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Light Source Detection for Digital Images in Noisy Scenes: A Neural Network Approach

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Abstract To produce realistic synthetic images, it is important to shade objects based on real illumination conditions of a scene. Estimating the direction of the light source of the scene is a key factor to achieve this. Properly estimating this source under noisy conditions is very challenging and it is a subject of intense research. Computational intelligence techniques offer promising way of tacking this problem. This paper presents a novel neural network based approach to recovering light source direction in relation to the viewpoint direction of a graphical image in noisy environments. The estimated light source direction can be used for the generation of 3D images from 2D ones. Experiments are performed using both synthetic and real images in noisy scenes. Four synthetic surfaces where generated with varying light source directions for a total of 12 images. Three real images were also used with varying degrees of noise. The experimental results show that the proposed approach is robust and provides a good level of accuracy.

1 Introduction

In computer vision, estimating illuminant directions and/or intensities is an important topic and has achieved many applications in domains such as shape from shading [1, 2, 3], augmented reality [4], and image approximation [5]. Detecting light

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sources offers a way of automatically locating the precise positions of light sources in a photograph. It can be used to render synthetic objects and conduct the seamless insertion of artificial or real objects in the scene by illuminating them under the same light conditions. Automatic localisation of light source effectively accelerate and facilitate software development, such as in computer games. Shape from shading is used to recover the 3D shape of a surface from a gradual variation of shading in the image, while information about the light distribution around an object of interest can be used to reduce the ambiguities concerning the 3D shape of the same object or of surrounding objects. This is why it is important to obtain an appropriate photometric model of real objects. To this end, one has to investigate the surface reflectance properties of the objects and the properties of illumination particularly their directions. One commonly used image formation model for this purpose is the Lambertian model. In this model, the gray level of the image at each pixel depends on the light source direction and the surface normal. Given a gray level image, the aim of shape from shading is to recover the light source and the surface shape at each pixel in the image.

A review of the state-of-the-art on the topic of estimation of surface reflectance properties and lighting allows to distinguish two main categories of techniques used in this area: (1) approaches to detection of multiple light sources [6, 7, 8, 9, 10, 11, 12, 13, 14]; (2) approaches to detection of single light source [16, 17, 18, 20, 21]. In this paper, the focus is on the latter case associated with detection of light source direction from a single view, i.e., recovery of the surface reflectance, lighting, or texture, given an image or multiple images taken from a single view. If the light sources are visible in the image of the study, the problem of estimating the illumination distribution may not be too challenging. For example, some techniques require a specific object to be part of the scene [8] or a mirror-like object to be inserted in the scene [15] for estimating the illumination distribution, but for real-life scenarios their applicability is limited as light sources are often not visible in the scenes due to various reasons, such as noisy environments. With unknown light sources in the scenes, or light sources hidden in noisy scenes, it is often not clear what is depicted in the scene. In other words, it would be very difficult to have information about the geometry, surface texture and other properties of the visible objects. In these scenarios, finding the exact locations and intensities of the light sources can be tremendously challenging.

In this paper, we propose and evaluate a novel approach to detecting light sources from noisy scenes using a spherical Lambertian model. We define some scene features related to light sources and use neural network model to identify the precise locations of the light sources. Some real and synthetic images are used to evaluate the accurate performance of the proposed approach.

This paper is organised as follows. Section 2 reviews the existing research efforts on detecting light sources in various conditions. Section 3 defines the scene features used for light source detection and present the neural network model The experimental results are provided in Section 4 and Section 5 concludes this paper with some discussions.

