A Modified STRUCT Model for Efficient Engineering Computations of Turbulent Flows with Rotation and Curvature

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Abstract: A modified STRUCT turbulence model (MST model) for efficient engineering computations of turbulent flows with rotation and curvature is proposed in this paper. The MST model switches between URANS and LES-like modes using a new damping function to adjust the turbulent viscosity. Compared with the original STRUCT method, the modifications are as follows: (1) the BSL k-ω model with the Spalart-Shur correction is chosen as the new baseline to improve the sensitivity to rotation and curvature; (2) a new adaptive time-scale ratio is proposed to avoid the arbitrariness of geometric averaging operation in the original method; (3) the normalized helicity is introduced into the new damping function to detect the energy backscatter phenomenon. Five classical high Reynolds number flow cases are tested. The results show that the turbulent viscosity of flow in the massively separated regions modeled by the MST model is reasonably reduced, and LES-like mode is activated, which captures more turbulent vortices and fluctuations on the URANS grids. With high efficiency and robustness, the MST model inherits the advantages of the original STRUCT method and improves the prediction accuracy of turbulent flows with rotation and curvature, which enables efficient engineering computations of the turbulence in hydraulic rotating machinery.

Keywords: STRUCT, turbulence model, rotation and curvature, hydraulic rotating machinery

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HIGHLIGHTS

A modified STRUCT turbulence model, MST model, is proposed.

A new URANS baseline is used to improve the sensitivity to rotation and curvature.

A new time-scale ratio is used to improve the adaptability of damping function to flow fields.

The normalized helicity is introduced to detect the energy backscatter phenomenon.

A good accuracy-cost balance is achieved for simulation of hydraulic rotating machinery.
1 Introduction

Hydraulic rotating machineries, such as pumps and hydroturbines, play an important role in the energy industry, and their energy conversion characteristics are determined by the inner flows, whose Reynolds number is typically $10^3$–$10^6$ during operation. The inner turbulent flows in hydraulic rotating machineries are strongly affected by the rotation and curvature effects, and they are one of the most complicated flow cases in engineering. Two basic turbulence models, URANS (Unsteady Reynolds-averaged Navier-Stokes) and LES (Large Eddy Simulation), have been commonly used to simulate the inner turbulent flows in hydraulic rotating machineries [1]. The URANS method is based on the Reynolds averaging in the statistical sense of the instantaneous Navier-Stokes equations, the accuracy of which remarkably depends on the analyst’s experience and the choice of boundary conditions, and it is usually difficult to obtain high-precision prediction results of complex separation flows [2]. The LES approach is based on the spectral filtering of turbulence energy, and in great detail it captures the large eddies that carry most of the turbulent kinetic energy and are responsible for most of the momentum transportation and turbulence mixing [3]. LES is generally more accurate than URANS, but the high computational cost has hampered its practical application [4]. For a better tradeoff of prediction accuracy and computational cost, some hybrid approaches have been proposed, such as hybrid URANS/LES and the second generation URANS (2G-URANS) approaches.

The hybrid URANS/LES approach has been a popular topic in recent years. A transition from URANS to LES can be achieved in two ways [5-6]. One strategy is to use a weighted sum of the Reynolds stress from the URANS model and the subgrid-scale stress from the LES model (blending), and the other strategy is to use a pure LES model in one part of the domain and a pure URANS model in the remainder (interfacing). The detached eddy simulation (DES) method is one of the most popular hybrid URANS/LES approaches [7]. The original DES model switches from the URANS mode in the boundary layer to the LES mode in the main flow, and the switching criterion is based on the grid size and the wall distance. In order to solve the problems of “modeled stress depletion” and “log layer mismatch” of the original DES model, the DDES (add the full name here) and IDDES (add the full name here) models are proposed [8-9]. Although the above DES models have been successfully used for the simulation of separation flows, there are some major limitations of these models in engineering applications. First, the priori selection of the desired resolution level and the compatible meshing scheme are required, which is often difficult, if not impossible, for large-scale engineering applications. Secondly, the LES-like local grid refinement is required in the main flow, which consumes a lot of computation resources.

