Input-Delay Estimation for a Class of Affine Dynamical Systems Based on Taylor Expansion

***Abstract*—Time delay is an important factor that degrades the performance of control systems in practice. While there are many existing results addressing the control problem of dynamical systems with known input delay or unknown delay but with conservative conclusions, how to effectively estimate unknown input delay is still a challenging problem. In this paper, we propose a novel method based on the Taylor expansion for the estimation of general input delay for a class of affine dynamical systems. Under mild conditions, the proposed method guarantees the asymptotic convergence of the estimation error to zero. Illustrative simulation examples are given to validate the theoretical result and the performance of the proposed method. In addition, an application to the input delay estimation of a continuous stirred tank reactor system is also presented to further show the effectiveness of the proposed method and its potential in practical systems.**

***Index Terms*—Input-delay, estimation, Taylor expansion, affine dynamical system.**

I. Introduction

As an important factor that could severely degrade the performance of dynamical systems, the study of time delay has attracted a large amount of attention in recent years. For dynamical systems, related investigations mainly includes stability analysis of systems with time delay and controller design with robustness to time delay [1]–[5]. Time delay can exist in state variables [6] or input variables [7] of a dynamical system. As pointed out in [8], it is more challenging for design with input delay than that with state delay, as the input usually can be fed with non-delayed state information to dominate the delayed states for a system with state delay but no input delay. In this paper, we focus on the latter.

Many results have been reported for the case with known delay in the input variable of dynamical systems. A classical approach is to address this problem is the Smith predictor [9], for which the value of time delay is needed to facilitate the computation of predictions. Pyrkin and Bobtsov [7] addressed the output-feedback stabilization problem of linear systems with known and constant input delay and unknown harmonic disturbance. The method does not require the systems to be minimum-phase and the degrees of the systems are allowed to be arbitrary. Sanz *et al.* [10] proposed a robust control method for a nonlinear system with known and constant total delay by using a state predictor. Na *et al.* [36] addressed the tracking control problems of pure feedback systems subject to input delay, where a high-order neural network observer is used to perform system state prediction. Furtat *et al.* [12] proposed a method for compensation of unknown bounded smooth disturbances for linear time invariant systems with constant and known input delay, for which a novel disturbance prediction method was introduced based on current and delayed values of the disturbance. Lei and Khailil [13] proposed a novel method to address the feedback linearization of a single-input single-output nonlinear system with known time-varying input and output delay, where a high-gain observer serves as a predictor. Some relavant results were also reported for multi-agent systems. For example, Wang *et al.* [14] proposed a method to address the consensus of Lipschitz nonlinear multi-agent systems with known input delay, in which sufficient conditions for global consensus were provided. However, without knowing the value of input delay, the above methods cannot work.

Recently, efforts have also been devoted to the case with unknown delay. Léchappé *et al.* [15] proposed a novel predictive control scheme for linear time-invariant systems with parameter uncertainty, external disturbance, and unknown input delay. The scheme requires that the delayed value of input is available and the the estimation quality of input delay depends on the richness of the input signal. Delphine *et al.* [16] proposed an adaptive backstepping controller for linear systems with unknown input delay. Pade approximation is one of the widely used approaches to address this case. Based on Pade approximation, Li *et al.* [17] proposed an adaptive fuzzy backstepping controller for a strict-feedback nonlinear system with input delay. Pade approximation was also adopted by Li *et al.* [18] to address the adaptive control of strict-feedback nonlinear systems with state constraints and input delay. However, as claimed in [17], [18], Pade approximation only applies to small delay. Meanwhile, due to the existence of the approximation error, asymptotic stability of the controlled system is generally difficult to guarantee. Bresch-Pietri *et al.* [19] proposed a framework for the estimation of input delay of linear systems, which requires the initial estimation error to be small enough. Obuz *et al.* [20] proposed a tracking controller for a class of nonlinear systems with unknown slowly time-varying input delay and additive disturbances. The method requires that a sufficiently accurate constant estimate of input delay is available.

Based on existing literature, it is found that a computationally efficient method for accurate input delay estimation is demanded, which motivates our current research. In this paper, we propose a method to address the issue and the method only requires knowing the bound of input delay. As a primary work, we consider the case with constant input delay, and the case with time-varying input delay will be our future work. It should be noted that when input delay is slowly time-varying, it can be addressed by methods for constant input delay. The current work is based on Taylor expansion. In our previous work, Taylor expansion was introduced to facilitate the controller design for the near-optimal control of affine nonlinear systems without input delay [21]–[23]. Different from [21]–[23], in this paper, Taylor expansion is used to facilitate the input delay estimation. Since the accuracy of the expansion depends on the residual term, an additional constraint on the second-order derivative of the input is introduced. Based on the above steps, an auxiliary system is introduced for the estimation. The performance of the method is theoretically analyzed and validated via simulation examples. We also provide an application of the method to a continuous stirred tank reactor system. The contributions of this work mainly include the following.