2 Related Work

Because there are no reliable methods that can detect light sources in any arbitrary image, it is important to understand the strengths of the different approaches. Pentland[17] in the first place dealt with the problem of estimating a light source that is not directly visible in an image object. He used a statistical approach to recover the direction of a single light source for arbitrary scenes without assuming that an object with known geometry is visible, but assuming a uniform distribution of the directions of the surface normals was used based on the Lambertian reflectance model. However, this statistical approach is only applicable to the surface normals of the objects in the scene that are isotropically distributed, for example, the spherical objects and complex scenes with many randomly distributed objects, it may not be suited for images of flat or cylindrical surfaces. Sharing Pentland's spherical assumption, Lee and Rosenfeld proposed a light source detection approach when developing shape from shaping techniques[18]. Lee and Rosenfeld computed the slant and the tilt of the surface in the light source coordinate system using only the first derivative of the intensity in a clearer way, they indicated that the regression model using derivatives in many directions is not necessary. The major part in the implementation of Lee and Rosenfeld's algorithm is the rotation of the image from the viewer coordinates to the light source coordinates, and the computation of the intensity gradient in the light source coordinates. There are no parameters to be determined in this algorithm, but like the Pentland's method, the local spherical assumption of the surface is the limits of the Lee and Rosenfeld's algorithm. Instead of using Pentland's equations in the differential forms for slant estimation, Chojnacki etal [19] derived a slant estimator based on the integral form. Compared with the Pentland's method, in the integral form the direction of differentiation was clearly stated. Like Lee and Rosenfeld [18], Chojnacki et al [19] also noted that the partial derivatives of the image intensity in arbitrary directions is always a linear combination of the derivatives in the horizontal and vertical directions. Thus, the tilt can be determined without the need to use the regression model proposed by Pentland [17]. However, an assumption was made both in the Pentland's method [17] and the Chojnacki method [19] about slant estimation, which is that the slant estimator is independent of the choice for direction s. One arising issue is that there are no details provided about how to choose the direction s in the implementation, eventhough it was noticed that the variations of s had an influence on the detection results. By replacing the smoothness constraint with an intensity gradient constraint which requires the reconstructed surface gradient to be closer to the intensity gradient of the input image in both x and y directions, Zheng and Chellappa used the shading information along image tours to reconstruct the shape, illuminant direction, and texture from a single image of a Lambertian surface[20]. In their proposed scheme, Zheng and Chellappa simplified the Euler equation by taking the first order Taylor series of the reflectance map and representing the depth, the gradient and their derivatives in discrete forms. In this scheme, a new algorithm was derived to update the depth and the gradient quickly and simultaneously in a hierarchical structure. Sato *et al* proposed a method of estimating the illumination distribution of a real scene from the image brightness observed on a real object surface in that scene[22, 23], in which an iterative optimization framework was used to simultaneously estimate both the illumination distribution of the scene and the reflectance properties of the surface without assuming any particular reflectance properties of the surface inside the shadows. Hara *et al* [24] proposed two types of methods for recovering the surface reflectance property of an object and the position of a light source from a single view without the distant illumination assumption, so they allowed the image synthesis of the target object to be under arbitrary light source positions.

3 Neural Network Approach to Estimating Light Source Direction in Noisy Scenes

Previous work [25], has shown that neural networks can effectively be used to reconstruct 3D scenes from noisy images. In the previous work [25], as well as in this paper, the estimation of illumination direction and albedo from a noisy scene is conducted under the following three assumptions:

- All illumination originates from one light source in the scene;
- All objects have Lambertian reflectance surfaces;
- The normal surface of the objects in the scene complies with normal distribution.

The first two assumptions are made in many of the existing approaches. Although these assumptions are seemingly simple and limit the applications, the current research indicates that estimating illumination direction and properties based on these assumptions is a difficult task. In this section a neural network based approach is presented to estimate the light source direction from image in noisy scene and reduce the error produced by the noise, in which scene features on the noisy image are used as inputs to the neural network.

3.1 Scene Features of Noisy Images

Given a noisy image, the following features are extracted by a mathematical formula [26, 27]. These features will work as the inputs to the neural network to recover the original slant and tilt of the light source.

The first feature of a noisy image is the average of the image brightness:

$$E_1 = \frac{\sum_y \sum_x R_{x,y}}{T} \tag{1}$$

where $R_{x,y}$ corresponds to the image intensity value at pixel (x,y) and T denotes the total number of pixels in the image.

