

Phases of QCD₃ from non-SUSY Seiberg duality and brane dynamics

Adi Armoni^{1,*} and Vasilis Niarchos^{2,†}

¹*Department of Physics, College of Science Swansea University, Swansea SA2 8PP, United Kingdom*

²*Department of Mathematical Sciences and Center for Particle Theory Durham University, Durham DH1 3LE, United Kingdom*



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We consider a nonsupersymmetric USp Yang-Mills Chern-Simons gauge theory coupled to fundamental flavors. The theory is realised in type-IIB string theory via an embedding in a Hanany-Witten brane configuration which includes an orientifold and antibranes. We argue that the theory admits a magnetic Seiberg dual. Using the magnetic dual we identify dynamics in field theory and brane physics that correspond to various phases, obtaining a better understanding of three-dimensional bosonization and dynamical breaking of flavor symmetry in USp QCD₃ theory. In field theory both phases correspond to magnetic “squark” condensation. In string theory, they correspond to open string tachyon condensation and brane reconnection. We also discuss other phases where the magnetic ‘squark’ is massive. Finally, we briefly comment on the case of unitary gauge groups.

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I. INTRODUCTION AND CONCLUSIONS

Quantum chromodynamics in three spacetime dimensions (QCD₃) is an interesting variant of the more familiar QCD in four dimensions. In a sense, the three-dimensional theory is richer because three dimensions allow the addition of a topological Chern-Simons (CS) term which blocks the RG running of the Yang-Mills interaction towards strong coupling and alters the infrared (IR) dynamics.

In the past decade or so, there has been significant progress in understanding the dynamics of three-dimensional supersymmetric gauge theories including the dynamics of a large class of supersymmetric Chern-Simons theories coupled to different types of matter. Steps forward have also been achieved in the study of nonsupersymmetric Yang-Mills-Chern-Simons (YM-CS) theories like QCD₃. For example, consider a CS theory coupled to N_f Dirac fermions in the fundamental representation. A recent intriguing result is a conjectured duality between the following pairs of theories [1]

$$SU(N_c)_{K+\frac{N_f}{2}} \oplus N_f \text{ fermions} \leftrightarrow U\left(K + \frac{N_f}{2}\right)_{-N_c} \oplus N_f \text{ scalars.} \quad (1.1)$$

*a.armoni@swansea.ac.uk

†vasileios.niarchos@durham.ac.uk

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A variant of this duality postulates that

$$U(N_c)_{K+\frac{N_f}{2}, K+\frac{N_f}{2} \pm N_c} \oplus N_f \text{ fermions} \\ \leftrightarrow U\left(K + \frac{N_f}{2}\right)_{-N_c, -N_c \mp (K+\frac{N_f}{2})} \oplus N_f \text{ scalars.} \quad (1.2)$$

In these expressions, and in what follows, the notation $SU(N)_k$ quotes *bare* CS levels. These are written in terms of the shifted level K defined as $k_{\text{bare}} - \frac{N_f}{2}$. Similarly, in the notation $U(N)_{k,\ell}$ the first level refers to the bare level of the SU part and the second to the bare level of the $U(1)$ part. Since both of these dualities relate a theory with fermions to a theory with bosons they are frequently referred to as bosonization (in close analogy with the more familiar bosonization in two dimensions). The scalars on the bosonic side have quartic interactions tuned to a Wilson-Fisher fixed point. The duality holds for $\frac{1}{2}N_f \leq K$.

Versions of this duality for $SO(N_c)$ and $USp(2N_c)$ gauge groups have also been formulated [2]. In this paper, we will mostly focus on results that are very closely related to the $USp(2N_c)$ version of the duality

$$USp(2N_c)_{2K+N_f} \oplus 2N_f \text{ fermions} \\ \leftrightarrow USp(2K + N_f)_{-2N_c} \oplus 2N_f \text{ scalars.} \quad (1.3)$$

Similar to the $SU(N_c)$ case, the bosonic side requires that the scalars are at a $USp(2N_f)$ invariant Wilson-Fisher fixed point. This duality is also expected to be valid for $\frac{1}{2}N_f \leq K$.

More recently, Komargodski and Seiberg (KS) argued for a scenario [3] that describes the infrared dynamics of the

fermionic theory when $N_f > 2|K|$. According to that scenario there is a window, for $2|K| < N_f < N_*$, where the theory exhibits a phase of flavor symmetry breaking: $U(N_f) \rightarrow U(N_f/2 - K) \times U(N_f/2 + K)$ in the unitary case and $USp(2N_f) \rightarrow USp(N_f - 2K) \times USp(N_f + 2K)$ in the symplectic case. N_* is a critical number of flavors whose precise value is currently unknown. The evidence in favor of this scenario includes the matching of anomalies and consistency under several RG flows [3]. For $N_f > N_*$ it is conjectured [3,4] that the theory flows to some IR fixed point that does not exhibit flavor symmetry breaking.

A. Summary of results

1. Benefits of an ultra-violet embedding

In this paper, we provide new evidence in favor of the above scenario for QCD₃ (bosonization, symmetry breaking, CFT) by embedding the IR dynamics of QCD₃ in an ultra-violet (UV) YM-CS theory, which is a three-dimensional cousin of a four-dimensional nonsupersymmetric orientifold QCD theory [5]. The main part of the paper will focus on the case of $USp(2N_c)$ gauge group. In that case, the UV embedding involves a $USp(2N_c)$ YM-CS theory at bare level $2k$ coupled to a real scalar field in the 2-index symmetric representation (“scalar gaugino”), a Dirac fermion in the 2-index antisymmetric representation (“gaugino”), $2N_f$ scalars in the fundamental representation (“squarks”) and $2N_f$ fermions in the fundamental representation (“quarks”). We argue that there are regimes when $k \neq 0$ where the quantum effects lift the scalars and the IR behavior is dominated by standard QCD₃ physics. The level K that appears in KS [3], e.g. in Eq. (1.3), is related to k via the relation

$$K = |k + 1 - N_c| - \frac{N_f}{2}. \quad (1.4)$$

The combination $k + 1 - N_c$ arises naturally when we integrate out the antisymmetric gaugino and will appear frequently in our formulas. It is therefore convenient to introduce the integer

$$\kappa \equiv k + 1 - N_c, \quad (1.5)$$

in terms of which $K = |\kappa| - \frac{N_f}{2}$.

The great advantage of the additional degrees of freedom in the orientifold QCD theory is that they allow us to formulate a nonsupersymmetric version of Seiberg duality. For $N_f = 0$ this duality reduces to level-rank duality [6]. For general N_f , the dual “magnetic” theory is a $USp(2N_f + 2\kappa)$ gauge theory whose details will be described in detail in Sec. II. Four-dimensional versions of the duality were studied in [5]. In this paper, we will show that the finite N_f duality passes the standard checks of

consistency under RG flows, global symmetry and ’t Hooft anomaly matching. Further favorable evidence is provided by string theory. We will return to some of the salient features of the string theory embedding in a moment.

The electric-magnetic duality operates when the rank of the dual gauge group is positive, namely when $N_f + k + 1 > N_c$ (equivalently, when $N_f > -\kappa$). For values of k, N_c, N_f outside this window there is no magnetic dual and one has to analyze separately the strong coupling dynamics of the electric theory. It is unclear what the infrared physics of the theory are in this regime. In supersymmetric analogues, e.g. in three-dimensional $\mathcal{N} = 2$ SQCD theories, the supersymmetric vacuum is lifted by nonperturbative effects when the dual rank becomes negative. When the dual rank vanishes, the dual theory is a theory of free chiral multiplets.

In this paper, we will work exclusively in the regime where the dual magnetic description exists. The magnetic description will provide an illuminating perspective on the dynamics of the electric theory. In particular, we will find, under certain assumptions, that the magnetic theory leads to a rather natural universal description for all the phases of QCD₃ outlined above. We will show that when $N_f \leq 2|K|$ bosonization emerges naturally in the IR via magnetic squark condensation; a mechanism that reminds of monopole condensation and the dual Meissner effect in four-dimensional physics. We will not be able to prove conclusively the existence of magnetic squark condensation, but we will provide evidence indicating that it is a viable possibility and that it leads to a suggestive consistent synthesis of known results.

This mechanism provides a new explanation of three-dimensional bosonization. Unlike previous explanations based on deformations of supersymmetric three-dimensional Seiberg dualities and mirror symmetry (see e.g. [7] for beautiful work in these directions) in this mechanism bosonization arises dynamically in the infrared as a consequence of a nonsupersymmetric version of Seiberg duality. These effects describe the part of the phase diagram denoted as I in the diagrams of Fig. 1.

Once magnetic squark condensation is assumed, the phase of symmetry breaking for $N_f > 2|K|$ follows naturally. There are two regions in the phase diagram of orientifold QCD₃, regions II and II’ in diagram B of Fig. 1, that describe in the IR the physics of QCD₃ for $N_f > 2|K|$.

Interestingly, the magnetic description in regions II and II’ is not identical. Consider first the situation in region II’ where $-N_f < \kappa < 0$. In this case, squark condensation leads to a complete Higgsing of the gauge group leaving behind a sigma model of Goldstone bosons for the dynamically broken symmetry. The magnetic theory provides an explicit description of how the sigma model arises.

