains turn monolayer $TX_2$ into indirect band gap semiconductors and the amplitude of the CBM keep decline. (3) Under red dot chain line, the compressive strains turn the CBM lower than 0 eV.
As seen from Figure 6, we depict the spatial distribution of the charge density for the monolayer MoS\(_2\), MoSe\(_2\), WS\(_2\) and WSe\(_2\) under the strain-free state. All of four TMDs are formed by covalent bonding with four electrons from external shells of metal atoms being donated to the counterpart non-metal atoms. The binding energy (\(E\)) of these four materials are therefore predominately determined by the bond length (\(L\)) between adjacent atoms. Figure 7 shows that the calculated \(X-X\) bond length of monolayer MoS\(_2\), MoSe\(_2\), WS\(_2\) and WSe\(_2\) become closer due to Poisson contraction. The \(X-X\) bond lengths become smaller by about 0.024 \(\text{Å}\), 0.025 \(\text{Å}\), 0.023 \(\text{Å}\) and 0.021 \(\text{Å}\) per +1\% biaxial strains for monolayer MoS\(_2\), MoSe\(_2\), WS\(_2\) and WSe\(_2\), respectively. Moreover, the \(X-X\) bond length nearly decreases linearly with the deformation from -10\% to +10\%. Figure 7 shows a trend of \(L_{\text{MoS}_2} < L_{\text{WS}_2} < L_{\text{MoSe}_2} < L_{\text{WSe}_2}\) from -1\% to +5\% deformation, which is the the most deformation change width of direct band gap region of the CBM-strain curve in Figure 5. From this result, we can
deduce $E_{\text{MoS}_2} > E_{\text{WS}_2} > E_{\text{MoSe}_2} > E_{\text{WSe}_2}$ between -1% and +5% deformation because the binding energy and the bond length has an inverse relationship.

Figure 6. The isosurface plot of the charge density of monolayer MoS$_2$ and MoSe$_2$ (a), WSe$_2$ and WS$_2$ (b). The isosurface value was taken as 0.05 e/Å$^3$.

Figure 7. Evolution of the $X$-$X$ bond length with respect to strains.

Young’s modulus ($Y$) of all four TMDs ($Y_{\text{MoS}_2}$, $Y_{\text{MoSe}_2}$, $Y_{\text{WS}_2}$ and $Y_{\text{WSe}_2}$) can be calculated using the first-principles methods[26, 29, 36]. Simulation results in Figure 8 shows that stress-strain curve firstly display a nonlinear characteristic, indicating Young’s modulus has a
reducing trend for the tensile strains and an increasing trend for the compressive strains, representing the material softening effect for the tensile strains and hardening effect for the compressive strains. It is interpreted that the material softening is due to the relation between the bonding force \( F \) and the atomic separation \( r \), where, \( F \propto 1/r^2 \). By stretching the material, atomic separation increases, leading to reduced \( F \), which is macroscopically reflected the reduced elastic modulus. The material softening of the monolayer MoS\(_2\) has been reported previously in\[29, 40\]. Second, Figure 8 shows that in the small strain region (from -0.012 to 0.012), the elastic modulus for these materials are calculated to be \( Y_{\text{MoS}_2}=217.6\) GPa, \( Y_{\text{MoSe}_2}=171.4\) GPa, \( Y_{\text{WS}_2}=218.8\) and \( Y_{\text{WSe}_2}=201.5\) GPa, which demonstrate a trend of \( Y_{\text{WS}_2}>Y_{\text{MoS}_2}>Y_{\text{WSe}_2}>Y_{\text{MoSe}_2}\).

![Stress-strain curve for monolayer MoS\(_2\), MoSe\(_2\), WS\(_2\) and WSe\(_2\).](image)

Figure 8. Stress-strain curve for monolayer MoS\(_2\), MoSe\(_2\), WS\(_2\) and WSe\(_2\).

Macroscopically the inter-atomic bonding force is reflected by the Young’s modulus, characteristic of material stiffness defined as stress per unit strain. It is a logical elucidation that a higher Young’s modulus leads to a higher inter-atomic bonding force, hence under an identical strain, the material that has a higher Young’s modulus will experience a greater stress. For example, the Young’s modulus of WS\(_2\) is higher than MoS\(_2\) than WSe\(_2\) than MoSe\(_2\) in Figure 8. Application of tensile strains on WS\(_2\) decreases the S-S bond length. The effect is a reduction in the W-\( d \) orbital and S-\( p \) orbital coupling, which is revealed as the
reduces band gap. Furthermore, the electronics from WS$_2$ need more energy than MoS$_2$ than WSe$_2$ than MoSe$_2$ from the increased electrical field causing the band gap change from direct to indirect because $E_{g-WS_2} > E_{g-MoS_2} > E_{g-WSe_2} > E_{g-MoSe_2}$ (strain-free state). This trend demonstrates WS$_2$ has the lowest stability and resulting in the narrowest deformation region of the CBM-strain curve in Figure 5. Results from the above analysis and data of the Young’s modulus, it is deduced that $E_{g-WS_2} > E_{g-MoS_2} > E_{g-WSe_2} > E_{g-MoSe_2}$ and $R_{d-WS_2} < R_{d-MoS_2} < R_{d-WSe_2} < R_{d-MoSe_2}$, which is exactly matching with the results from the first-principles simulations about the band gap and the direct band gap region. It can be concluded that a TX$_2$ having smaller young’s modulus will be more immune to mechanical strains, exhibiting greater electro-mechanical stability.