The second feature of a noisy image is the average of the squared image brightness:

$$E_2 = \frac{\sum_y \sum_x ({R_{x,y}}^2)}{T} \tag{2}$$

The third and fourth features of noisy images are the averages of the horizontal and vertical components of the direction of the noisy image spatial gradient separately:

$$E_x = \frac{\sum_y \sum_x (R_{x,y} - R_{x,y-1})}{T} \tag{3}$$

$$E_y = \frac{\sum_y \sum_x (R_{x,y} - R_{x-1,y})}{T} \tag{4}$$

The fifth and sixth features are the contaminated slant (CS) and tilt (CT) angles extracted from the noisy image.

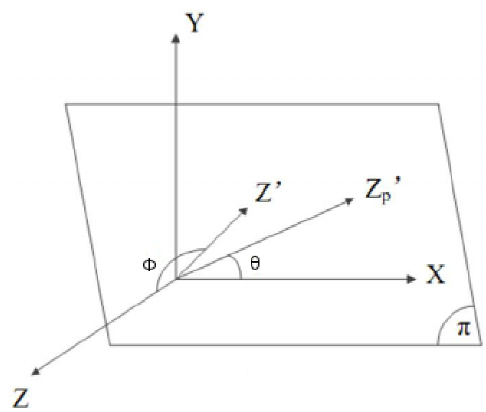


Fig. 1 Slant θ and Tilt Angle ϕ

Given the Lambertian reflectance model whose surface reflects light in all directions, the slant angle θ [26, 27] shown in Figure 1 is the angle between the camera axis and the light source direction, while the tilt angle ϕ [26, 27] is the angle between the x-axis and the projection light in the same direction. The CT is computed by:

$$Ct = tan^{-1}\left(\frac{E_x}{E_y}\right) \tag{5}$$

while the CS is given by:

$$Cs = \begin{cases} 1 & if\ \alpha \geq 1 \\ cos^{-1}\left(\frac{4E_1}{g}\right) & if\ \alpha < 1 \end{cases} \tag{6}$$

where

$$g = \sqrt{6\pi^2 E_2 - 48E_1^2} \tag{7}$$

and

$$\alpha = \frac{4E_1}{g} \tag{8}$$

3.2 Structure of The Proposed Neural Network

Given a noisy scene, the information available is limited to the features extracted in the above subsection. The task is to recover the original slant angle and tilt angle of light source and to calculate the light source direction. It is known in the signal processing community that it is rather challenging to recover the original signal from data highly contaminated by noise without prior knowledge. So, a neural neural network model is proposed to fulfil this task. The structure of the proposed neural network is shown in the Figure 2. This corresponds to a 3 layer feed-forward network with six input neurons and two output neurons. The six input neurons are merely "fan-out" units accepting the six features extracted above from noisy images. No processing takes place in these units. The sigmoid activation function is used in this feed-forward network with one hidden layer of units. The net input to the *k*th units in hidden layer is given by:

$$net_k^h = \sum w_{ik}^h x_i + b_k^h \tag{9}$$

where w_{ik}^h are the activation weights of the *k*th units in the hidden layer to the *i*th units in the input layer and b_k^h are the activation bias of the *k*th units in the hidden layer. The output o_k^h of the *k*th units in hidden layer is obtained by a sigmoid function, as given below:

$$o_k^h = \frac{1}{1 + \exp\left(-net_k^h\right)} \tag{10}$$

The two output neurons represent the recovered slant and tilt angles of the light source.