Region II of the phase diagram of orientifold QCD₃ refers to the parameter regime $N_f > \kappa > 0$. The dictionary (1.4) implies that this region describes the same IR physics as QCD₃ with $N_f > 2|K|$. Naively, it looks like a natural

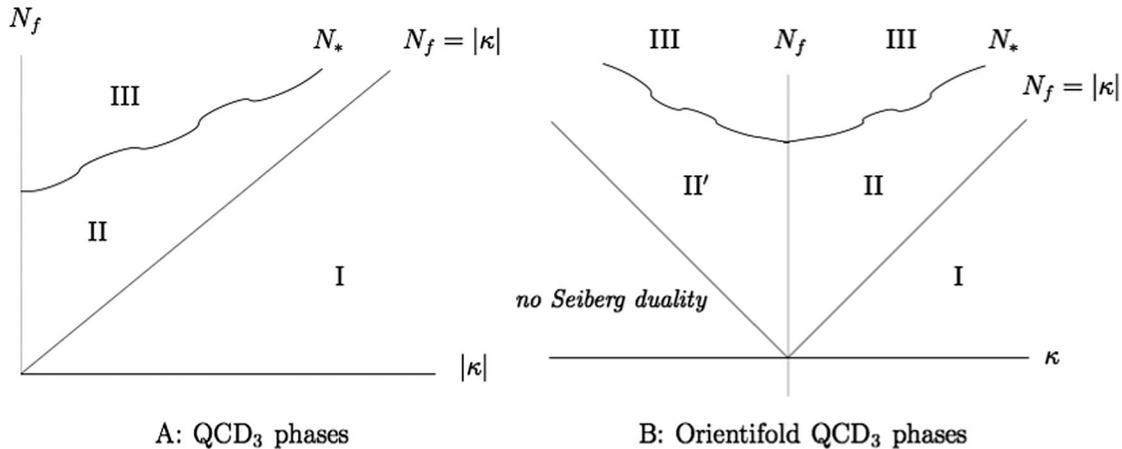


FIG. 1. Diagram A represents the phases of QCD₃ anticipated by [3]. Region I is the phase of bosonization, region II the phase of symmetry breaking and region III a phase with an IR CFT description. The curve separating the regions II and III occurs at some N_* which is potentially a function of both κ and N_c . The wiggly features of this curve are not a statement about its actual shape, but rather a symbolic depiction of our ignorance about its precise form. A constraint on the shape of this curve was determined in Ref. [3]. Diagram B represents the phases of orientifold QCD₃ that arise naturally (under a certain set of assumptions) from the magnetic description of the theory. With the exception of the region $\kappa < -N_f$ where no Seiberg duality is available the remaining regions I, II/II' and III, are expected to describe the same IR physics as the corresponding regions in diagram A.

continuation of the bosonization regime where squark condensation leads to a theory of $2N_f$ bosons. It is obvious, however, that this theory cannot be the same as the bosonic dual of region I, because bosonization is inconsistent for $N_f > 2|K|$, which is also clear in the behavior of the electric and magnetic orientifold QCD theories under mass deformations. This suggests that $N_f = \kappa$ is a critical point above which the magnetic theory reverts to the symmetry breaking phase. We propose a particular scenario for the mechanism behind this phase transition. This scenario is consistent with expectations about the massive deformations of the fermionic (electric) theory and predicts naturally a phase transition to a topological sector as the mass of the fermions is increased. A dual bosonic formulation appears at the transition point as anticipated by KS [3].

The symmetry pattern for sufficiently large N_f persists as long as the magnetic squarks are tachyonic and condense. Competing effects in the one-loop mass normalization of the squarks suggest the possibility that when N_f is large enough, $N_f \geq N_*$ for some $N_*(k, N_c)$, the squarks become massive. Depending on the mass squared of the elementary magnetic meson fields, Seiberg duality suggests a specific description for the IR theory. We argue that the most likely scenario is one where the elementary magnetic mesons are tachyonic. The condensation of the elementary magnetic mesons leads to an IR theory of free fermions and light mesons, a scenario that agrees with the large- N_f results in [4].

2. Lessons from string theory

It is useful to consider a further UV embedding of the orientifold QCD theory into a nonsupersymmetric brane

configuration in string theory. The immediate benefits of this embedding are

- String theory is a useful guide to the spectrum and tree-level Lagrangian of the magnetic dual and provides motivating evidence for a potential nonsupersymmetric Seiberg duality that is the center-point of this paper.
- Nontrivial effects in field theory have a natural description in terms of brane physics (frequently as a geometric rearrangement of the brane configuration). For example, the magnetic squark condensation that drives the UV theory to bosonization in the IR has a natural interpretation in brane physics as open string tachyon condensation and brane reconnection.¹

In this paper, we are interested in nonsupersymmetric brane configurations in a Hanany-Witten brane setup in type-IIB string theory along the lines of the Giveon-Kutasov analysis [9]. In order to break supersymmetry, we consider a combination of an $O3$ plane, anti-D5 branes and anti-D3 branes suspended between two 5-branes, one of which is a bound state of an NS5 brane and anti-D5 branes.² The mutual presence of antibranes with an orientifold plane breaks the $\mathcal{N} = 2$ supersymmetry completely. Supersymmetry is restored asymptotically when N_c , N_f and k are taken to infinity. This feature is useful.

¹A different embedding of three-dimensional bosonization in string theory was recently proposed in ref. [8] which describes via holography effects in gauge theory in the large- N limit.

²A similar four-dimensional brane configuration in tt-III string theory and a corresponding nonsupersymmetric Seiberg duality was proposed in [5].

The large- N regime is a technically convenient regime where supersymmetry is softly broken by $1/N$ effects.³

The broken supersymmetry leads to nontrivial potentials between different components of the brane configuration. On the electric side, the potentials are attractive and lead to a stable brane configuration. On the magnetic side, however, the potentials can be repulsive, leading to open string tachyon condensation and brane reconnection between the flavor and color branes. The rich phase diagram that arises in this manner is literally a web of phases driven by the presence or absence of open string instabilities. The result translates in gauge theory either as bosonization or dynamical symmetry breaking, or a CFT.

B. Comments on the scales of the setup, RG flows and nonsupersymmetric duality

In the absence of supersymmetry, there are quantum effects that generate nontrivial RG flows. The details of the RG flow and the resulting IR physics depends on the specification of the UV parameters of the theory. How are these parameters treated in the formulation of the electric/magnetic orientifold QCD theories, the statement of non-supersymmetric Seiberg duality between them and the embedding in open string theory that was described above?

In this paper, the nonsupersymmetric orientifold QCD theories are formulated at a UV cutoff scale Λ . When we discuss an embedding in open string theory we identify Λ with the natural scale in open string theory, $\frac{1}{\sqrt{\alpha'}}$. In the quantum regime, there are effects taking place at scales of the order of $\frac{1}{\sqrt{\alpha'}}$ and effects below that scale that remain finite when we take the field theory limit $\alpha' \rightarrow 0$. Our reference to open string theory is limited exclusively to effects that occur at the string scale. We use such effects as a partial guide to the ultra-violet physics of interest in the nonsupersymmetric field theories that we want to study.

In particular, the orientifold QCD theories contain scalars, fermions and gauge bosons. For scalars, other than Nambu-Goldstone bosons, the quantum effects induce either a positive or a negative (tachyonic) mass,² which behaves as a positive power of the cutoff. In that case, we assume that a corresponding effect is present in open string theory, and do not remove it by a corresponding counter-term in field theory (see [10] for a related discussion based on an earlier calculation in string theory in [11] and Sec. III for further comments). We could apply this prescription directly in field theory without reference to open string theory, but we choose to keep the string theory language because it makes the reference to certain effects more natural. In this context, when the scalars are massive they decouple from the low-energy dynamics and play no role in

³In this paper, the conjectured nonsupersymmetric Seiberg duality is applied at finite- N and the statements resulting from this application are also intended as statements for the finite- N theory.

TABLE I. The matter content of the electric theory. The $SO(2N_f)$ global symmetry of the brane construction enhances to $USp(2N_f)$ in the IR field theory.

Electric theory		
	$Sp(2N_c)$	$SO(2N_f)$
A_μ	$\square\square$ $N_c(2N_c + 1)$	\bullet
σ	$\square\square$ $N_c(2N_c + 1)$	\bullet
λ	\square $N_c(2N_c - 1)$	\bullet
Φ	\square $2N_c$	\square $2N_f$
Ψ	\square $2N_c$	\square $2N_f$

the IR physics. Although such scalars may appear in our list of fields (see Tables I and II), we do not consider them as part of the spectrum of the IR field theory. On the electric side, this effect leads naturally to a connection with QCD₃. On the magnetic side, we argue that the pattern is more involved and the precise physics has a more sensitive dependence on the regime of parameters.

The quantum effects also generate masses for the fermions. The ‘‘gluino,’’ which has already a tree-level Chern-Simons mass, and the quarks obtain a mass shift proportional to the square of the gauge coupling times a logarithm of the UV cutoff. Since this mass does not come from a power divergence, fermions can participate actively in the IR dynamics in our scheme.

Still, the above prescription does not specify completely the UV parameters of the orientifold QCD theories. When we make a statement about nonsupersymmetric Seiberg

TABLE II. The matter content of the magnetic theory. The dual rank is expressed in terms of $\tilde{N}_c = N_f + k + 1 - N_c$.

Magnetic theory		
	$Sp(2\tilde{N}_c)$	$SO(2N_f)$
a_μ	$\square\square$ $\tilde{N}_c(2\tilde{N}_c + 1)$	\bullet
s	$\square\square$ $\tilde{N}_c(2\tilde{N}_c + 1)$	\bullet
l	\square $\tilde{N}_c(2\tilde{N}_c - 1)$	\bullet
ϕ	\square $2\tilde{N}_c$	\square $2N_f$
ψ	\square $2\tilde{N}_c$	\square $2N_f$
M	\bullet	\square \square $N_f(2N_f - 1)$
χ	\bullet	$\square\square$ $N_f(2N_f + 1)$

duality between two theories we assume that there is a specific tuning of UV parameters, e.g. masses and other couplings, that drives that electric and magnetic theories to identical IR physics. This is a tuning that refers to couplings that remain finite as the UV cutoff Λ is sent to infinity. The main point of the paper is that the magnetic theory exhibits all the phases needed to reproduce known dualities and phases in QCD₃. In that sense, in this paper we do not generate new IR dualities and the UV tunings of mass parameters and other couplings that we need to do in the orientifold QCD theories are assumed to be closely related to the ones needed in previous treatments of QCD₃. The novelty here is that we find a single description (following from the magnetic theory) for all phases and dual descriptions of QCD₃.