Aiming at resolving the substantial turbulent structures, such 2G-URANS models as VLES (add the full name here) [10-12], PANS (add the full name here) [13-15], SAS (add the full name here) [16-18] and FBM (add the full name here) [19-20] models, have emerged, and they do are not explicitly dependent on the computation grid like the DES models. The VLES model can be obtained by rescaling a conventional URANS model through damping the Reynolds stress with a function that controls the ratio of modeled energy to resolved energy. The PANS model is based on the self-similarity scale assumption in the physical space, and a seamless transition from URANS to DNS (direct numerical simulation) can be realized by using a damping function to adjust the given URANS model. The SAS model introduces the Von Kármán length scale and is able to switch from the URANS mode to the SAS mode where the related physical length scale is reduced yielding a lower turbulent viscosity and high fluctuations. The FBM model introduces the ratio of a
turbulence length scale to a filter size and aims to obtain a lower turbulent viscosity. Although the aforementioned 2G-URANS models do not explicitly depend on the computation grid, the grid space scale is still needed to calculate the damping function, which may cause requirements on grid conditions and affect the simulation efficiency and robustness.

Different from the above hybrid approaches that bridge URANS and LES using the space scale of grids, the STRUCT (add full name here) method [22-23] uses the turbulent timescale to determine the damping function and it maintains the efficiency and robustness of URANS while extending the ability to locally resolve complex turbulent vortices. A variety of flow cases have demonstrated the LES-like capability of STRUCT on coarse grids, with a significant reduction of computation time. In addition, a consistent accuracy improvement can be achieved by increasing the mesh resolution [22,24]. These features make the STRUCT method an attractive approach for efficient computation of turbulent flows in hydraulic rotating machineries.

However, the original STRUCT method is not without limitation. Firstly, the turbulent flows in hydraulic rotating machineries are strongly affected by the rotation and curvature effects, which may result in a remarkable energy backscatter phenomenon [25]. The original STRUCT method does not consider the energy backscatter appropriately, which may cause local over-dissipation and give unreasonable results. Secondly, as a key activation parameter of the original STRUCT method, the modeled timescale can be determined by means of the geometric averaging operation or the additional convective-diffusive transport equation [22-23]. The former is more suitable for efficient engineering computations, but it may lead to an unsuitable value due to the arbitrary identification threshold of flow structures. Thirdly, the damping function in the near wall region always goes to 1, where the ordinary URANS mode is employed. As the base URANS model of the original STRUCT method, the standard k-ε model is not an ideal choice. To address these limitations, this paper presents a modified STRUCT model (MST), and five classical flow cases with high Reynolds number are employed to test the performance of the new model, including the turbulent swirling flow through an abrupt axisymmetric expansion, flow around a hydrofoil, flow past a cylinder, Taylor-Couette flow and flow in a centrifugal pump impeller. For comparison, the classical SST (add full name here) k-ω model is also applied to these test cases.

2 Mathematical Formulation

2.1 The OST model

The original STRUCT method (OST) switches between URANS and hybrid modes by using a timescale-based damping function to adjust the turbulent viscosity. The standard k-ε model is used for the baseline URANS, and the transport equations of turbulent kinetic energy k and dissipation rate ε are expressed as [23]

\[
\begin{align*}
\rho \frac{\partial k}{\partial t} + \rho u_j \frac{\partial k}{\partial x_j} &= \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma} \right) \frac{\partial k}{\partial x_j} \right] + G_k - \rho \epsilon \\
\rho \frac{\partial \epsilon}{\partial t} + \rho u_j \frac{\partial \epsilon}{\partial x_j} &= \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma} \right) \frac{\partial \epsilon}{\partial x_j} \right] + C_{\epsilon 1} \frac{\epsilon}{k} G_k - C_{\epsilon 2} \rho \frac{\epsilon^2}{k}
\end{align*}
\]

where \( \bar{u}_j \) is average velocity components, \( \rho \) is the water density, \( t \) is time coordinates, \( x_j \) is spatial coordinates, \( \mu \) is dynamic viscosity, \( \mu_t \) is turbulent viscosity, \( \sigma \) is the Prandtl number of turbulent
kinetic energy, $G_k$ is the production term of turbulent kinetic energy, $\sigma_\varepsilon$ is the Prandtl number of turbulent dissipation rate, and $C_{1}\varepsilon$ and $C_{2}\varepsilon$ are closure constants.