Slant output neuron:

$$\begin{aligned} net_s &= sum(w_k^s * o_k^h + b^s) \\ o_s &= f(o_s) = net_s(identity function) \end{aligned}$$

Tilt output neuron:

$$\begin{aligned} net_t &= sum(w_k^t * o_k^t + b^t) \\ o_t &= f(o_t) = net_t(identity function) \end{aligned}$$

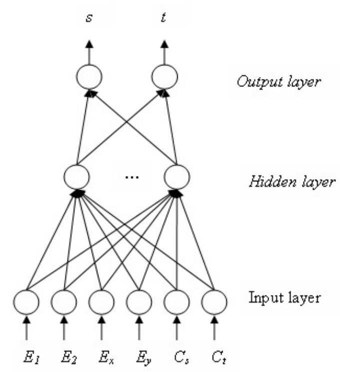


Fig. 2 Architecture of the multilayer feedford NN for estimating Light Source Direction

The most important issue is how to effectively train the network from a noisy scene for recover the original slant and tilt of light source. To this end, we need to create the training data set. Given N initial images from scenes that are less contaminated by noise , their corresponding slant s_i and tilt angles t_i of the light source are calculated in a similar way as did by (6) and (5) separately with $y^{(i)} = (s^{(i)}, t^{(i)})^T$. Gaussian noises are added to these N images, and six above scene features are extracted from the noise added images with $x^{(i)} = (E_1^i, E_2^i, E_x^i, E_y^i, Cs^{(i)}, Ct^{(i)})^T$. The data set $\left\{ \left(x^{(i)}, y^{(i)} \right) \right\}_{i=1}^{N}$ is then used to train the neural network shown in Figure 2 by the back-propagation algorithm. The back-propagation algorithm works as follows [28]. At the output layer, the output vector is compared with the desired output. The error is calculated from the delta rule and is propagated back through the network to adjust the weights with the idea of minimising the difference between the network outputs and the desired outputs. Such networks can learn arbitrary associations by using differentiable activation functions (10). A theoretical foundation of back-propagation can be found in [29] and [30].

The rationale of the neural network's merit in reducing noise lies in that the output of every neuron in the hidden layer and output layer is obtained by having to perform the weighted averaging of inputs from the previous layer like the ones shown in (9), which acts as a low-pass filter. So, the noise from the input layer will be cancelled much through the connections between the input layer and the hidden layer, and reduced further through the connections between the hidden layer and the output layer.

The neural network was trained multiple times under different starting conditions in order to avoid any local minimums. Furthermore, In order to find the best number of neurons for the hidden layer, pruning method was used. The neural network will start with a high number of neurons, and decrease the number of neurons until the optimal number is found.

3.3 Estimation of Light Source Direction

Given a noisy image, the recovered slant s and tilt t can be obtained by applying the trained neural network to the features. Then the light source direction in (S_x, S_y, S_z) can be estimated as follows [26, 27]:

$$S_x = cos(t) \cdot cos(s) \tag{11}$$

$$S_y = sin(t) \cdot sin(s) \tag{12}$$

$$S_z = cos(s) \tag{13}$$

4 Experimental Results

In this section, the proposed model is evaluated by estimating the light source directions of some images in noisy scenes. However, to find a good set of images to train and test the neural network models is very difficult, because the ideal image has to satisfy the previous assumption set for the neural network based model, for example all images must reflect a Lambertian reflectance model surface and have constant albedo values. In our experiments, the coordinate S_z is always set to be 1, which corresponds to the fact that all the images have infinite point source illumination. So, we only focus on the construction of a neural network model based on the proposed scheme to estimate (S_x, S_y) off the noisy scenes. In the next subsection, some experimental images are described. The light source directions for all is given in table 1.