We will not attempt to embed the above fine tunings of the field theory UV parameters to open string theory. In that sense, there is no claim in this paper about a rigorous embedding of nonsupersymmetric Seiberg duality in open string theory and no such claim is necessary for our purposes. Such a claim, which may or may not be possible, would require a better understanding of quantum effects in open string theory that goes beyond the scope of the present paper. Nevertheless, despite the lack of such a statement we argue that classical open string theory provides a useful starting point in a field theory discussion of the electric and magnetic orientifold QCD theories and that it gives motivating evidence for a potential IR duality (see Sec. II C).

C. Outline of the paper

The paper is organized as follows. In Sec. II, we present the orientifold field theories of interest, describe how they are embedded in suitable brane configurations in ten-dimensional type-II B string theory, and formulate the nonsupersymmetric electric-magnetic duality that they are conjectured to obey. Motivating evidence in favor of the duality is summarized in a separate subsection.

In Sec. III, we discuss perturbative nonsupersymmetric effects in both the electric and magnetic gauge theories. We discuss the expected behavior of the electric theory in the IR and its relation to QCD₃. A description of the IR physics from the magnetic theory point of view is relegated to the subsequent sections.

In Sec. IV A, we consider the parameter regime where $k + 1 - N_c \geq 0$. In that case, the magnetic squarks can lead to “full color-flavor recombination” (color-flavor locking) on the magnetic side. This is translated to three-dimensional bosonization of the electric side. The magnetic description does not provide only the correct matter content for bosonization, but also the requisite $USp(2N_f)$ -invariant quartic interaction needed for the Wilson-Fischer fixed point. It is argued that this picture is consistent for $N_f \leq k + 1 - N_c$, but fails for larger values of N_f , where the most natural scenario, in accordance with expectations from QCD₃, is a scenario of a symmetry breaking phase.

The other regime is when $-N_f \leq k + 1 - N_c < 0$. In that regime, the squark condensation Higgses the magnetic gauge group completely and leads to “partial color-flavor recombination.” Section IV B provides a detailed description of the effects that take place in this regime. On the electric field theory side, these effects are translated as dynamical symmetry breaking. The Nambu-Goldstone bosons are identified as massless modes in the low-energy spectrum of the open string theory on the branes and as massless modes after squark condensation in the magnetic Lagrangian.

In Sec. IV C, we discuss the possibility of a phase where the magnetic squark becomes massive. This leads to a phase with a CFT description in the IR in accordance with field theory expectations in QCD₃ [4]. The magnetic dual provides a specific prediction for the IR CFT. This possibility requires the existence of a critical number N_* with the new phase being realized when $N_f \geq N_*$.

In the above analysis, it is always assumed that the bare CS level of the orientifold QCD theory k is nonzero. When $k = 0$ the IR physics is dominated by the YM interaction. The IR physics of this theory, which besides the effects of the fundamental fermions involves nontrivial dynamics from Dirac fermions in the 2-index antisymmetric representation as well as additional scalar fields, is an interesting question that has not been explored in the past. This situation is discussed in Sec. V. A recent discussion of CS theories with matter in the adjoint representation but no fundamentals can be found in [12].

D. Open problems

In this paper, we focus on the case of (orientifold) QCD₃ theories with symplectic gauge group. This choice is dictated by the fact that this case is the most straightforward one from the viewpoint of the string theory construction that underlies part of this work. Since the analysis of unitary groups with even rank, $U(2N)$, shares many similarities with the analysis of the $USp(2N)$ case (and is related to it by a nonperturbative planar equivalence), it is rather natural to put forward a corresponding picture for $U(2N)$ (orientifold) QCD₃. Preliminary comments in this direction are summarized in Appendix B.

The formulation of nonsupersymmetric Seiberg duality for orientifold QCD₃ theories with general unitary gauge groups and orthogonal groups remains an open problem. We hope to return to these cases in a future publication.

E. Summary of conventions

In what follows, when we mention the level of a UV gauge theory, we will always refer, unless otherwise stated, to the bare level k (or $2k$ in the USp gauge theories). k is always an integer—it is the same integer (or related to the integer) that appears in the 5-brane bound states in our brane configurations. This convention is to be contrasted with other notation in the literature, e.g. [3] that is closely

related to our discussion, where the quoted level is $K = k_{\text{bare}} + k_{\text{quantum}}$ with $k_{\text{quantum}} = -\frac{N_f}{2}$.

When we integrate out massive particles the CS level shifts. In the YM-CS regularization (which we follow), only the fermions contribute to the shift. We will use conventions natural in string theory where the integration of a single Dirac fermion with positive mass m leads to the IR level

$$k_{\text{IR}} = k + 1. \quad (1.6)$$

A list of possible mass deformations, and their string theory interpretation, is summarized in Appendix A 2.

Another useful fact in our discussion is the fact that when we integrate out a gaugino (in the 2-index antisymmetric representation) in the $USp(2N)$ theory we obtain the CS level shift

$$k_{\text{IR}} = k - \text{sgn}(k)(N - 1). \quad (1.7)$$

II. NONSUPERSYMMETRIC USp SEIBERG DUALITY

The gauge theories of interest can be phrased independently of string theory. Nevertheless, since string theory provides a convenient organising principle for the non-supersymmetric duality of interest we will present in parallel the gauge theories in question and their embedding in classical open string theory. We consider the USp YM-CS theories that describe classically the IR physics on the Hanany-Witten brane configurations depicted in Figs. 2, 3. As we emphasized in subsection IB, the construction in this section should not be viewed as a rigorous claim towards an embedding of nonsupersymmetric Seiberg duality in open string theory, but rather as a useful starting point of a study of an IR duality in field theory.

A. Electric theory

Let us start with a description of the electric theory. The brane configuration that engineers the theory consists of N_c anti-D3 branes suspended between an NS5 brane and a tilted $(1, 2k)$ five-brane. In addition, there is an orientifold $O3$. The orientifold plane changes from $O3^+$ to $O3^-$ when it crosses the five-brane. This is identical to the brane configuration of [6]. In addition, we add N_f anti-D3 branes

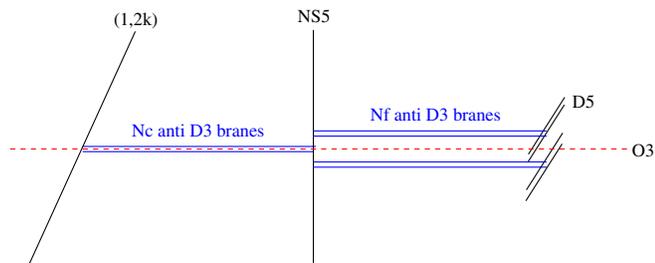


FIG. 2. The electric theory. It is a nonsupersymmetric $USp(2N_c)$ Yang-Mills theory with a level $2k$ Chern-Simons term and $2N_f$ flavors.

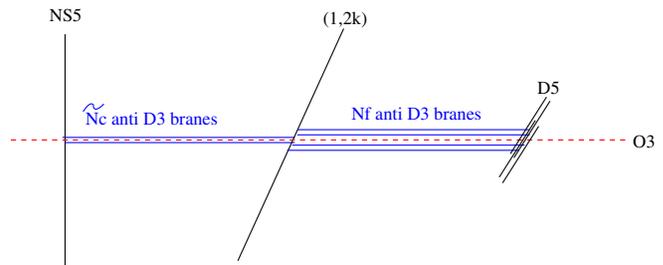


FIG. 3. The magnetic theory. It is a nonsupersymmetric $USp(2N_f + 2k - 2N_c + 2)$ Yang-Mills theory with a level $-2k$ Chern-Simons term.

which are attached to the right of the NS5 brane and end on D5 branes. The resulting brane configuration is depicted in Fig. 2. It consists of

- (i) an NS5 brane along the 012345 directions,
- (ii) a tilted $(1, 2k)$ five-brane along 012(37)89 directions. The latter five-brane is a bound state of an NS5 brane and $2k$ anti-D5 branes and is tilted in the (37) plane by an angle θ such that $\tan(\theta) = -2g_s k$. With this choice of angle, we would preserve $\mathcal{N} = 2$ supersymmetry, had we not had an orientifold. The minus sign reflects the presence of antibranes in the construction.
- (iii) an $O3$ plane along the 0126 directions, which is $O3^-$ at $x^6 = \pm\infty$,
- (iv) N_c (color) anti-D3 branes (and their mirrors) along the 012|6| directions (|6| denotes that the branes have a finite extent in the 6-direction),
- (v) N_f (flavor) anti-D3 branes (and their mirrors) along the 012|6| directions. The flavor branes end on anti-D5 branes, which are oriented along the 012789 directions.

The matter content of the theory is similar to the matter content of the supersymmetric theory, except that the fields transform differently with respect to the gauge group, due to the presence of the orientifold. The full matter content of the electric theory is given in Table I below. Note that the gauge fields A_μ and the scalar gaugino σ transform in the two-index symmetric (adjoint) representation of the gauge group, while the Dirac gaugino λ transforms in the two-index antisymmetric representation. The complex scalars Φ (squarks) and the Dirac fermions Ψ (quarks) are both in a fundamental pseudoreal representation of $USp(2N_c) \times SO(2N_f)$. Note that although the global symmetry on the brane is $SO(2N_f)$, the global symmetry of the low energy QCD theory is $USp(2N_f)$. This is due to irrelevant interactions inherent to the string realization of the theory.