1) Input delay captures the fact of actuation latency relative to system output. it widely exists in practice, e.g., engine combustion control, process control, but largely ignored in theory. This work presents a theoretically provable adaptive scheme that can learn unknown time delay in runtime and penetrates the power of adaptive control from parameter adjustment to transmission delay estimation.
2) Without delay estimation, conventional methods rely on the range of time delay to draw a conservative conclusion. With the presented solution, improved control performance can be achieved by including the estimated time delay in the feedback loop.

The reminder of the paper is organized as follows. In Section II, the problem investigated in this paper is described. In Section III, the proposed method is illustrated with the corresponding theoretical results given, followed by the corresponding simulation validations given in Section IV. In Section V, an application to the input delay estimation of a continuous stirred tank reactor system is shown to further show the effectiveness of the proposed method and its potential in practical systems. Then, conclusions are given in Section VI.

II. Problem Description

In this section, the problem investigated in this paper is illustrated in details.

Consider the following affine dynamical system:

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t)) + g(\mathbf{x}(t))\mathbf{u}(t-\tau), \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$ denotes the state variable, t denotes the time variable, $\dot{\mathbf{x}} = \mathrm{d}\mathbf{x}/\mathrm{d}t$, $\mathbf{u} \in \mathbb{R}^m$ denotes the input variable, and $\tau > 0 \in \mathbb{R}$ denotes unknown constant input delay. Functions $f(\mathbf{x}) \in \mathbb{R}^n$ and $g(\mathbf{x}) \in \mathbb{R}^{n\times m}$ are continuously differentiable, with $g(\mathbf{x})$ being column full-rank and $n \geq m$. In this paper, we are interested in the estimation of input delay τ via the measurement of state variable $\mathbf{x}$. Let $\hat{\tau}(t)$ denote the estimation of τ at time t. Our goal is to make $\hat{\tau}$ asymptotically converges to τ, i.e., $\lim_{t\to+\infty}(\tau - \hat{\tau}(t)) = 0$.

Many physical systems can be modeled as an affine dynamical system, such as such as four-bar linkage systems [24], rotational/translational actuators [25], continuous stirred tank reactors [26], robot manipulators [27], and DC motors [15]. When controllers and systems are connected through networks, the controller signals are transmitted through a network. As a result, network-induced delay causes input delay formulated in (1) [28].

For input delay τ, we have the following assumption.

Assumption 1: The unknown constant input delay τ satisfies $\tau_{\text{min}} \leq \tau \leq \tau_{\text{max}}$, where $\tau_{\text{min}} > 0 \in \mathbb{R}$ and $\tau_{\text{max}} > 0 \in \mathbb{R}$ are two known constants.

Regarding Assumption 1, we have the following remark.

Remark 1: In practice, although we may not known the exact value of input delay, input delay are generally bounded. A very simple way is to set the value of τ_{max} to be extremely large and the value of τ_{min} to be extremely small but larger than zero. However, as what happens in adaptive control, if we have a better knowledge about τ_{min} and τ_{max}, faster parameter convergence can be achieved. Note that Assumption 1 was also adopted in relevant literature [15], [19].

III. Proposed Method

In this section, a novel method based on Taylor expansion and an auxiliary system is proposed for the input-delayed affine dynamical system stated above.

A. Taylor Expansion

According to Taylor expansion, we have

$$\mathbf{u}(t-\tau) = \mathbf{u}(t-\hat{\tau}) + (\hat{\tau}-\tau)\dot{\mathbf{u}}(t-\hat{\tau}) + \frac{(\hat{\tau}-\tau)^2}{2}\ddot{\mathbf{u}}(\chi),$$

where $\chi \in \mathbb{R}$ lies between $t-\tau$ and $t-\hat{\tau}$. Substituting the above equation into system (1) yields

$$\begin{aligned}\dot{\mathbf{x}}(t) =& f(\mathbf{x}(t)) + g(\mathbf{x}(t))\Big(\mathbf{u}(t-\hat{\tau}) + (\hat{\tau}-\tau)\dot{\mathbf{u}}(t-\hat{\tau}) \\ &+ \frac{(\hat{\tau}-\tau)^2}{2}\ddot{\mathbf{u}}(\chi)\Big).\end{aligned} \tag{2}$$