Following the hybrid approach [26], the turbulent viscosity of the OST model is directly adjusted by a damping function $D_f$:

\[
\begin{cases}
\mu_t = D_f \cdot \rho C_\mu \frac{k_m^2}{\varepsilon} \\
D_f = \min \left( \frac{t_r}{at_m}, 1 \right)
\end{cases}
\]

where $k_m$ is the modeled turbulent kinetic energy, $t_r$ is the resolved timescale, $t_m$ is the modeled timescale, and $C_\mu$ and $a$ are empirical constants. The damping function $D_f$ can also be interpreted as the ratio of modeled turbulent kinetic energy to the total amount of turbulent kinetic energy, and the hybrid mode of OST model is activated when the resolved timescale is lower. In particular, the definitions of the two timescales are expressed as

\[
\begin{cases}
t_r = \frac{1}{f_r} = \frac{1}{\sqrt{\langle Q \rangle}} \\
t_m = \frac{1}{f_m} = \frac{k_m}{\varepsilon}
\end{cases}
\]

where $f_r$ is the frequency of $t_r$, $f_m$ is the frequency of $t_m$, $\langle \cdot \rangle$ represents a generic averaging operation in the concerned region to estimate the modeled timescale, and $Q$ is the second invariant of the resolved velocity gradient tensor given by

\[
Q = \frac{1}{2} \left( \|\Omega\|_F^2 - \|D\|_F^2 \right) = -\frac{1}{2} \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i}
\]

where $\|\Omega\|_F$ is the Frobenius norm of rotation rate tensor, and $\|D\|_F$ is the Frobenius norm of strain rate tensor. Due to the outstanding advantages, Galilean invariance and local description in elementary motions, this parameter is suitable for describing properties of the resolved flow field.

Fig. 1 shows an illustrative comparison between the STRUCT strategy and other turbulence approaches. The STRUCT method models less scales than LES, thus achieving lower grid requirements and computation cost. Meanwhile, its turbulence capturing ability is greater than URANS, thus obtaining higher simulation accuracy and more turbulent structures.

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**Simulation Accuracy**

URANS  STRUCT  LES  DNS

**Computation Cost**

Figure 1. Illustration of STRUCT and other turbulence modelling strategies
2.2 The MST model

We propose the MST model to overcome the limitations of the OST model discussed in Section 1, and the detailed derivation is explained in this section.

- **Baseline Model**

In order to improve the sensitivity to adverse pressure gradients in the turbulent flows with rotation and curvature, the BSL $k$-$\omega$ model with the Spalart-Shur correction is employed as the new baseline URANS model. This maintains the robust and accurate formulation of the standard $k$-$\omega$ model in the near wall region and takes advantage of the freestream independence of the standard $k$-$\epsilon$ model in the main flow region [27-28]. The transport equations of turbulent kinetic energy $k$ and specific dissipation rate $\omega$ of the BSL $k$-$\omega$ model are expressed as [27]

$$
\rho \frac{\partial k}{\partial t} + \rho u_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_{k3}} \right) \frac{\partial k}{\partial x_j} \right] + G_k - \beta' \rho \omega
$$

$$
\rho \frac{\partial \omega}{\partial t} + \rho u_j \frac{\partial \omega}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_{\omega3}} \right) \frac{\partial \omega}{\partial x_j} \right] + 2 \rho \left( 1 - F_i \right) \frac{1}{\sigma_{\omega2}} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} + \alpha_3 \frac{\partial \omega}{\partial x_j} \frac{\partial \omega}{\partial x_j} + \alpha_3 \frac{\partial \omega}{\partial x_j} G_k - \beta \rho \omega^3
$$

where $\bar{u}_j$ is average velocity components, $\rho$ is the water density, $t$ is time coordinates, $x_j$ is spatial coordinates, $\mu$ is dynamic viscosity, $\mu_t$ is turbulent viscosity, $G_k$ is the production term of turbulent kinetic energy, $F_i$ is a weighted function, and $\sigma_{k3}, \sigma_{\omega2}, \sigma_{\omega3}, \alpha_3, \beta_3$ and $\beta'$ are closure parameters.

- **Adaptive Time-scale Ratio**

In order to determine an adaptive (automatic and dynamic) modeled timescale $t_m$, the specific dissipation rate $\omega$ with the dimension of “frequency” is directly used in this research. It is obtained from the transport equations and avoids the arbitrariness of the geometric averaging operation and the cost of additional convective-diffusive transport equation [22-23]. Accordingly, the modeled-to-resolved timescale ratio $R_i$ is given by

$$
R_i = \frac{t_m}{t_r} = \frac{1}{\varphi \omega} \sqrt{\left| Q \right|}
$$

where $\varphi$ is an empirical constant, whose value is set to 0.085 in this study. The turbulent flows in hydraulic rotating machineries are generally incompressible, and the second invariant $Q$ is still employed to determine the resolved timescale $t_r$.