The classical Lagrangian of the electric theory is

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{matter}}. \quad (2.1)$$

The gauge part is Yang-Mills-Chern-Simons theory at level $2k$

$$\mathcal{L}_{\text{gauge}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{CS}}, \quad (2.2)$$

$$\mathcal{L}_{\text{YM}} = \frac{1}{g^2} \text{Tr} \left[-\frac{1}{4} (F_{\mu\nu})^2 + \frac{1}{2} (D_\mu \sigma)^2 + i \bar{\lambda} \not{D} \lambda - i \bar{\lambda} [\sigma, \lambda] + \frac{1}{2} D^2 \right], \quad (2.3)$$

$$\mathcal{L}_{\text{CS}} = \frac{k}{4\pi} \text{Tr} \left[\varepsilon^{\mu\nu\rho} \left(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho \right) + 2D\sigma - 2\bar{\lambda}\lambda \right]. \quad (2.4)$$

D_μ is the standard gauge covariant derivative and the contraction of spinor indices has been kept implicit. For the matter part

$$\begin{aligned} \mathcal{L}_{\text{matter}} = & D_\mu \bar{\Phi}^{ai} D^\mu \Phi_{ai} + i \bar{\Psi}^{ai} \not{D} \Psi_{ai} - \bar{\Phi}^{ai} (\sigma^2)_a{}^b \Phi_{bi} \\ & + \bar{\Phi}^{ai} D_a{}^b \Phi_{bi} - \bar{\Psi}^{ai} \sigma_a{}^b \Psi_{ai} \\ & + i \bar{\Phi}^{ai} \bar{\lambda}_a{}^b \Psi_{bi} - i \bar{\Psi}^{ai} \lambda_a{}^b \Phi_{bi}, \end{aligned} \quad (2.5)$$

where $a, b, \dots = 1, \dots, 2N_c$ are color indices and $i, j, \dots = 1, \dots, 2N_f$ are flavor indices. The pseudoreality condition imposes on bosons and fermions

$$\bar{\Phi}^{bj} = \Phi_{ai} \Omega^{ab} \tilde{\Omega}^{ij}, \quad (2.6)$$

with $\Omega, \tilde{\Omega}$ the $USp(2N_c), USp(2N_f)$ symplectic tensors.

As we discussed in Sec. IB, the tree-level couplings in the above Lagrangian are modified by nontrivial quantum effects. We will later adopt a scheme where some of the fields acquire large positive masses of the order of the cutoff. Other couplings will be implicitly assumed to be suitably fine-tuned in order to achieve the IR physics of interest.

B. Magnetic theory

We define the proposed magnetic dual classically as the theory that describes the infrared dynamics of the brane configuration in Fig. 3, which results from that in Fig. 2 by swapping the NS5 and $(1, 2k)$ 5-branes across the x^6 direction. In the presence of the orientifold, $k+1$ additional D3 branes are created during this swap. As a result, we find

$$\tilde{N}_c = N_f + k + 1 - N_c = N_f + \kappa$$

D3 branes suspended between the NS5 and $(1, 2k)$ 5-branes.

The dual theory is a $USp(2\tilde{N}_c)$ gauge theory at bare CS level $-2k$. It exists when $\tilde{N}_c \geq 0$. Note that classically the flavor anti-D3 branes can slide freely between the tilted five-brane bound state and the N_f D5 branes (which share the common directions (89))⁴ leading to corresponding elementary meson and mesino degrees of freedom. The meson M transforms in the 2-index antisymmetric of

⁴As we discuss later, the moduli associated with motion in the (89) plane are lifted by quantum effects. Hence, they are pseudomoduli.

$SO(2N_f)$ and the mesino χ in the 2-index symmetric representation of $SO(2N_f)$. The full matter content of the magnetic theory is summarized in Table II.

Let us discuss the classical Lagrangian of the magnetic theory. Besides the gauge interactions $\mathcal{L}_{\text{gauge}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{CS}}$ for gauge group $USp(2\tilde{N}_c)$

$$\mathcal{L}_{\text{YM}} = \frac{1}{g^2} \text{Tr} \left[-\frac{1}{4} (f_{\mu\nu})^2 + \frac{1}{2} (D_\mu s)^2 + i \bar{l} \not{D} l - i \bar{l} [s, l] + \frac{1}{2} D^2 \right], \quad (2.7)$$

$$\mathcal{L}_{\text{CS}} = -\frac{k}{4\pi} \text{Tr} \left[\varepsilon^{\mu\nu\rho} \left(a_\mu \partial_\nu a_\rho - \frac{2i}{3} a_\mu a_\nu a_\rho \right) + 2Ds - 2\bar{l}l \right] \quad (2.8)$$

a part of the Lagrangian includes the same terms

$$\begin{aligned} \mathcal{L}_{\text{matter},1} = & D_\mu \bar{\phi}^{ai} D^\mu \phi_{ai} + i \bar{\psi}^{ai} \not{D} \psi_{ai} - \bar{\phi}^{ai} (s^2)_a{}^b \phi_{bi} \\ & + \bar{\phi}^{ai} D_a{}^b \phi_{bi} - \bar{\psi}^{ai} s_a{}^b \psi_{ai} \\ & + i \bar{\phi}^{ai} \bar{\lambda}_a{}^b \psi_{bi} - i \bar{\psi}^{ai} \lambda_a{}^b \phi_{bi} \end{aligned} \quad (2.9)$$

for the magnetic squarks ϕ^i , and quarks ψ^i as the electric theory. In addition, it includes kinetic terms for the mesons $M^{[ij]}$ and mesinos $\chi^{(ij)}$

$$\mathcal{L}_{\text{matter},2} = \partial_\mu \bar{M}_{ij} \partial^\mu M^{ij} + i \bar{\chi}_{ij} \not{D} \chi^{ij} \quad (2.10)$$

and a set of interactions that constitute the nonsupersymmetric orientifold version of the supersymmetric cubic superpotential between mesons/mesinos and squarks/quarks.

The complete matter Lagrangian is

$$\begin{aligned} \mathcal{L}_{\text{matter}} = & D_\mu \bar{\phi}^{ai} D^\mu \phi_{ai} + i \bar{\psi}^{ai} \not{D} \psi_{ai} - \bar{\phi}^{ai} (s^2)_a{}^b \phi_{bi} \\ & + \bar{\phi}^{ai} D_a{}^b \phi_{bi} - \bar{\psi}^{ai} s_a{}^b \psi_{ai} + i \bar{\phi}^{ai} \bar{\lambda}_a{}^b \psi_{bi} \\ & - i \bar{\psi}^{ai} \lambda_a{}^b \phi_{bi} + \partial_\mu \bar{M}_{ij} \partial^\mu M^{ij} + i \bar{\chi}_{ij} \not{D} \chi^{ij} \\ & - \frac{1}{4} y^2 \phi_{ai} \phi_j^a \bar{\phi}_b{}^i \bar{\phi}^{bj} - y^2 \bar{M}_{ik} M^{ij} \bar{\phi}_a{}^k \phi_j^a \\ & - 2y \psi_{ai} \chi^{ij} \phi_j^a - y \psi_{ai} M^{ij} \psi_j^a \end{aligned} \quad (2.11)$$

and the total Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{matter}}. \quad (2.12)$$

y is a coupling with mass dimension 1/2 that appears in front of the cubic superpotential interactions in the supersymmetric version of the theory. In the nonsupersymmetric theory at hand, the RG flow will not respect the relations between the couplings that appear in front of the terms in the last line of (2.11). The expression (2.11) is written here only as a specific bare Lagrangian that follows from its

supersymmetric ancestor by appropriately modifying the representations of certain fields.

C. Evidence for duality

We claim that with a suitable definition of the UV parameters the above electric and the magnetic theories form a Seiberg dual pair, namely they lead to the same IR physics. The construction in classical open string theory based on swapping five-branes is the first weak evidence for a potential relation between the two theories. Independently, in Appendix A we provide standard evidence in the form of 't Hooft anomaly matching and a preliminary map between deformations and RG flows.

In addition, we note that at large N_c , N_f and k the two theories become supersymmetric. The reason is that in this limit there is no difference between the two-index symmetric and the two-index antisymmetric representations. In the brane picture, this statement translates to the fact that the orientifold (the Möbius amplitude) is a $1/N$ effect. Therefore, in the large N limit the electric and magnetic theories are dual to each other as a supersymmetric pair similar to the one analyzed in [9].

It is not straightforward to make a claim of a rigorous embedding of nonsupersymmetric Seiberg duality in open string theory and in this paper we do not attempt to put forward such a claim. Nevertheless, it is interesting to note here another argument from string theory in favor of the duality, due to Sugimoto.⁵ This argument does not involve five-brane swapping. It relies on the SUSY duality and, under certain assumptions, leads to the non-SUSY duality. The idea is that the same operation (adding $N_f + k$ antibranes) on a pair of dual supersymmetric theories leads to a duality between nonsupersymmetric electric and magnetic theories. The electric SUSY theory becomes the magnetic non-SUSY theory and the magnetic SUSY theory becomes the electric non-SUSY theory.

More explicitly, start from a supersymmetric Giveon-Kutasov brane configuration that realizes a USp electric-magnetic pair [9]. Consider the electric side with $N_c - 1$ color branes and the magnetic side with $N_f + k - N_c$ color branes. Both sides contain N_f flavor branes. Add $N_f + k$ infinite anti-D3 branes on top of the SUSY electric theory and $N_f + k$ infinite anti-D3 branes on top of the SUSY magnetic theory. On both sides of the duality, the $N_f + k$ antibranes extend over the flavor branes, the color branes and beyond them where there are no D3 branes.

On the original electric side of the duality, we obtain $N_f + k - N_c + 1$ anti-D3 branes as color branes. Let us split the $N_f + k$ flavor segment of the antibranes into N_f and k antibranes. The N_f antibranes annihilate the N_f branes and the k antibranes make the five-brane bound state

an NS5 brane. Similarly, on the “other side” of the brane configuration the $N_f + k$ antibranes become N_f flavor antibranes and a tilted five-brane bound state. The result is the magnetic $USp(2(N_f + k - N_c + 1))$ non-SUSY theory.

In a similar way, adding $N_f + k$ antibranes to the color part of the SUSY magnetic theory with $N_f + k - N_c$ color branes results in N_c antibranes. The annihilation of branes in the other segments of the configuration leads to the $USp(2N_c)$ non-SUSY electric theory.

In this way, we obtained the nonsupersymmetric duality between the electric $USp(2N_c)$ theory and the magnetic $USp(2(N_f + k - N_c + 1))$ theory from the SUSY pair. This argument does not explain the full role of non-supersymmetric quantum effects in the duality and its potential embedding in string theory, but it provides a potentially useful, complementary viewpoint in our discussion. It will be interesting to repeat this exercise in a pure field theory language.