Given that the maximal Euclidean norm of $\ddot{\mathbf{u}}$, which is denoted by $\|\ddot{\mathbf{u}}\|_{\text{max}}$, satisfies the following equation:

$$\frac{(\hat{\tau}-\tau)^2}{2}\|\ddot{\mathbf{u}}(\chi)\|_{\text{max}} \leq \frac{d_0}{\|g(\mathbf{x}(t))\|_2}, \tag{3}$$

where $d_0 > 0 \in \mathbb{R}$ is small enough, the dynamics of the system is dominated by the first two terms. In inequality (3), $\|\cdot\|_2$ denotes the Euclidean norm and $|\cdot|$ denotes the absolute value. Regarding (3), it can be realized by the following way. The input signal to the system is given at the acceleration level, i.e., through $\ddot{\mathbf{u}}$. To achieves (3), we can make

$$\ddot{\mathbf{u}} = h(\dot{\mathbf{u}}, \mathbf{u}, \mathbf{x}, t)/\varepsilon, \tag{4}$$

where $h(\dot{\mathbf{u}}, \mathbf{u}, \mathbf{x}, t)$ is a function and parameter ε is given as follows:

$$\varepsilon \geq \frac{\|h(\dot{\mathbf{u}}, \mathbf{u}, \mathbf{x}, t)\|_2(\hat{\tau}-\tau)^2\|g(\mathbf{x})\|_2}{2d_0},$$

which together with Assumption 1 leads to

$$\varepsilon \geq \frac{\|h(\dot{\mathbf{u}}, \mathbf{u}, \mathbf{x}, t)\|_2 \max\{(\hat{\tau}-\tau_{\text{min}})^2, (\hat{\tau}-\tau_{\text{max}})^2\}\|g(\mathbf{x})\|_2}{2d_0}. \tag{5}$$

In the implementation of the proposed method, the value of ε can be set according to equation (5).

B. Auxiliary System Design

Based on equation (2), to estimate the value of input delay τ, we have the following auxiliary system:

$$\begin{aligned}\dot{\hat{\mathbf{x}}}(t) =& f(\mathbf{x}(t)) + g(\mathbf{x}(t))\mathbf{u}(t-\hat{\tau}(t)) - \kappa_1 \text{sgn}(\hat{\mathbf{x}}(t)-\mathbf{x}(t)),\\ \dot{\hat{\tau}}(t) =& \kappa_2 \dot{\mathbf{u}}^{\text{T}}(t-\hat{\tau}(t)) g^{\text{T}}(\mathbf{x}(t))(\hat{\mathbf{x}}(t)-\mathbf{x}(t)),\end{aligned} \tag{6}$$

where $\hat{\mathbf{x}}$ is the state variable of the auxiliary system; $\kappa_1 > 0 \in \mathbb{R}$ and $\kappa_2 > 0 \in \mathbb{R}$ are design parameters; $\text{sgn}(\cdot)$ is the sign function for which

$$\text{sgn}(y) = \begin{cases} 1 \text{ if } y > 0, \\ 0 \text{ if } y = 0, \\ -1 \text{ if } y < 0. \end{cases}$$

Intuitively, in the design of the auxiliary system, the co-state $\hat{\mathbf{x}}$ is used to capture the influence of delayed input $u(t-\hat{\tau})$ on the state of the system.

Based on (2), it can by easily found that, when the input delay is accurately estimated and the co-state $\hat{\mathbf{x}}$ converges to $\mathbf{x}$, i.e., $\hat{\tau}(t) = \tau$ and $\hat{\mathbf{x}}(t) = \mathbf{x}(t)$, the first equation of (6) reduces to (1). Although the accurate input delay τ is unknown, we cannot directly measure the difference between $\hat{\tau}(t)$ and τ. However, the measurement for the difference between $\hat{\mathbf{x}}(t)$ and $\mathbf{x}(t)$ is available. As seen from (6), the auxiliary system is totally driven by the difference between $\hat{\mathbf{x}}(t)$ and $\mathbf{x}(t)$, which is further driven by the difference between $\hat{\tau}(t)$ and τ. Regarding the two parameters κ_1 and κ_2, to ensure that the change of $\hat{\tau}$ takes an dominant role in the dynamics of the auxiliary system, we need to set the value of κ_1 to be much larger than that of κ_2.

C. Theoretical Analysis

In this subsection, theoretical results about the proposed method are given to guarantee its performance in addressing the input-delayed affine dynamical system (1). About the performance of the auxiliary system, we have the following theorem.

Theorem 1: Given that $\kappa_1 > d_0 > 0$ and the second-order derivative of input u satisfies (3), the state variable $\hat{\mathbf{x}}$ of auxiliary system (6) and the state variable $\mathbf{x}$ of system (1) satisfies $\lim_{t\to+\infty}(\hat{\mathbf{x}}(t)-\mathbf{x}(t)) = 0$.