- **Consideration of Turbulence Energy Backscatter**

In order to take the energy backscatter phenomenon into account, the helicity, a topological measure of the intertwining of vortices and an invariant even in the viscous fluid [29-30], is introduced. The turbulent viscosity diminishing is found to be connected with the limitations of the energy transfer from the larger scales to smaller scales in the presence of helicity [31]. The regions of high helicity fluctuations are found to be closely correlated with the regions of low energy flux owing to the reduction of nonlinear interactions [32]. Moreover, the energy backscatter is stronger than the forward dissipation when the value of normalized helicity $H_h$ exceeds 0.7, and a modified Spalart-Allmaras model with consideration of the energy backscatter is obtained by using the
normalized helicity, which can improve the simulation accuracy of the corner separation flows in the compressor [25]. The normalized helicity $H_n$ is also employed:

$$H_n = \frac{V \cdot (\nabla \times V)}{|V| \cdot |\nabla \times V|} = \cos [V, \nabla \times V]$$

(7)

where $V$ is velocity vector, $\nabla \times V$ is the vorticity vector, and $[\cdot]$ represents the vector angle operation. The new damping function of the MST model is redefined as

$$\left\{ \begin{array}{c}
\mu_f = D_f \cdot \frac{k_m}{\omega} \\
D_f = \frac{1}{\ln \left[ ch (\alpha \cdot R_i) + \beta \right] + \gamma \cdot |H_n|^\lambda} 
\end{array} \right. $$

(8)

where empirical constants, $\alpha=1.571, \beta=1.718, \gamma=0.727, \lambda=0.865$, are optimized to these values based on the relevant references [22,25], and they are suitable for solving the flow fields with water medium.

In summary, the new $D_f$ of the MST model not only overcomes the limitations of OST but it also activates smoothly, thereby avoiding the step problem caused by the piecewise function of the OST model. The theoretical formulation of MST makes it a more suitable simulation approach for turbulent flows with rotation and curvature.

3 Applications of the MST Model

Five classical flow cases are employed to test the performance of the proposed MST model, and the include turbulent swirling flow through an abrupt axisymmetric expansion, flow around a hydrofoil, flow past a cylinder, Taylor-Couette flow and flow in a centrifugal pump impeller.

3.1 Swirling flow through an abrupt axisymmetric expansion

Turbulent swirling flow through an abrupt axisymmetric expansion is a complex flow case possessing lots of dynamic phenomena such as vortex breakdown, detachment and reattachment, which is similar to the vortex rope in the draft tube of hydroturbine. The computation domain of the abrupt expansion is shown in Fig. 2, corresponding to the experiment [33]. For this flow case, the Reynolds number based on the inlet diameter $D$ and the bulk velocity is approximately $3 \times 10^4$, and the swirl number is approximately 0.6.

![Figure 2. Computation domain and local coordinate frame of the abrupt expansion](image)
Figure 3. Mean and RMS values of axial velocity
As for the meshing scheme, high quality hexahedral grids are adopted to discretize the domain. The discretization mesh is consistent with those used in literatures for this flow case [14,34-35]. In terms of boundary conditions, the velocity inlet condition is used according to the radial profiles obtained from experiment data, and a high inlet turbulence intensity (10%) is also estimated. The pressure outlet condition is adopted, and the no-slip condition is employed for wall surfaces. The key parameter $t_m$ of the OST model is set to 0.5s. Transient simulation is carried out, and the time step is selected to meet the CFL condition. The computation time is not less than 25 times the throughput-flow period (the period required by the mean flow to pass through the total domain once), and the results below are obtained by using the time-weighted average of the last 12 periods.
Fig. 3 and Fig. 4 show the distributions of the mean and RMS values of axial velocity $V_z$ and circumferential velocity $V_\theta$ on four cross sections behind the expansion, corresponding to the $z/D$ values of 0.25–1.0. The results of the MST model are compared with the experiment data as well as those of the OST model and SST $k-\omega$ model. It is observed that the SST $k-\omega$ model gives poor results of mean and RMS velocity profiles, and especially it captures little turbulent fluctuation information, which implies that the prediction results of this model converge to a steady simulation. The accuracy of OST model is better, but some deviations still exist near the pipe center ($r=0$), which indicates that the original STRUCT method is insufficient to simulate turbulent flows with rotation and curvature. By contrast, the simulation results of MST model are in good agreement with the experimental data. Moreover, the power spectrum density (PSD) of the MST model is shown in Fig. 5. The resolved scales are asymptotic to an inertial subrange, reasonably close to the classical $-5/3$ scaling, which implies that the LES-like mode is activated efficiently and the expected improvements are achieved on the basis of the original STRUCT method.