III. PERTURBATIVE DYNAMICS OF THE ELECTRIC AND MAGNETIC THEORIES

Given that the electric and magnetic theories are non-supersymmetric we anticipate potentials for the various scalars.⁶ The potentials are due to nonplanar effects ($1/N$ effects) and they vanish in the large- N limit where both the electric and magnetic theories become a dual pair with $\mathcal{N} = 2$ supersymmetry. In some cases, the field theory potentials have an interpretation as potentials between branes in the brane configuration. The potentials depend on the UV cut-off of the theory. Within field theory we can remove this dependence by renormalization. The embedding in string theory, however, provides a natural UV cut-off, $\Lambda^2 = \frac{1}{\alpha'}$, and a physical meaning to the potentials.

In all cases, the effects that we consider are due to the difference between the representations of bosons and fermions. Similar dynamics and considerations were involved in a proposal of a nonsupersymmetric S-duality [10].

Consider a scalar propagator: in perturbation theory the difference between a bosonic loop and a fermionic loop will produce either a massive scalar or a tachyonic scalar, depending on whether there are more bosonic degrees of freedom or more fermionic degrees of freedom, as depicted schematically in Fig. 4 below.

If a scalar is massive it will not acquire a vev and the perturbative expansion near the origin (zero vev) is stable. If a scalar is tachyonic we anticipate a new minimum where a vev is acquired and symmetries (global or local) may be broken.

⁵Private discussions with AA about four-dimensional non-supersymmetric Seiberg duality, IPMU, 2013.

⁶Potentials, e.g. mass terms, are also generated for the fermions, as we noted in subsection IB. These are typically subleading as a function of the UV cutoff. In this section, we focus on the scalar potentials, which, by definition, can affect the stability of the theory.

A. Electric theory

The electric theory contains two scalars, the squark Φ and the scalar gaugino σ .

The scalar gaugino couples to itself, to the gauge boson and to the fermionic gaugino λ . The generated mass due to the bosonic and fermionic loops is

$$M_\sigma^2 = g^2(2N_c + 2) \int^\Lambda \frac{d^3k}{(2\pi)^3} \frac{1}{k^2} - g^2(2N_c - 2) \times \int^\Lambda \frac{d^3k}{(2\pi)^3} \frac{1}{k^2} \sim g^2\Lambda \quad (3.1)$$

namely $M_\sigma^2 > 0$. When $\Lambda \rightarrow \infty$ the mass of the field σ becomes infinitely heavy and the field decouples from the low-energy physics. The one-loop correction (3.1) can be trusted in the limit $\frac{g^2}{\Lambda} \ll 1$, since higher loop corrections are suppressed in this regime by positive powers of the dimensionless ratio g^2/Λ . In particular, the two-loop corrections are $\mathcal{O}(g^4)$. In field theory, we keep these perturbative, power-divergent corrections as a specific choice of scheme.

A similar one-loop computation in weakly coupled string theory would produce the potential $V(\sigma) \sim M_\sigma^2 \sigma^2$. $M_\sigma^2 > 0$ has an interpretation in the brane picture: the N_c color branes are attracted to the orientifold plane and the brane configuration is stable (nontachyonic). The field theory computation (3.1) is a hint that $M_\sigma^2 > 0$ also in string theory.

A similar analysis can be performed for the squark Φ . The squark couples to the ‘‘gauge multiplet’’, namely to A_μ , σ , λ . Since there are more bosonic than fermionic degrees of freedom perturbation theory suggests that $M_\Phi^2 > 0$. The squark therefore decouples from the low energy physics.

As a result, the low energy field theory contains a $USp(2N_c)$ gauge field A_μ , a gaugino λ and $2N_f$ quarks Ψ .

When $k \neq 0$ there are Chern-Simons terms. The Chern-Simons terms provide a mass $M_{CS} = g^2 k$ to the gauge field and to the gaugino. We therefore anticipate that the IR dynamics will be dominated by the topological Chern-Simons theory coupled to the quarks.⁷ In that sense, for $k \neq 0$, the electric theory is very similar to QCD₃.

The case $k = 0$ is special, as there is no Chern-Simons term. The IR theory involves the strong coupling dynamics

⁷Integrating out the gaugino to obtain QCD₃ in the IR is most straightforward in the semiclassical regime $k \gg 1$. Away from this regime the relation with QCD₃ in the IR is harder to establish, but the expectation is that the IR phases are the same even for finite k unless something drastic happens in the infrared behavior of OQCD₃. The breakdown of the magnetic description outside the window of Seiberg duality could be such an effect. Inside the window of Seiberg duality with $k \neq 0$ there are currently no indications of drastic effects that would lead to significant deviations from QCD₃ dynamics.



FIG. 4. Perturbative contributions to a scalar mass.

of the Yang-Mills interaction between the gauge field, the gaugino and the quarks.

B. Magnetic theory

Similarly to the electric theory, the scalar gaugino of the magnetic theory acquires a mass and decouples. The color branes are therefore attracted to the orientifold plane.

The dynamics of the squarks and the mesons is more complicated. Let us focus on the squarks.

At the one-loop level there are effects due to the coupling with the gauge multiplet and effects due to the coupling with the meson multiplet. Let us denote the magnetic gauge coupling by g_m and the coupling to the meson multiplet by y .

There are more bosonic than fermionic degrees of freedom in the gauge multiplet and more fermionic than bosonic degrees of freedom in the meson multiplet. As a result,

$$M_\Phi^2 \sim (-y^2 + g_m^2)\Lambda. \quad (3.2)$$

As in (3.1), this correction is dominant in the limit $g^2/\Lambda \ll 1$. There are two competing effects in (3.2) and the squark may become either massive or tachyonic. In field theory, what happens depends on the choice of UV parameters. In string theory, however, the result is a matter of computation, which, unfortunately, is not immediately obvious. In what follows, we will consider both possibilities and identify the various phases associated with each one of them. We note that at large- k the gauge field becomes very massive and decouples, therefore we anticipate that in this limit the dynamics is dominated by a tachyonic squark.

The magnetic theory includes a coupling of the form

$$y^2 \bar{M}_{ik} M^{ij} \bar{\phi}_a^k \phi^a_j. \quad (3.3)$$

If the meson field acquires a vev of the form $\langle \bar{M}_{ik} M^{ij} \rangle = v^2 \delta_k^j$ the squark field becomes massive. If the squark field acquires a vev of the form $\langle \phi^a_j \rangle = v \delta^a_j$, and if flavor symmetry is not broken (bosonized phase), the mesons become massive. We propose that the most likely scenario in the context of electric-magnetic duality is one where in all phases

$$M_\phi^2 \times M_M^2 < 0. \quad (3.4)$$

In the following sections, we will discuss several phases of the magnetic theory, their realization in the brane picture and their relation to the electric theory.

IV. PHASES OF ORIENTIFOLD QCD₃ AND QCD₃

A. Bosonization

We focus on the magnetic description of the orientifold QCD₃ theory. The first region of parameter space under consideration is the region where $\kappa \equiv k + 1 - N_c \geq N_f$, namely region I in diagram B of Fig. 1. The rank of the magnetic gauge group, $\tilde{N}_c = N_f + \kappa$ is automatically positive in this regime. The one-loop computation of the previous section suggests that the $2N_f$ squarks are tachyonic, at least for sufficiently small N_f at fixed κ . Henceforth, we will operate under this assumption for the whole region I in diagram B of Fig. 1.

We assume that the magnetic squarks condense. Let us examine first what happens from the viewpoint of the brane configuration that describes the magnetic theory, Fig. 3. The magnetic squarks are low-lying modes of the open strings that stretch between the $N_f + \kappa$ color D3s to the N_f flavor D3s (and their images). In the language of string theory, condensation of the squarks is open string tachyon condensation through a process that reconnects the $2N_f$ color D3s with $2N_f$ flavor D3s.

After reconnection $2N_f$ D3 branes are stretching between the NS5 brane and the $2N_f$ D5 branes, and 2κ color D3 branes are stretching between the NS5 brane and the $(1, 2k)$ five-brane, see Fig. 5.

Since the $2N_f$ reconnected D3 branes have no common directions with the five-branes on which they end their low energy spectrum does not have a gauge field, gaugino, mesons or mesinos. The only low lying modes arise from the strings on the 2κ color D3s and the strings stretching between the color D3s and the reconnected D3s. The theory on the color D3s is a $USp(2\kappa)$ gauge theory at level $-2k$ with a massive 2-index antisymmetric gaugino. The mass of the gaugino remains proportional to the level $2k$. At energies below this mass the gaugino can be integrated out. In addition, the field theory analysis below suggests that the modes of the strings stretching between the color and reconnected flavor D3s are $2N_f$ scalars in the fundamental representation of the $USp(2N_f)$ gauge group. The reconnection preserves the original $SO(2N_f)$ global symmetry which enhances to $USp(2N_f)$ in the low energy theory.

In this manner, brane reconnection implies that the IR physics is dominated by a $USp(2N_f)$ CS theory coupled to $2N_f$ bosons. Deformations of the brane setup that induce a nonvanishing mass to the quarks in the electric description show that the magnetic theory has the right interactions to be the bosonic theory appearing in bosonization. The

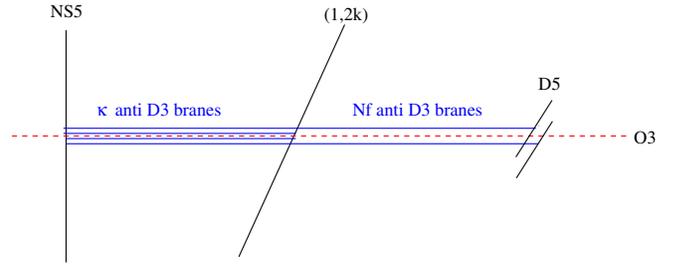


FIG. 5. A realization of the bosonized phase in the magnetic theory. N_f flavor branes reconnect with color branes. The gauge group is $USp(2\kappa)$.

deformation 2 in Fig. 7 leaves the level unchanged and Higgses the dual gauge group. The deformation 4 shifts the level but does not Higgs the dual gauge group.