Proof: Since system (1) is equivalent to system (2), we only need to prove that the state variable $\hat{\mathbf{x}}$ of auxiliary system (6) asymptotically converges to $\mathbf{x}$ of system (2). Let $\tilde{\mathbf{x}}(t) = \hat{\mathbf{x}}(t) - \mathbf{x}(t)$ and $\tilde{\tau}(t) = \hat{\tau}(t) - \tau$. From equations (6) and (2), we have

$$\begin{aligned}\dot{\tilde{\mathbf{x}}}(t) =& -g(\mathbf{x}(t))\tilde{\tau}(t)\dot{u}(t-\hat{\tau}(t)) - g(\mathbf{x}(t))\frac{\tilde{\tau}^2(t)}{2}\ddot{u}(\chi)\\ &- \kappa_1 \text{sgn}(\tilde{\mathbf{x}}(t)),\\ \dot{\tilde{\tau}}(t) =& \kappa_2 \dot{\mathbf{u}}^{\text{T}}(t-\hat{\tau}(t)) g^{\text{T}}(\mathbf{x}(t))\tilde{\mathbf{x}}(t).\end{aligned} \tag{7}$$

Consider the following Lyapunov candidate function:

$$V(t) = \frac{1}{2}\tilde{\mathbf{x}}^{\text{T}}(t)\tilde{\mathbf{x}}(t) + \frac{1}{2\kappa_2}\tilde{\tau}^2(t).$$

Evidently, $V(t) \geq 0$, $\forall t > 0$, and $V(t) = 0$ only when $\tilde{\mathbf{x}}(t) = 0$ and $\tilde{\tau}(t) = 0$. The derivative of $V(t)$ along the state trajectory of system (7) is calculated as follows:

$$\begin{aligned}\dot{V}(t) &= \tilde{\mathbf{x}}^{\text{T}}(t)\dot{\tilde{\mathbf{x}}}(t) + \frac{1}{\kappa_2}\tilde{\tau}(t)\dot{\tilde{\tau}}(t)\\ &= \tilde{\mathbf{x}}^{\text{T}}(t)\bigg(-g(\mathbf{x}(t))\tilde{\tau}(t)\dot{\mathbf{u}}(t-\hat{\tau}) - g(\mathbf{x}(t))\frac{\tilde{\tau}^2(t)\ddot{u}(\chi)}{2}\\ &\quad - \kappa_1\text{sgn}(\tilde{\mathbf{x}}(t))\bigg) + \tilde{\tau}(t)\dot{\mathbf{u}}^{\text{T}}(t-\hat{\tau}(t))g^{\text{T}}(\mathbf{x}(t))\tilde{\mathbf{x}}(t)\\ &= -\tilde{\mathbf{x}}^{\text{T}}(t)g(\mathbf{x}(t))\frac{\tilde{\tau}^2(t)\ddot{\mathbf{u}}(\chi)}{2} - \kappa_1\tilde{\mathbf{x}}^{\text{T}}\text{sgn}(\tilde{\mathbf{x}}(t))\\ &= -\tilde{\mathbf{x}}^{\text{T}}(t)g(\mathbf{x}(t))\frac{\tilde{\tau}^2(t)\ddot{\mathbf{u}}(\chi)}{2} - \kappa_1\|\tilde{\mathbf{x}}\|_1\\ &\leq \|\tilde{\mathbf{x}}(t)\|_2\|g(\mathbf{x}(t))\|_2\frac{\tilde{\tau}^2(t)\|\ddot{\mathbf{u}}(\chi)\|_{\max}}{2} - \kappa_1\|\tilde{\mathbf{x}}\|_1,\end{aligned}$$

where $\|\cdot\|_1$ denotes the 1-norm. Together with inequality (3), we further have

$$\dot{V}(t) \leq d_0\|\tilde{\mathbf{x}}(t)\|_2 - \kappa_1\|\tilde{\mathbf{x}}(t)\|_1.$$

Note that, for any $\tilde{\mathbf{x}}$, we have $\|\tilde{\mathbf{x}}(t)\|_2 \leq \|\tilde{\mathbf{x}}(t)\|_1$ (pp. 53 of [29]). As a result,

$$\dot{V}(t) \leq d_0\|\tilde{\mathbf{x}}(t)\|_1 - \kappa_1\|\tilde{\mathbf{x}}(t)\|_1 = (d_0 - \kappa_1)\|\tilde{\mathbf{x}}(t)\|_1, \tag{8}$$

which, together with $\kappa_1 > d_0$, yields

$$\dot{V}(t) \leq 0.$$

Thus, based on (8) and the definition of $V(t)$, there exists an invariant set at which $\|\tilde{\mathbf{x}}\|_1 = 0$. It follows that $\lim_{t\to+\infty}(\hat{\mathbf{x}}(t) - \mathbf{x}(t)) = 0$. The proof is complete. □

About the above theorem, we have the following remark about its underlying intuition.