4.1 Experimental Images

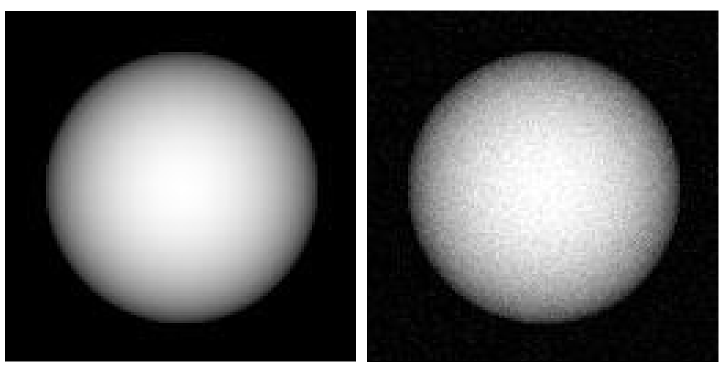
4.1.1 Synthetic images

The synthetic images described in [2, 32] were created by using the original depth maps from range data obtained from a range laser finder (see [2]). To create realistic synthetic images, the virtual objects have to be shaded consistently under the real illumination condition of the scene. To this end, the forward discrete approximation of depth is used to calculate the surface gradient. Shaded images are generated using the Lambertian reflectance model with different light conditions and source directions of the same surface, which present an advantage of synthetic images over real ones. It is known that real images usually do not perfectly meet the above assumptions of the reflectance model due to errors that can not be measured or corrected.

In this paper, four synthetic surfaces are generated, they are: Sphere, synthetic Vase, Penny and Mozart. Figures 3.a, 4.a and 5.a show three different views of a synthetic sphere image as taken from [2]. These images correspond to the same surface, from the same view point direction, but with different light source directions. Their corresponding noisy images from the same view point with different light source directions are independently shown in Figures 3.b, 4.b and 5.b.

Table 1 Original Light Source Directions

Image	Light Source Direction		
	S_x	S_y	S_z
Sphere 1	0	0	1
Sphere 2	0.5	0.5	1
Sphere 3	-0.5	-0.5	1
Mozart 1	0	0	1
Mozart 2	1	0	1
Penny 1	0	0	1
Penny 2	1	0	1
Synthetic Vase 1	0	0	1
Synthetic Vase 2	1	0	1
Lenna	1.5	0.866	1
Pepper	0.767	0.642	1
Real Vase	-0.939	1.867	1



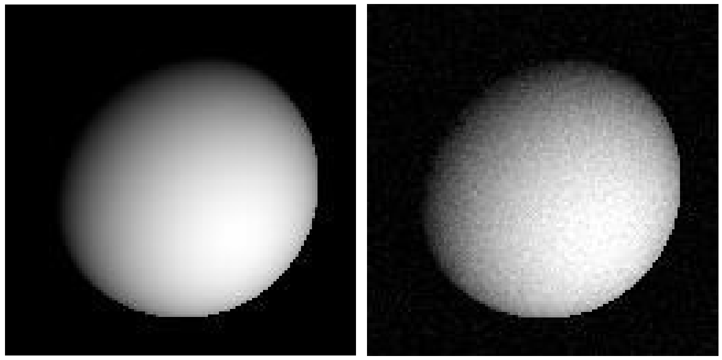
(a) (b)

Fig. 3 Synthetic sphere 1 image (normal (a) and noisy (b))

The synthetic vase images displayed in Figures 6.a and 7.a were generated using the equations suggested by Ascher and Carter [2, 31] as follows:

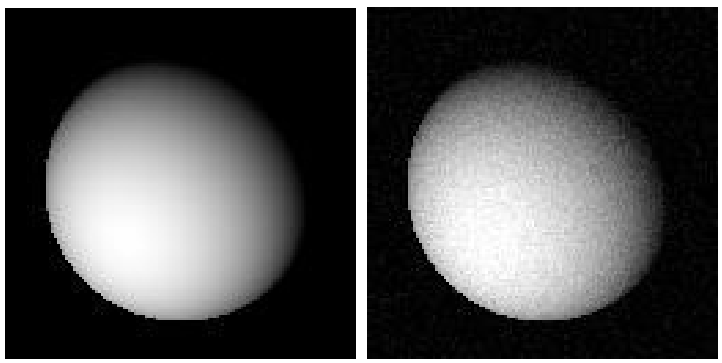
$$Z_{(x,y)} = \sqrt{f(y)^2 - x^2} \tag{14}$$

where



(a) (b)

Fig. 4 Synthetic sphere 2 image (normal (a) and noisy (b))



(a) (b)