Let us take a closer look at the features of the magnetic field theory. In terms of the low-energy magnetic theory of Sec. II B, it is easy to see that the color-flavor locking vev of the magnetic squarks Higgses the color gauge group from

$$USp(2N_f + 2\kappa) \rightarrow USp(2\kappa) \quad (4.1)$$

and leaves behind the gauginos in the 2-index antisymmetric of the Higgsed gauge group and $2N_f$ fundamental squarks. The $2N_f$ magnetic quarks become massive in the presence of the squark vev because of the Yukawa coupling between the squarks, the gauginos and the quarks. Similarly, both the elementary mesons and mesinos obtain masses and decouple at low energies.

The resulting low energy theory has global symmetry $USp(2N_f)$. After integrating out the massive gaugino (with the assumption that $k \neq 0$ everywhere in this section), we obtain a $USp(2\kappa)$ CS theory at level $-2N_c$ coupled to $2N_f$ squarks. The magnetic tree-level Lagrangian that was Higgsed leads automatically to a $USp(2N_f)$ -invariant quartic interaction for the squarks of the form

$$\mathcal{L}_{\text{quartic}} = (\bar{\phi}^{ai} \phi_{aj})(\bar{\phi}^{bj} \phi_{bi}). \quad (4.2)$$

This is one of the two possible quartic scalar interactions that are $USp(2N_f)$ -invariant (see [2] for related comments; the interaction $\tilde{\Omega}^{ij} \phi_{aj} \Omega^{ab} \phi_{bk} \tilde{\Omega}^{kl} \phi_{cl} \Omega^{cd} \phi_{di}$ is the second possibility). The renormalization group will naturally induce this second interaction, as well as the $USp(2N_f)$ -invariant mass term $\sum_{a,i} |\phi_{ai}|^2$. As in previous studies of bosonization we assume there is an appropriate fine-tuning of the UV parameters that allows a match between the fermionic and bosonic theories. The match of massive deformations in the electric and magnetic theory above via suitable brane motions reinforces the picture that the bosonic theory has the right ingredients for the

existence of a Wilson-Fischer fixed point as required by bosonization.

To summarize, after integrating out the gaugino, in the IR of the electric theory we have

$$USp(2N_c)_{2\kappa} \oplus 2N_f \text{ quarks.} \quad (4.3)$$

In the IR of the magnetic theory, we have

$$USp(2\kappa)_{-2N_c} \oplus 2N_f \text{ scalars.} \quad (4.4)$$

In the regime of this subsection, the nonsupersymmetric Seiberg duality implies the three-dimensional bosonization duality (1.3) with the identification (1.4), $K = \kappa - \frac{N_f}{2}$.

B. Symmetry breaking

Next we discuss the regions II and II' in the phase diagram B in Fig. 1. We will argue in favor of a symmetry breaking scenario in both regions. Region II is characterized by the inequalities $N_f > \kappa > 0$ and region II' by the inequalities $-N_f < \kappa < 0$. It is convenient to consider first the region II'.

1. Region II'

$\kappa < 0$ means that there are more flavor D3 branes than color D3 branes. As a result, at most $2N_f + 2\kappa$ D3 branes can reconnect. The color group is fully Higgsed and one is left with $2|\kappa|$ flavor D3s stretching between the (1, 2k) and the flavor D5s over an O3⁻ plane in addition to the reconnected D3s, see Fig. 6.

The elementary mesons on the $2|\kappa|$ D3s are massive. The flavor D3s stay attached to the orientifold and their global symmetry remains $SO(2|k|)$.

At the same time there is a global $SO(2(N_f + \kappa))$ symmetry associated with the $2(N_f + \kappa)$ D3s. We deduce that in string theory there is a breaking of the global $SO(2N_f)$ symmetry of the form

$$SO(2N_f) \rightarrow SO(2(N_f + \kappa)) \times SO(2|\kappa|). \quad (4.5)$$

In the IR field theory, the global symmetries enhance and it is not hard to show that the color-flavor locking vev of the squarks breaks the global $USp(2N_f)$ in the following way

$$USp(2N_f) \rightarrow USp(2N_f + 2\kappa) \times USp(2|\kappa|). \quad (4.6)$$

This is exactly the same pattern of symmetry breaking

$$USp(2N_f) \rightarrow USp(N_f - 2K) \times USp(N_f + 2K) \quad (4.7)$$

anticipated in [3] with the identification (1.4), $K = |\kappa| - \frac{N_f}{2}$. This identification is consistent with massive deformations of the theory.

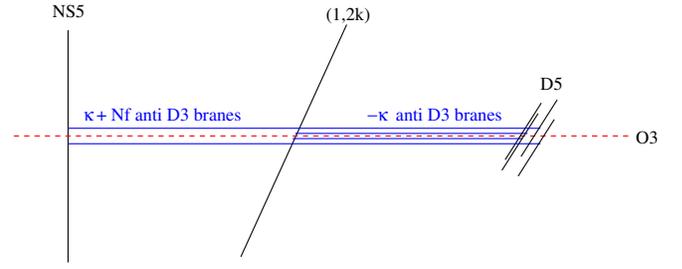


FIG. 6. A realization of the flavor breaking phase in the magnetic theory. $N_f + \kappa$ flavor branes reconnect with color branes. The magnetic gauge group is completely broken.

Consequently, the IR physics of this phase is described by the coset σ -model of the Nambu-Goldstone modes associated with the breaking (4.5). There are

$$\begin{aligned} & \frac{1}{2}2N_f(2N_f - 1) - \frac{1}{2}(2N_f + 2\kappa)(2N_f + 2\kappa - 1) \\ & - \frac{1}{2}2|\kappa|(2|\kappa| - 1) = 4|\kappa|(N_f + \kappa) \\ & = (N_f - 2K)(N_f + 2K) \end{aligned} \quad (4.8)$$

massless Nambu-Goldstone bosons, which arise as massless modes on the open strings stretching between the $2|\kappa|$ D3-branes and the $2(N_f + \kappa)$ D3-branes, see Fig. 6. As a check, notice that the result (4.8) (derived from (4.5)) agrees with the counting of Nambu-Goldstone bosons in QCD₃ (4.7).

The order parameter for the breaking is $\langle \phi_{ai} \Omega^{ab} \phi_{bj} \rangle$.

2. Region II

The region II, characterized by the parameter regime $N_f > \kappa > 0$, is more intriguing. On one hand, this is a case where, at first sight, all $2N_f$ flavor D3s can reconnect with color D3s leading to an IR $USp(2\kappa)$ theory coupled to $2N_f$ bosons implying bosonization and no global symmetry breaking. On the other hand, the dictionary (1.4) implies that this phase should still be describing the symmetry breaking phase of QCD₃ with $N_f > 2|K|$ where bosonization is inconsistent. It is hard to believe that the association of the orientifold QCD₃ with QCD₃ breaks down in this region because we can go to a limit of large k where the gaugino becomes arbitrarily massive and can be integrated out safely. The only logical conclusion seems to be that something critical is happening to the brane configuration as one crosses the threshold $N_f = 2\kappa$.

A natural guess is the presence of additional instabilities in the brane setup that describes the magnetic theory. Previously, in region I, it was sensible to expect that the brane reconnection leads to a stable vacuum. We would like to propose that the vacuum after brane reconnection is unstable, namely the $2N_f$ scalars in the resulting IR description have a negative mass squared. In that case, it

seems likely that open string condensation could proceed by annihilating the 2κ color D3s against another set of 2κ color D3s eventually leaving behind $2(N_f - \kappa)$ reconnected D3s (between $2(N_f - \kappa)$ D5s and the NS5) and 2κ flavor D3s (between 2κ D5s and the $(1, 2k)$ bound state). This is identical to the symmetry breaking configuration in region II'.

As a check of this scenario let us consider a deformation of the configuration with N_f reconnected D3s and κ D3s between the $(1, 2k)$ bound state and the NS5 brane that adds mass to the scalars (equivalently, a real mass to the fermion quarks in the electric description). The deformation (see deformation 4 in Fig. 7 in Appendix A) is implemented by bringing the $2N_f$ D5s on top of the $(1, 2k)$ bound state and breaking them up to two half-D5s separated symmetrically in the x^3 direction. The $(1, 2k)$ bound state connects with the broken D5s to create a $(1, 2k + 2)$ five-brane bound state. The reconnected D3s are attached to the broken D5s and dragged along the x^3 direction. Hence, for a finite separation the low-lying scalar modes on the strings stretching between the color D3s and the reconnected D3s acquire a positive mass squared shift. If they are tachyonic at zero separation, as the above scenario dictates, then there is a critical distance where they become massless. Precisely at that point there is a dual description of the system in terms of the bosonic degrees of freedom. For larger separations the scalars are massive and can be integrated out to recover a topological QFT in the infrared. This picture reproduces nicely all the features of the phase transition in Fig. 6 of Ref. [3]. Similar statements can be made for the mass deformation represented by the brane move 2 in Fig. 7 in Appendix A.

The above-mentioned checks imply an overall picture which is consistent with the proposed scenario of a transition in brane physics at $N_f = \kappa$. It would be interesting to find further evidence in favor of this scenario with a more explicit computation in string theory.

C. Scenario of symmetry restoration

In QCD₃, it is expected [4] that there is some $N_*(k, N_c)$ such that when $N_f \geq N_*$ the IR CFT is interacting and there is no symmetry breaking. Can this possibility arise naturally in our framework? We would like to argue that this could be described in the magnetic brane setup by a regime where the magnetic squarks are massive.