Figure 9 and Figure 10 show the band structures evolution with the asymmetrical biaxial compressive and the tensile strains along the armchair ($\varepsilon_a$) and zigzag ($\varepsilon_z$) directions on the monolayer TX$_2$ orthorhombic cell. We investigate the CBM of the band due to strains in directions between $\varepsilon_a$ and $\varepsilon_z$. Firstly, the simulation was conducted by setting the $\varepsilon_a$-direction strains to increase while keeping the $\varepsilon_z$ unchanged. Then we repeat the calculation when $\varepsilon_z$-direction strains increases by a small step. In this way, the strains direction between $\varepsilon_a$- and $\varepsilon_z$- that leads to the most notable change of the CBM can be obtained. We define a total strains of $\varepsilon_t = \sqrt{\varepsilon_a^2 + \varepsilon_z^2}$. It is seen from the simulation results in the Figure 9 that for the compressive strains, the reduction of the CBM under $\varepsilon_a$-direction is faster than $\varepsilon_z$-direction. The $\varepsilon_t$ threshold deformation of the CBM reaching to 0 eV are roughly 13%, 9% and 11% for monolayer MoS$_2$, MoSe$_2$ and WSe$_2$, respectively. The CBM declines more quickly under a pure $\varepsilon_a$- or $\varepsilon_z$- direction deformation than the deformations along the 45° angle between $\varepsilon_a$- and $\varepsilon_z$- direction, which displays a strong directional dependence effect. However, Figure 10 shows for the tensile strains, the reduction of the CBM under uniaxial $\varepsilon_a$- or $\varepsilon_z$- direction is smaller than biaxial symmetrical deformation both in $\varepsilon_a$- and $\varepsilon_z$- direction. From Figure 10, it can be concluded that for the tensile strains, the $\varepsilon_t$ threshold deformation for the CBM reaching to 0 eV are roughly 22.6%, 21.2% and 19.8% for monolayer MoS$_2$, MoSe$_2$ and WSe$_2$, respectively. For the tensile strains, the effect on the band structures is almost independent on the direction of the strains.
Figure 9. CBM variation of the monolayer MoS$_2$ (a), MoSe$_2$ (b) and WSe$_2$ (c) orthorhombic cell with compressive strains.

Figure 10. CBM variation of the monolayer MoS$_2$ (a), MoSe$_2$ (b) and WSe$_2$ (c) orthorhombic cell with tensile strains.

4 Conclusion

To summarize, the first-principles methods have been used to investigate the electronic and mechanical properties of four monolayer TMD materials - MoS$_2$, MoSe$_2$, WS$_2$ and WSe$_2$. Hexagonal and orthorhombic cell structures of all these materials have been built and subsequently simulated when subjecting to tensile and compressive strains. Main conclusion is deduced that the region of the direct band gap in the CBM-strain curves exhibiting a trend of $R_{d-WS2} < R_{d-MoS2} < R_{d-WSe2} < R_{d-MoSe2}$, which has been systematically underpinned by further first-principles studies on the stress-strain relation. In the Young’s modulus and band gap study, it has been analyzed that the Young’s modulus and band gap of these four materials demonstrate a pattern of $Y_{WS2} > Y_{MoS2} > Y_{WSe2} > Y_{MoSe2}$ and $E_{g-WS2} > E_{g-MoS2} > E_{g-WSe2} > E_{g-MoSe2}$. Therefore, it is reasonable to conclude that “softer” monolayer TX$_2$ have wider direct band gap regions and higher band gap, which is a desirable property for photoluminescence applications. Then, for the orthorhombic cell, the CBM declines more quickly under a pure $\varepsilon_\alpha$ or $\varepsilon_z$ direction compressive strains than the strains in between $\varepsilon_\alpha$ and $\varepsilon_z$, which displays a strong directional dependence effect. However, the strains effect on the band structures is
almost independent on the direction of the tensile strains. These conclusions pave the way for further exploitation of 2D TMDs in tunable electronic devices.

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References
The range of direct bandgap of 2D materials can be tuned by the mechanical strain. The material with a larger Young's modulus has a narrower direct bandgap range. For the orthorhombic cell, the conduction band minimum displays a strong directional dependence effect to the compressive strain.