Remark 2: According to Theorem 1, we have $\lim_{t\to+\infty}(\hat{\mathbf{x}}(t) - \mathbf{x}(t)) = 0$ under the same input. In other words, the input-to-state response of the auxiliary system (6) is asymptotically equivalent to the considered one, i.e., system (1). Consequently, if we can design a controller for auxiliary system (6) with a fixed constant $\hat{\tau}$ (i.e., excluding the update for $\hat{\tau}$) to achieve certain objective such as output tracking, the controller will also work for system (1) with the update for $\hat{\tau}$. It can be expected that although there may be some differences in the transient behavior, the long-term behavior of the states of the two systems will be almost the same. This kind of design is similar to the model-reference adaptive control, which was shown to be effective in [30], [31] for the case without delay. To sum up, with the proposed design, the control problem of a system with unknown input delay can be addressed by controllers originally designed for the corresponding systems with known input delay.

Apart from the benefits shown in Remark 2, we are also interested in the convergence of $\hat{\tau}(t)$ to τ. About this issue, we have the following theorem.

Theorem 2: Given that, $\forall \varpi \in [\tau_{\min}, \tau_{\max}]$, $\exists k > 0$ and $T > 0$ such that $\int_t^{t+T}((g(\mathbf{x})\dot{\mathbf{u}}(t'-\varpi))^{\text{T}}g(\mathbf{x})\dot{\mathbf{u}}(t'-\varpi))\text{d}t' \geq k$, the state variable $\hat{\tau}(t)$ of auxiliary system (6) satisfies $\lim_{t\to+\infty}(\hat{\tau}(t) - \tau) = 0$, i.e., input delay estimation $\hat{\tau}(t)$ asymptotically converges to actual input delay τ of system (1).

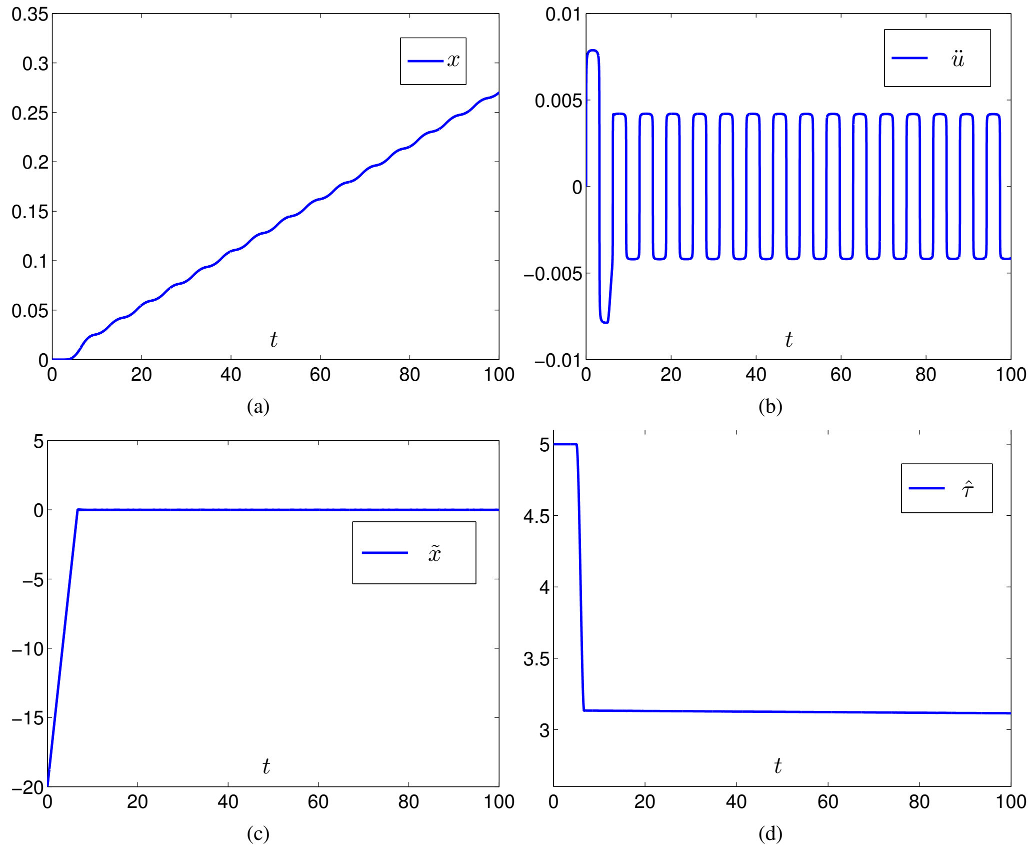

Fig. 1. Data profiles when auxiliary system (6) is used for system (10) with $h(\dot{u},u,\mathbf{x},t)$ in equation (4) set to $h(\dot{u},u,\mathbf{x},t)=\sin(t)$. (a) Profile of state variable $x(t)$ of system (10). (b) Profile of the second-order derivative $\ddot{u}(t)$. (c) Profile of state difference $\tilde{x}(t)=\hat{x}(t)-x(t)$. (d) Profile of state variable $\hat{\tau}(t)$ of auxiliary system (6).