Assume that quantum effects make the squarks massive (namely, their mass squared is positive). Integrating out the massive squarks, we obtain

$$\begin{aligned} \mathcal{L}_{\text{matter}} = & i\bar{\psi}^{ai} \not{D}\psi_{ai} + \partial_\mu \bar{M}^{ij} \partial^\mu M_{ij} + i\bar{\chi}^{ij} \not{\partial}\chi_{ij} \\ & - M^i{}_j \psi_{ai} \bar{\psi}^{aj} - \mu^2 \bar{M}^{ij} M_{ij} + \dots, \end{aligned} \quad (4.9)$$

where the dots indicate higher-dimension interactions including an M^4 interaction. μ^2 is an induced tachyonic

mass squared to the mesons. To write this mass term we employed the proposed relation (3.4) between the signs of the mass squared of the squarks and the mesons, which suggests that the mesons are tachyonic in this case. Then, as the tachyonic mesons M condense the quarks become massive. In the IR, we obtain the sum of a topological QFT with free mesinos. In addition, at the true vacuum of the theory we have massive mesons, with $M^2 \sim 1/N_f$. Indeed, in the large- N_f limit the theory on the flavor branes acquires supersymmetry because there is no distinction between the symmetric and the antisymmetric representations. $1/N_f$ corrections should give a small mass to the mesons.

V. THE SPECIAL CASE OF YANG-MILLS THEORY WITHOUT A CHERN-SIMONS TERM

Consider the special case where $k = 0$. In the absence of a bare CS term, the IR of the field theory is dominated by the Yang-Mills interaction. The theory is parity invariant classically. Due to the Vafa-Witten theorem [13] that states that parity cannot be broken spontaneously, the theory is also parity invariant quantum mechanically.

The brane dynamics is also expected to be different with respect to the case $k \neq 0$. Instead of having configurations with an NS5 brane and a tilted five-brane, we have configurations with parallel NS5 branes.

The magnetic theory is $USp(2N_f - 2N_c + 2)$. For $N_c \geq 1$, we have $N_f \geq N_f - N_c + 1$, hence brane dynamics implies that the bosonization phase cannot occur.

According to the philosophy advocated in this paper, whether flavor symmetry is broken or not depends on whether the magnetic squarks condense or become massive.

Brane dynamics suggests that if the squarks condense the flavor symmetry is broken $USp(2N_f) \rightarrow USp(2N_f - 2N_c + 2) \times USp(2N_c - 2)$. This pattern, however, is not consistent with the Vafa-Witten theorem, because such a breaking pattern does not preserve parity.

According to Eq. (3.2), the squarks may become tachyonic or massive. For large enough k they become tachyonic. We propose that for $k = 0$ the squarks are massive.

When the squarks become massive flavor symmetry is not broken. The dynamics is the same as that of the case $N_f \geq N_*$, which was described in Sec. IV C. This result is different from the one for QCD₃ with $K = 0$, as described in the literature [4]. For sufficiently small N_f QCD₃ without a Chern-Simons term is expected to break the flavor symmetry in a pattern consistent with parity $USp(2N_f) \rightarrow USp(N_f) \times USp(N_f)$. This disparity does not lead to an obvious inconsistency, because the present electric theory contains a *massless* gluino. The gluino does not acquire a CS mass and its presence can alter the IR dynamics.

We therefore make a prediction, using brane dynamics, that three-dimensional Yang-Mills theory with a fermion in

the antisymmetric representation and $2N_f$ fundamental fermions does not break the USp flavor symmetry, as opposed to the theory without the antisymmetric fermion.

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APPENDIX A: EVIDENCE FOR NONSUPERSYMMETRIC SEIBERG DUALITY

1. 't Hooft anomaly matching

In this subsection, we consider 't Hooft anomaly matching between the electric and magnetic sides of the orientifold QCD theory in Sec. II. Both sides have a global $USp(2N_f)$ symmetry. We can gauge this symmetry by coupling to background $USp(2N_f)$ gauge fields. We want to identify the CS counterterms for these fields. We follow the steps outlined for USp bosonization in [2] (related discussions about this type of anomaly can be found in [14]).

On the electric side, we have $USp(2N_c)_{2k} \times USp(2N_f)_{2k_e}$.⁸ On the magnetic side, we have $USp(2\tilde{N}_c) \times USp(2N_f)_{2k_m}$. Both $k_e, k_m \in \mathbb{Z}$ and the second USp factor is classical. Implementing the mass deformation 4 of Fig. 7 we get $USp(2N_c)_{2(k+N_f)} \times USp(2N_f)_{2(k_e+N_c)}$ on the electric side. On the magnetic side, we get $USp(2\tilde{N}_c)_{-2(k+N_f)} \times USp(2N_f)_{2(k_m+\tilde{N}_c)}$. Integrating out the gaugino on the dynamical $USp(2N_c)$ and $USp(2\tilde{N}_c)$ factors, we recover level-rank duality in the IR. The match of the CS levels of $USp(2N_f)$ on both sides requires

$$k_e + N_c = k_m + \tilde{N}_c. \quad (\text{A1})$$

The global symmetry that acts faithfully on local operators is $USp(2N_f)/\mathbb{Z}_2$. This puts restrictions on the CS counterterms. Consistency with the \mathbb{Z}_2 quotient on the electric side requires

$$N_c k + N_f k_e \in 2\mathbb{Z}. \quad (\text{A2})$$

On the magnetic side, it requires

$$-\tilde{N}_c k + N_f k_m \in 2\mathbb{Z}. \quad (\text{A3})$$

We notice that

⁸In our notation, $2k$ and $2k_e$ are the bare Chern-Simons levels.

$$\begin{aligned} & -\tilde{N}_c k + N_f k_m \\ & = N_c k + N_f k_e + (N_c + \tilde{N}_c)(N_f - k) - 2\tilde{N}_c N_f \\ & = N_c k + N_f k_e + (N_f + k + 1)(N_f - k) - 2\tilde{N}_c N_f. \end{aligned} \quad (\text{A4})$$

Since $(N_f + k + 1)(N_f - k) \in 2\mathbb{Z}$ identically, we deduce that the conditions (A2) and (A3) are automatically satisfied together, which is a good consistency check.

2. RG flows

In the tree-level orientifold QCD₃ theory we can consider four types of mass deformations. In this appendix, we mostly ignore the potential quantum-induced instabilities. When such instabilities exist we implement the deformations at the unstable vacuum. It is convenient to see first how the deformations of interest map between the classical electric and magnetic descriptions in the brane setup. See [9] for analogous deformations in a $d = 3$ $\mathcal{N} = 2$ setup. The brane deformations are summarized pictorially in Fig. 7.

The first deformation, number 1 in Fig. 7, moves an image pair of D5s in the (45) plane. On the electric side, this reduces $N_f \rightarrow N_f - 1$ while $N_c \rightarrow N_c$ and $k \rightarrow k$. On the field theory side, this involves a complex mass deformation of the form

$$\delta\mathcal{L}_{electric} = m^{11}\bar{m}_{11}\bar{\phi}^{a1}\phi_{a1} - m^{11}\psi_1^q\psi_{a1} - \bar{m}_{11}\bar{\psi}^{a1}\bar{\psi}_1^1, \quad (\text{A5})$$

which indeed removes one flavor, $N_f \rightarrow N_f - 1$, but does not shift the level k or the rank of the gauge group N_c . On the magnetic side, the deformation has the effect $N_f \rightarrow N_f - 1$, $\tilde{N}_c \rightarrow \tilde{N}_c - 1$, $k \rightarrow k$. The corresponding deformation gives a vev to ϕ_{a1} so that

$$\phi_{a1}\phi_1^a = -\bar{m}_{11}. \quad (\text{A6})$$

The vev of ϕ_{a1} breaks the magnetic gauge group, $\tilde{N}_c \rightarrow \tilde{N}_c - 1$, removes one flavor ($N_f \rightarrow N_f - 1$) and leaves k invariant as anticipated.

The second deformation, number 2 in Fig. 7, moves an image pair of D5s in the 3 direction dragging along a pair of flavor D3s stretching between a D5 and the NS5. On the electric side, this reduces $N_f \rightarrow N_f - 1$ while $N_c \rightarrow N_c$ and $k \rightarrow k$. On the field theory side this involves equal and opposite sign masses for one flavor

$$\begin{aligned} \delta\mathcal{L}_{electric} & = m\bar{\psi}^{a,N_f+1}\psi_{a1} + m\bar{\psi}^{a1}\psi_{a,N_f+1} \\ & = m\psi_1^q\psi_{a1} - m\psi^{a,N_f+1}\psi_{a,N_f+1}. \end{aligned} \quad (\text{A7})$$

The scalars are not involved in this deformation. Indeed, (A7) removes the fermions of one flavor and after