According to the previous researches, the phenomenon, “a precessing vortex core rotates around the geometrical axis and subsequently breaks down to small coherent structures”, is remarkable. As shown in Fig. 6, the vortical structures are visualized by the isosurfaces of Liutex criterion [36]. The SST $k-\omega$ model captures a large scale vortex rope, which cannot simulate self-induced unsteady motions of the “vortex breakdown” and results in the above poor results. Both the OST model and the MST model capture the “vortex breakdown”, and the latter captures more detailed vortical structures.

The average turbulent viscosity ratio and damping function on the plane of $z/D=1.0$ are shown in Table 1. Compared with the OST model, the damping function of the MST model is reasonably reduced, which effectively adjusts the turbulent viscosity and helps to capture more turbulent...
information. The switch between URANS and LES-like modes is more efficient for the MST model, thereby allowing more turbulent vortices and fluctuations to be captured on the same grids.

Table 1. Comparison of the average turbulent viscosity ratio and damping function

<table>
<thead>
<tr>
<th>$z/D=1.0$</th>
<th>Viscosity Ratio $\mu_i/\mu$</th>
<th>Damping Function $D_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MST</td>
<td>2.83</td>
<td>0.4</td>
</tr>
<tr>
<td>OST</td>
<td>52.72</td>
<td>0.1</td>
</tr>
<tr>
<td>SST</td>
<td>177.39</td>
<td>1.0</td>
</tr>
</tbody>
</table>

3.2 Flow around a hydrofoil

Flow around a hydrofoil is a classical case possessing many dynamic phenomena such as flow transition and vortex shedding, which is of great significance to analyze the flow field in the impeller and guide-vane domains in hydraulic rotating machineries. The computation domain of a NACA0009 hydrofoil is shown in Fig. 7, corresponding to the experiment [37]. For this flow case, the Reynolds number based on the chord length $L$ and the free-stream velocity is approximately $2 \times 10^6$. A 2D flow obstacle made up of glue and sand combination is placed near the leading edge, which is designated as the “tripped transition” in the experiment [38].

As for the meshing scheme, high quality hexahedral grids are adopted to discretize the domain. The discretization mesh is consistent with those reported in literatures [39-40]. In terms of boundary conditions, the velocity inlet condition is used according to the chord Reynolds number, and a medium inlet turbulence intensity (5%) is also estimated. The pressure outlet condition is adopted, and no-slip condition is employed for wall surfaces. The symmetry boundary is also used to reduce the computation cost. The key parameter $\Delta t_m$ of the OST model is set to 0.001s. Transient simulation is carried out, and the time step is selected to meet the CFL condition. The computation time is not less than 6 times the through-flow period (the period required by the mean flow to pass through the total domain once), and the results below are obtained by using the time-weighted average of the last 3 periods. Moreover, the classical $\gamma$-Reθ transition model is activated for the SST and MST model, both of which are based on the $k$-$\omega$ model and ready to be combined with the transition model in engineering computations.

Figure 7. Computation domain and local coordinate frame of the hydrofoil
Figure 8. Mean and RMS values of streamwise velocity

(a) EXP
The distributions of the mean and RMS values of streamwise velocity $V_x$ on three measured lines in the $xoy$ plane, corresponding to the $(x-L)/h$ values of 1.0–3.0, are shown in Fig. 8. The SST $k-\omega$ model gives poor results of mean and RMS velocity profiles, and little turbulent fluctuation information is captured. The OST model gives better results, but some deviations still exist in the predictions of RMS velocity profiles. By contrast, the accuracy of the MST model is significantly improved. In addition, a comparison of the vortex shedding frequency is shown in Table 2. The relative errors of vortex shedding frequency obtained by the MST and OST models are within 2%, while the value obtained by the SST model is much higher than that of the experiment.