Proof: According to Theorem 1, there exists an invariant set at which $\tilde{\mathbf{x}}(t)=0$ and $\dot{\tilde{\mathbf{x}}}(t)=0$. Together with equations (1) and (6), we have $g(\mathbf{x}(t))\mathbf{u}(t-\tau)-g(\mathbf{x}(t))\mathbf{u}(t-\hat{\tau}(t))=0$. By the mean value theorem, we further have

$$g(\mathbf{x}(t))\dot{\mathbf{u}}(t-\varpi)\tilde{\tau}(t)=0 \tag{9}$$

with $\varpi\in[\tau_{\text{min}},\tau_{\text{max}}]$. It follows from (9) that

$$(g(\mathbf{x}(t))\dot{\mathbf{u}}(t-\varpi))^{\mathrm{T}}g(\mathbf{x}(t))\dot{\mathbf{u}}(t-\varpi)\tilde{\tau}(t)=0$$

Integrating both sides of the above equation, with the fact that $\tilde{\tau}(t)$ is a constant in the invariant set, over a time region $[t,t+T]$, with $T>0$, we have

$$\tilde{\tau}(t)\int_t^{t+T}(g(\mathbf{x})\dot{\mathbf{u}}(t'-\varpi))^{\mathrm{T}}(g(\mathbf{x})\dot{\mathbf{u}}(t'-\varpi))\mathrm{d}t'=0.$$

Consequently, given that, $\forall\varpi\in[\tau_{\text{min}},\tau_{\text{max}}]$, $\exists k>0$ and $T>0$ such that $\int_t^{t+T}((g(\mathbf{x})\dot{\mathbf{u}}(t'-\varpi))^{\mathrm{T}}g(\mathbf{x})\dot{\mathbf{u}}(t'-\varpi))\mathrm{d}t'\geq k$, we have $\tilde{\tau}(t)=0$. It follows that input delay estimation $\hat{\tau}(t)$ asymptotically converges to actual input delay τ of system (1) by LaSalle's invariance principle [33]. The proof is complete. □

About Theorem 2, we have the following remark about how to satisfy the stated condition about input $\mathbf{u}$ in practice.

Remark 3: In practice, to satisfy the PE condition, we can introduce noise into the input channel. Assume we have noise $\omega\in\mathbb{R}^m$ and $\nu\mathbb{R}^m$ such that $\dot{\omega}=\nu$ with ω being zero-mean, i.e., $E(\omega)=0$. Let $\dot{\mathbf{u}}=\dot{\mathbf{u}}_1+k_1\omega$ and $\ddot{\mathbf{u}}=\ddot{\mathbf{u}}_1+k_1\nu$ with k_1 being a small positive number. Then, the condition that $\int_t^{t+T}((g(\mathbf{x})\dot{\mathbf{u}}(t'-\varpi))^{\mathrm{T}}g(\mathbf{x})\dot{\mathbf{u}}(t'-\varpi))\mathrm{d}t'=\int_t^{t+T}((g(\mathbf{x})(\dot{\mathbf{u}}_1(t'-\varpi)+k_1\omega(t'-\varpi)))^{\mathrm{T}}g(\mathbf{x})(\dot{\mathbf{u}}_1(t'-\varpi)+k_1\omega(t'-\varpi)))\mathrm{d}t'\geq k$ can be easily satisfied due to the randomness of ω.

IV. Simulation Validations

In this section, simulative examples are presented to validate the effectiveness of the proposed method.

A. Validation of Theorem 1

We consider the following nonlinear dynamical system:

$$\dot{x}(t)=-4x(t)+0.1x^3+\sin(x)+u(t-\tau), \tag{10}$$

where $\tau=3$. We assume that we only know that $\tau\in[1,10]$, i.e., $\tau_{\text{min}}=1$ and $\tau_{\text{max}}=10$. To verify Theorem 1, $h(\dot{u},u,\mathbf{x},t)$ in equation (4) is set to $h(\dot{u},u,\mathbf{x},t)=\sin(t)$. Without generality, the initial state of system (10) is set to $x(0)=0$. For the auxiliary system (6), the initial states are set to $\hat{x}(0)=-20$ and $\hat{\tau}(0)=5$. The design parameters are set to $d_0=0.1$, $\kappa_1=3$, and $\kappa_2=100$. The reason why we set the value of κ_2 to be much larger than that of κ_1 is to guarantee that the learning of input delay takes a dominant role in forcing the state difference to converges to zero. Meanwhile,