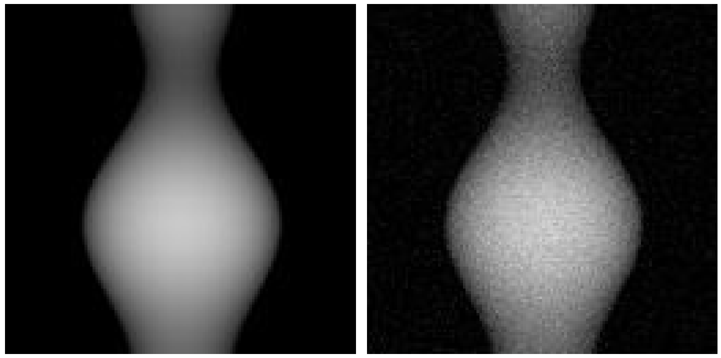
Fig. 5 Synthetic sphere 3 image (normal (a) and noisy (b))

$$f(y) = 0.15 - 0.1y(6y+1)^2(y-1)^2(3y-2)$$
$$-0.5 \leq x \leq \ 0.5, and 0 \leq x \leq \ 1 \tag{15}$$

while the corresponding noisy images from the same view point with different light directions are shown in Figures 6.b and 7.b separately.

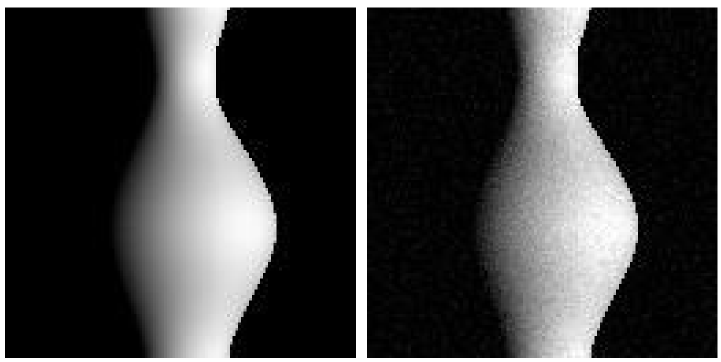
The Penny images shown in Figures 8.a and 9.a have the light source directions of $(S_x, S_y, S_z) = (0,0,1)$ and $(1,0,1)$ separately, while Figures 8.b and 9.b illustrate their corresponding noisy images from the same view point.

The Mozart images illustrated in Figures 10.a and 11.a are generated with the light source directions $(S_x, S_y, S_z) = (0,0,1)$ and $(1,0,1)$ separately, and the noisy images from the same view point with different conditions are shown in Figures 10.b and 11.b.



(a) (b)

Fig. 6 Synthetic Vase Image 1 (normal (a) and noisy (b))



(a) (b)

Fig. 7 Synthetic Vase Image 2 (normal (a) and noisy (b))

(a) (b)

Fig. 8 Real Penny Image 1 (normal (a) and noisy (b))

(a) (b)

Fig. 9 Real Penny Image 2 (001 (a) and 101 (b))

(a) (b)

Fig. 10 Synthetic Mozart Image 1 (normal (a) and noisy (b))

(a) (b)

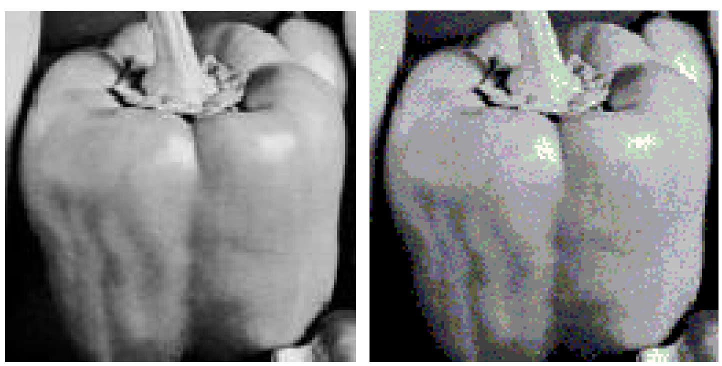
Fig. 11 Synthetic Mozart Image 2 (normal (a) and noisy (b))

4.1.2 Real images

Three real images [2, 32] were also used for training and testing the neural network model, they are: Lenna, Pepper and Real Vase as shown in Figures 12.a, 13.a and 14.a. The light source directions of these three surfaces are specified with the images [2, 32]. Figures 12.b, 13.b and 14.b depicts the noisy real images.