As shown in Fig. 9, the vortical structures are visualized by the isosurfaces of Liutex criterion. According to the experiment observation, “the vortices essentially exhibit a parallel vortex shedding mode for the tripped transition” [37]. The features of Karman vortex street obtained by MST model are closer to the experimental image. It proves again that the expected improvements by MST.

Table 2. Comparison of the vortex shedding frequency

<table>
<thead>
<tr>
<th>Vortex shedding frequency / (Hz)</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXP 1078</td>
<td>-</td>
</tr>
<tr>
<td>MST 1058.16</td>
<td>1.84%</td>
</tr>
<tr>
<td>OST 1092.14</td>
<td>1.31%</td>
</tr>
<tr>
<td>SST 1354.62</td>
<td>25.66%</td>
</tr>
</tbody>
</table>

3.3 Flow past a cylinder

Flow past a cylinder is also a classical case possessing many important phenomena and has been intensively investigated [41-43]. In this study, the computation domain of a circular cylinder is shown in Fig. 10, corresponding to the experiment [41]. For this flow case, the Reynolds number
based on the diameter $D$ and the free-stream velocity is approximately $1.4\times10^5$.

High quality hexahedral grids are adopted to discretize the domain, and the discretization mesh is consistent with those reported in literatures [42,44]. The spanwise extension is chosen to be $z=D$. As for boundary conditions, the velocity inlet condition is used according to the Reynolds number, and a low inlet turbulence intensity (1%) is also estimated. The pressure outlet condition is adopted, and the no-slip condition is employed for the wall surfaces. Two symmetry boundary conditions are applied far away from the cylinder ($S_1$ & $S_2$), and the translational periodicity condition is employed in the spanwise direction. The key parameter $t_m$ of the OST model is set to 0.001s. Transient simulation is carried out, and the time step is selected to meet the CFL condition. The computation time is not less than 12 times the through-flow period (the period required by the mean flow to pass through the total domain once), and the results below are obtained by means of the time-weighted average of the last 6 periods. The classical $\gamma$-$Re_{\theta}$ transition model is also activated for the SST and MST models.

![Graphs showing comparisons of the vortex shedding frequency and flow separation position](image)

(Table 3. Comparisons of the vortex shedding frequency and flow separation position)

<table>
<thead>
<tr>
<th></th>
<th>Vortex shedding frequency / (Hz)</th>
<th>Relative error</th>
<th>Flow separation position / (°)</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXP</td>
<td>38.47</td>
<td>-</td>
<td>77</td>
<td>-</td>
</tr>
<tr>
<td>MST</td>
<td>38.33</td>
<td>0.36%</td>
<td>78.76</td>
<td>2.29%</td>
</tr>
<tr>
<td>OST</td>
<td>44.99</td>
<td>16.95%</td>
<td>83.11</td>
<td>7.94%</td>
</tr>
<tr>
<td>SST</td>
<td>49.98</td>
<td>29.92%</td>
<td>81.66</td>
<td>6.05%</td>
</tr>
</tbody>
</table>

The distributions of dimensionless mean streamwise velocity $V_s/V_0$, mean normal velocity $V_y/V_0$ and RMS Reynolds stress $\nu_s V_s/V_0^2$ on the measured line ($x=D$) are shown in Fig. 11. It is observed that the accuracy of MST model is better than the OST and SST models. In addition, the comparisons of the vortex shedding frequency and flow separation position are shown in Table 3.
The relative error of vortex shedding frequency obtained by the MST model is within 0.5%, while the values obtained by the OST and SST models are much higher. The relative error of flow separation position obtained by MST model is approximately 2%, which is the lowest and acceptable for engineering computations. Moreover, as shown in Fig. 12, the vortical structures are visualized by the isosurfaces of Liutex criterion. More detailed structures of vortex street are captured by the MST model.

![Vortex street visualized by Liutex isosurfaces](image)

(a) MST  (b) OST  (c) SST

Figure 12. Vortex street visualized by Liutex isosurfaces (Liutex=300s⁻¹)

### 3.4 Taylor-Couette flow

Taylor-Couette flow is one of the paradigmatic systems of the physics of fluids, which is characterized by the flow between two coaxial and independently rotating cylinders. The computation domain used for this flow case is shown in Fig. 13. The inner cylinder is rotating at a constant circumferential velocity, and the outer cylinder is at rest. The Reynolds number based on the inner radius \( R_1 \), outer radius \( R_2 \) and circumferential velocity of inner wall \( U_0 \) is 8000, and the Taylor number is \( 6.4 \times 10^7 \).