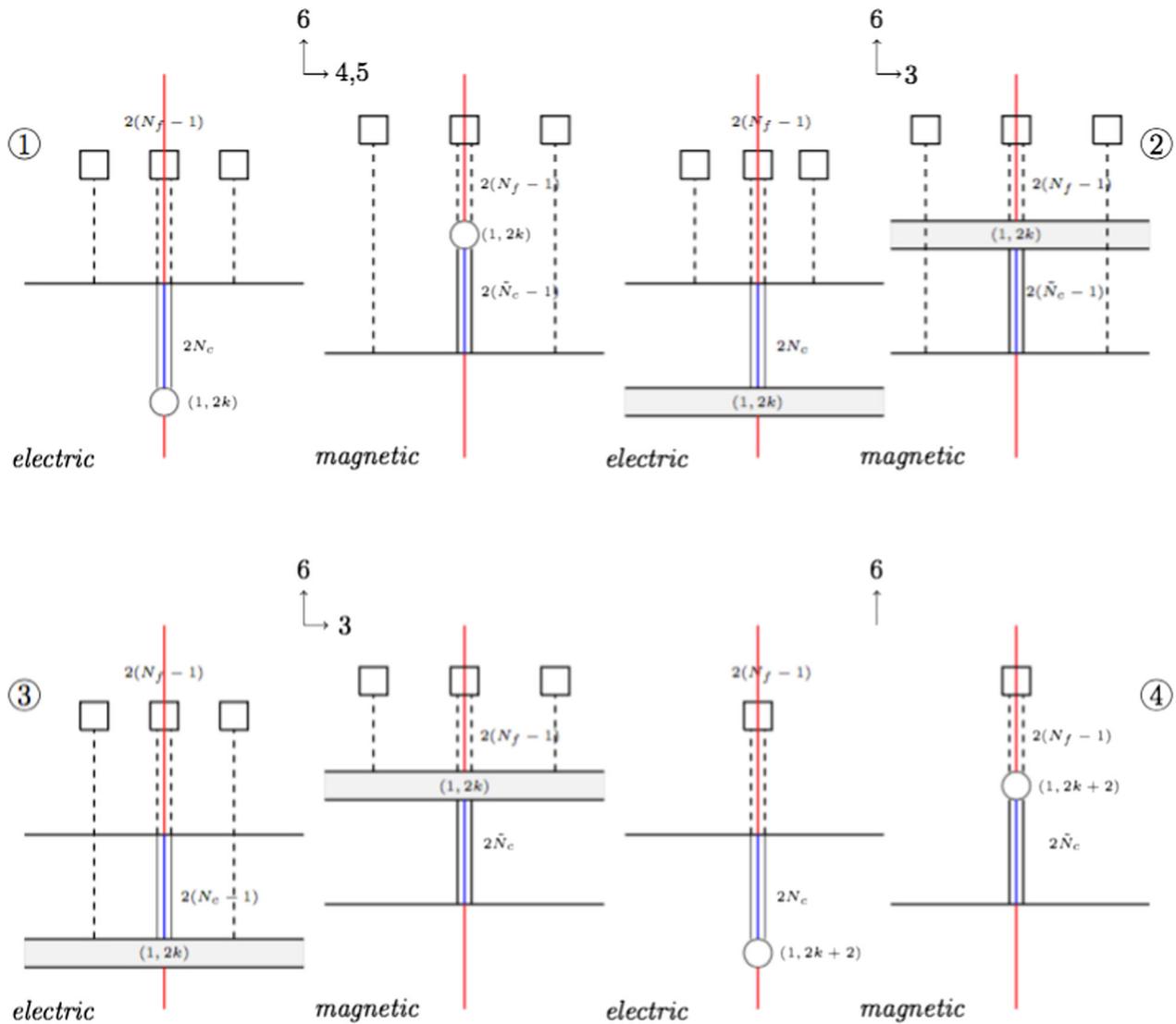


FIG. 7. Summary of mass deformations in the electric and magnetic brane setups. The red lines denote the $O3^-$ planes and the blue lines the $O3^+$ planes. The horizontal solid black line in the middle of each diagram is the NS5 brane. The circle in diagrams 1 and 4, and the grey belt in diagrams 2 and 3 is the five-brane bound state. The vertical solid black lines are the color D3 branes and the vertical dashed black lines are the flavor D3 branes. The vertical direction is always along x^6 and the horizontal along the (45) plane in diagram 1 and along x^3 in diagrams 2 and 3.

integrating out all auxiliary fields we find a potential for the scalars ϕ_{a1} with a quartic and sextic term. There are two potential vacua for this potential. One at the origin and one away from the origin. The deformation number 2 in Fig. 7 does not Higgs the gauge group so it corresponds to the vacuum at the origin. On the magnetic side the moved flavor D3s reconnect with 2 image color D3s and stretch between the D5s and the NS5. This reduces $N_f \rightarrow N_f - 1$, $\tilde{N}_c \rightarrow \tilde{N}_c - 1$ and keeps $k \rightarrow k$.

The third deformation, number 3 in Fig. 7, moves an image pair of D5s in the 3 direction but now in the electric side the corresponding flavor D3s reconnect with the $(1, 2k)$ 5 brane. This deformation reduces $N_f \rightarrow N_f - 1$,

$N_c \rightarrow N_c - 1$ and keeps $k \rightarrow k$. On the field theory side this is still a case of equal and opposite sign masses for one flavor; see Eq. (A6). In this case, the scalars get a vev and the gauge group is Higgsed, i.e. $N_c \rightarrow N_c - 1$. On the magnetic side the moved flavor D3s stretch between the D5s and the $(1, 2k)$ bound state. This reduces $N_f \rightarrow N_f - 1$ but leaves \tilde{N}_c and k intact. Notice that this move is only possible if the flavor and color D3s are not reconnected. If the magnetic squarks are tachyonic, the move would have to be implemented on an unstable vacuum.

The final deformation, number 4 in Fig. 7, moves a pair of flavor D5s along the 6 direction on the $(1, 2k)$ bound state where they merge to form a $(1, 2k + 2)$ bound state.

This operation reduces $N_f \rightarrow N_f - 1$ but $N_c \rightarrow N_c$ and $k \rightarrow k + 1$. On the field theory side, this operation involves giving same sign real masses to one flavor

$$\delta\mathcal{L}_{electric} = m\bar{\psi}^{a1}\psi_{a1} + m\bar{\psi}^{a,N_f+1}\psi_{a,N_f+1} = 2m\bar{\psi}^{a1}\psi_{a1}. \quad (\text{A8})$$

On the magnetic side, the magnetic quarks acquire corresponding masses and as these masses are sent to infinity $N_f \rightarrow N_f - 1$, $\tilde{N}_c \rightarrow \tilde{N}_c$ and $k \rightarrow k + 1$ consistently with the duality.

APPENDIX B: UNITARY GROUPS

In this section, we propose a duality for a $U(2N_c)$ orientifold QCD₃ theory. In order to have a string theory realization of a nonsupersymmetric theory and a Seiberg duality, we follow [6] where the $N_f = 0$ case was considered (see also [15,16] for the analogous four-dimensional theory).

The field theory lives on a type-0B brane configuration. The electric and magnetic brane configurations are identical to those we considered in Sec. II. The orientifold projection is special to type-0 strings: it decouples the bulk tachyon from the brane and projects out half of the RR fields [17].⁹ The matter content of the electric theory on the brane configuration is given in Table III. In the planar (large- N) limit, the theory is equivalent to the corresponding $\mathcal{N} = 2$ $U(2N)$ gauge theory that lives on type-IIB branes. Note also that the $U(2N_f)$ global symmetry is the symmetry of the IR theory.

By swapping the five-branes, we arrive at the magnetic theory. Its matter content is given in Table IV.

The dynamics of the theory based on unitary groups is similar to the dynamics of the theory based on USp groups. This is not surprising: there exists nonperturbative planar equivalence between $USp(2N)$ and $U(2N)$ gauge theories.

On the electric side, the scalars (squark and scalar gaugino) acquire mass and decouple. The CS level is shifted due to the antisymmetric gluino.

The IR levels of the electric theory $2K_{\text{SU}}$, $2K_{U(1)}$ are given by

$$K_{\text{SU}} = |k + 1 - N_c| - \frac{N_f}{2}; K_{U(1)} = k + 1 - \frac{N_f}{2} \quad (\text{B1})$$

⁹A more careful treatment of this setup requires its formulation in a *noncritical* type-0 string theory with Sagnotti-type orientifolds. This construction was performed in the context of the analogous four-dimensional theory in [15] following earlier work on nonsupersymmetric noncritical strings in [18]. In that construction, the noncritical type-0 string theory is classically stable and is not plagued by closed string tachyons. The corresponding formulation of a noncritical string theory for the three-dimensional theory at hand would require exact world-sheet techniques for backgrounds with RR fluxes, which are currently beyond reach.

TABLE III. The matter content of the electric theory.

Electric theory		
	$U(2N_c)$	$U(2N_f)$
A_μ	adjoint ($2N_c$) ²	•
σ	adjoint ($2N_c$) ²	•
λ	\square $2N_c(2N_c - 1)$	•
Φ	\square $2N_c$	\square $2N_f$
Ψ	\square $2N_c$	\square $2N_f$

TABLE IV. The matter content of the magnetic theory. $\tilde{N}_c = N_f + k - N_c + 1$.

Magnetic theory		
	$U(2\tilde{N}_c)$	$U(2N_f)$
a_μ	adjoint ($2\tilde{N}_c$) ²	•
s	adjoint ($2\tilde{N}_c$) ²	•
l	\square $2\tilde{N}_c(2\tilde{N}_c - 1)$	•
ϕ	\square $2\tilde{N}_c$	\square $2N_f$
ψ	\square $2\tilde{N}_c$	\square $2N_f$
M	•	\square $2N_f(2N_f - 1)$
χ	•	adjoint ($2N_f$) ²

thanks to the fundamental fermions and the antisymmetric fermion that shift the bare level $2k$. Note that in the bosonized phase $K_{U(1)} = K_{\text{SU}} + N_c$.

In the preliminary discussion of this appendix, we single out the case with even color, $2N_c$ and even flavor, $2N_f$, which proceeds as in the USp case.

When $N_f \leq 2K_{\text{SU}}$ the squarks condense, the Higgs mechanism takes place, and we find a dual bosonized theory of the form $U(2K_{\text{SU}} + N_f)$ with shifted levels

$$\tilde{K}_{\text{SU}} = -N_c; \tilde{K}_{U(1)} = -N_c + K_{\text{SU}} + \frac{N_f}{2}. \quad (\text{B2})$$

When $2K_{\text{SU}} \leq N_f \leq N_*$ the reconnected branes suggest that $U(2N_f) \rightarrow U(N_f + 2K_{\text{SU}}) \times U(N_f - 2K_{\text{SU}})$, in agreement with the conjecture by KS [4]. There are

$$(2N_f)^2 - (2N_f + 2\kappa)^2 - (2\kappa)^2 = 8|\kappa|(N_f + \kappa) = 2(N_f - 2K_{\text{SU}})(N_f + 2K_{\text{SU}}) \quad (\text{B3})$$

massless Nambu-Goldstone bosons that corresponds to massless modes of the oriented open strings stretched between the $2|\kappa|$ branes and $2(N_f + \kappa)$ branes

The case $k = 0$ (or $K_{\text{SU}} = 0$) is special. As in the USp theory we propose that there is no symmetry breaking and the theory is described by the magnetic theory of $N_f \geq N_*$. For $N_f \geq N_*$ the magnetic theory consists of a massless mesino and a light meson of mass square $1/N_f$.

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