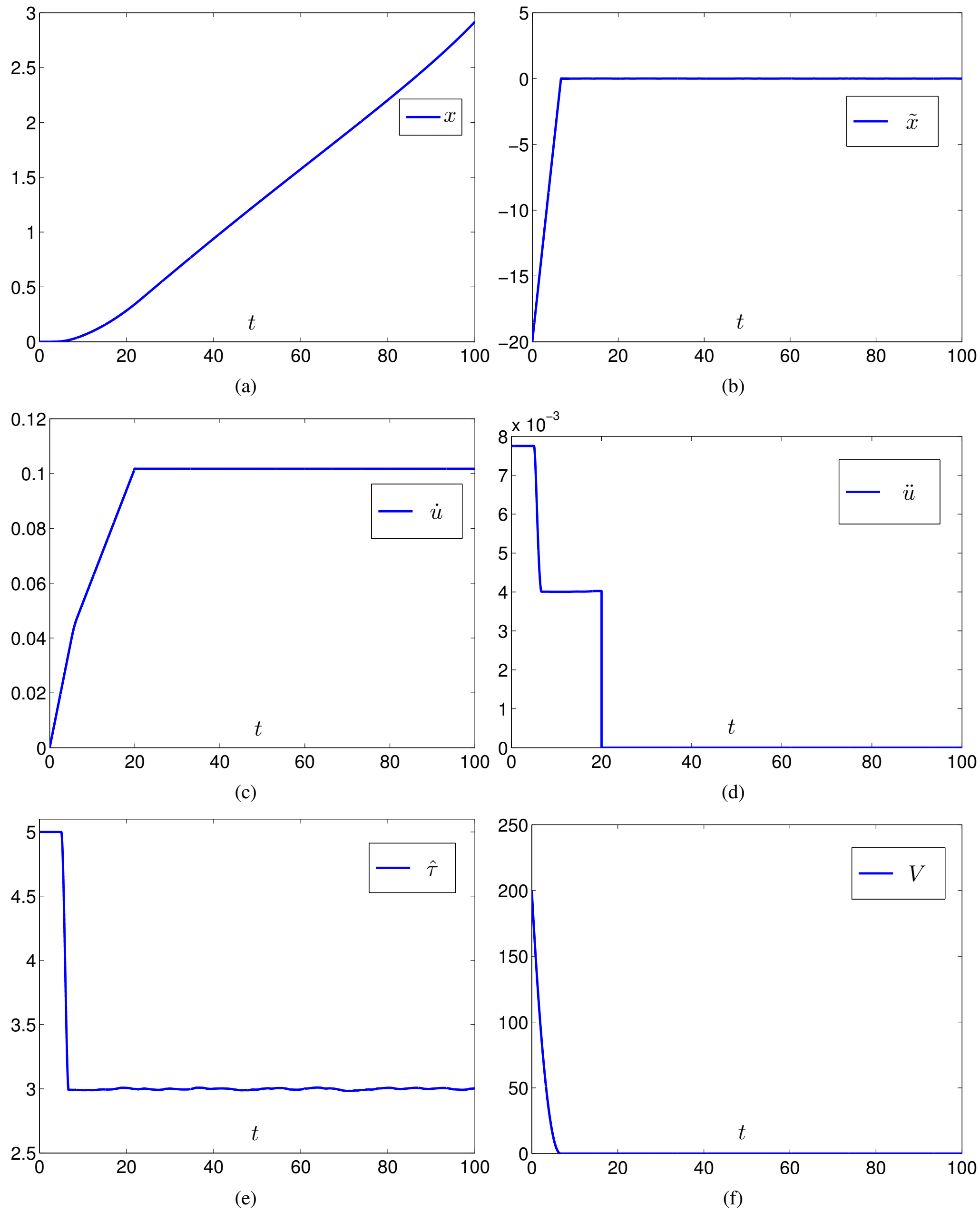

Fig. 2. Data profiles when auxiliary system (6) is used for system (10) with $h(\dot{u}, u, \mathbf{x}, t)$ in equation (4) set as equation (11). (a) Profile of state variable $x(t)$ of system (10). (b) Profile of state difference $\tilde{x}(t) = \hat{x}(t) - x(t)$. (c) Profile of the first-order derivative $\dot{u}(t)$. (d) Profile of the second-order derivative $\ddot{u}(t)$. (e) Profile of state variable $\hat{\tau}(t)$ of auxiliary system (6). (f) Profile of the associated Lyapunov function $V(t)$.