High quality hexahedral grids consistent with literatures [45-46] are adopted to discretize the domain. The periodic conditions are employed in the axial direction, and the no-slip conditions are used for the two cylinder surfaces. The key parameter \( t_m \) of the OST model is set to 16s.

The distribution of RMS circumferential velocity is shown in Fig. 14. Again, the accuracy of MST model is significantly better than the OST and SST models, capturing more turbulent fluctuations.

![Computation domain of TC flow](image)

Figure 13. Computation domain of TC flow

![RMS circumferential velocity](image)

Figure 14. RMS circumferential velocity

### 3.5 Flow in a Centrifugal Pump Impeller

Turbulent flow in a centrifugal pump impeller is a typical flow case of hydraulic rotating machineries, which is strongly affected by the effects of rotation and curvature. The computation
domain adopted in this study is shown in Fig. 15, corresponding to the experiment [47-48]. The Reynolds number based on the impeller inlet diameter $D_1$ and the rated flowrate $Q_0$ is approximately $5.5 \times 10^4$. The circumferential velocity of impeller outlet $U_2$ based on the impeller outlet diameter $D_2$ and the rotating speed $n$ is approximately 7.21 m/s.

High quality hexahedral grids are adopted to discretize the domain, and the number of grids are approximately 1.60 million, which is determined on the basis of previous simulations of this flow case [48-49]. As for boundary conditions, the mass flowrate inlet condition with a low turbulence intensity is employed, while the pressure outlet condition and the no-slip wall condition are adopted.

Under the $1.0Q_0$ condition, blade-to-blade distributions of the mean relative circumferential velocity $U_0/U_2$ (0.5-spanwise height), corresponding to the streamwise coordinate $D/D_2$ values of 0.5 and 0.75, are shown in Fig. 16. Compared with the experimental data in two adjacent flow passages [47], the MST model gives a reasonably accurate prediction.

![Figure 15. Computation domain of the centrifugal pump impeller](image)

![Figure 16. Mean relative circumferential velocity](image)

Velocity contours in the impeller (0.5-spanwise height) are shown in Fig. 17. Compared with the experimental results in two adjacent flow passages [47,50], typical features of velocity distribution are well predicted by the MST model. For example, the low speed areas caused by rotation instability near the blade pressure surfaces are captured, and the high speed areas near the blade suction surfaces and impeller outlet are resolved correctly. Moreover, under the $0.25Q_0$ condition, the important phenomenon, “alternate stall”, is also well captured by the MST model, and the prediction results of stall cells are in agreement with those simulated by the LES method [48-49]. Again, it proves that the MST model is suitable for the computations of turbulent flows in the impeller, a core component of hydraulic rotating machineries.
In summary, according to the test results of the above five classical flow cases, the MST model is more suitable for the numerical simulation of turbulent flows with rotation and curvature than the original STRUCT method, and it greatly improves engineering computations of turbulent flows in hydraulic rotating machineries.

4 Conclusions

In this study, a modified STRUCT model (MST) for the turbulent flows with rotation and curvature is proposed, and the conclusions are drawn as follows.

(1) The MST model, using the BSL $k-\omega$ model with the Spalart-Shur correction as the baseline, switches between URANS and LES-like modes by means of a new damping function $D_f$ to adjust the turbulent viscosity $\mu$. The new damping function $D_f$ introduces the normalized helicity $H_n$ to detect the energy backscatter phenomenon, and adopts a new adaptive timescale ratio $R_t$ to avoid the arbitrariness of geometric averaging operation in the original STRUCT method.

(2) Comprehensive tests are carried out using five classical high Reynolds number flow cases (turbulent swirling flow through an abrupt axisymmetric expansion, flow around a hydrofoil, flow past a cylinder, Taylor-Couette flow and flow in a centrifugal pump impeller), and they confirm that the turbulent viscosity $\mu$ of the MST model is reasonably reduced in the massively separated regions, and the LES-like mode is activated, which allows more turbulent fluctuations and vortical structures to be captured on the same URANS grids.

(3) The MST model improves the prediction accuracy of turbulent flow fields with rotation and
curvature, while inherits the advantages of the original STRUCT method (high efficiency and robustness). It provides a good foundation for efficient engineering computations of turbulent flows in hydraulic rotating machineries.

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