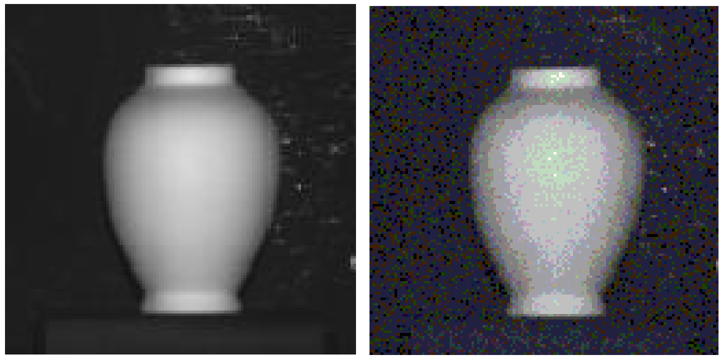
(a) (b)

Fig. 12 Real Lenna Images (normal (a) and noisy (b))



(a) (b)

Fig. 13 Real Pepper Images (normal (a) and noisy (b))



(a) (b)

Fig. 14 Real Vase Images (normal (a) and noisy (b))

4.2 Estimation of the Level of Performance of the Neural Network Model

By constructing the neural network model as described in the previous sections, seven synthetic images are going to be used for training. Table 2 shows the images used for training as well as the light source direction.

For testing and validation purposes, five images are going to be used. Three of the testing images used are real (Lenna, Pepper, Real Vase), and two are synthetic (Synthetic Vase 1, Synthetic Vase 2). Table 3 shows a summary of the results obtained in terms of the performance of the proposed neural network approach to estimating light source directions from the images in noisy scenes. Overall, the neural network model can achieve accurate estimation results on the noisy scenes both for synthetic and real images. Only for one of the testing real images resulted in inaccurate light source position. As the proposed method assumes that all objects have Lambertian reflectance surfaces, in a real world scenario, a 3D scene may include specular highlights [33] or be a mixture of different reflectance surfaces [33].

Table 2 Images Used for Training the Neural Network. All Images Used for Training are Synthetic.

Image	Expected LSDs
Sphere 1	(0, 0)
Sphere 2	(0.5, 0.5)
Sphere 3	(-0.5, -0.5)
Mozart 1	(0, 0)
Mozart 2	(1, 0)
Penny 1	(0, 0)
Penny 2	(1,0)

Table 3 Estimating light source directions (LSDs) for noisy scenes by the proposed scheme. The type of image is either *S* for synthetic or *R* for real.

Image	Type	Expected LSDs	Model Output LSDs
Lenna	R	(1.5, 0.866)	(1.53, 0.896)
Pepper	R	(0.767, 0.642)	(0.7053,0.662)
Real Vase	R	(-0.939, 1.867)	$\left(2.28e^{-6}, 3.27e^{-5}\right)$
Synthetic Vase 1	S	(0, 0)	$\left(1.75e^{-7}, 4.15e^{-8}\right)$
Synthetic Vase 2	S	(1, 0)	$\left(0.749, 2.6e^{-5}\right)$

5 Conclusions and Future Research

In this paper, a neural network based approach has been presented to estimate the light source directions from the images in noisy scenes, in which six scene features were suggested to work as the inputs. Some synthetic and real images were used to evaluate the proposed model, and the experimental results have shown that the neural network model can produce accurate estimation results in most cases. Further research needs to be performed for the proposed method to handle specular highlights, surfaces with a mixture of different reflectance types and multiple light sources. Our ultimate objective is to reconstruct 3D images from 2D ones in noisy environments and apply to medical domains.

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