as there are no projection terms added in the auxiliary system (6), the value of κ_2 cannot be too large so as to prevent the value of $\hat{\tau}(t)$ from decreasing to be negative. To guarantee the satisfaction of inequality (5), we simply set

$$\varepsilon = \frac{|h(\dot{u}, u, \mathbf{x}, t)| \max\{(\hat{\tau} - \tau_{\min})^2, (\hat{\tau} - \tau_{\max})^2\} \|g(\mathbf{x})\|_2}{2d_0} + 2.$$

Under the above setup, the simulation is conducted in Simulink, and the data of interest are shown in Fig. 1. As seen from Fig. 1(b), the second-order derivative of input, i.e., $\ddot{u}(t)$, is bounded in a small region. This is due to the effect of ε by referring to equation (4). As seen from Fig. 1(c), the state difference, i.e., $\tilde{x}(t)$, converges to zero with time, which is consistent with Theorem 1. However, as seen from Fig. 1(d), the state variable $\hat{\tau}$ of auxiliary system (6) does not converge to $\tau = 3$. This is due to the fact that $\ddot{u}$ does not satisfy the condition stated in Theorem 2.

B. Validation of Theorem 2

To verify Theorem 2, we further conduct a simulation with different setting of $h(\dot{u}, u, \mathbf{x}, t)$ and the other setups are the same with the aforementioned. Specifically, following Remark 3, $h(\dot{u}, u, \mathbf{x}, t)$ is set as follows:

$$h(\dot{u}, u, \mathbf{x}, t) = \begin{cases} 0.5, & 0 \le t \le 20 \\ 0, & t > 20. \end{cases} \tag{11}$$

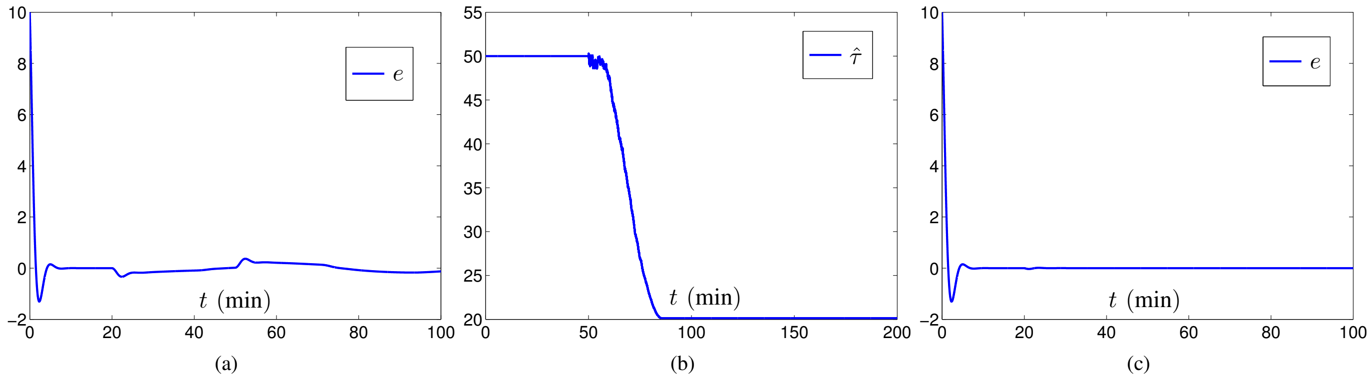

Fig. 3. Data profiles when the proposed method is combined with Smith predictor. (a) Regulation error $e(t)$ when Simith predictor enhanced PI controller is applied to system (12) with $\hat{\tau}=50$. (b) Delay estimation $\hat{\tau}(t)$ when the proposed method is used for system (12). (c) Regulation error $e(t)$ when Simith predictor enhanced PI controller is applied to system (12) with the obtained delay estimation.

Obviously, under this setting, we have $\lim_{t\to+\infty}\ddot{u}(t)=0$ and $\lim_{t\to+\infty}\dot{u}(t)>0$, by which the conditions stated in Theorem 2 are satisfied. As seen from Fig. 2(b), the state difference $\tilde{x}(t)$ converges to zero with time. This coincides with Theorem 1. As seen from Fig. 2(c) and Fig. 2(d), under such setting, the second-order derivative of input, i.e., $\ddot{u}(t)$, converges to zero and the first-order derivative of input, i.e., $\dot{u}(t)$, converges to a positive constant, which satisfy the conditions stated in Theorem 2. Besides, Fig. 2(e) shows that $\hat{\tau}(t)$ converges to 3, i.e., τ, and Fig. 2(f) shows that the associated Lyapunov function $V(t)$ converges to zero. These results coincide with Theorem 2.

C. Combination with Smith predictor

To validate the potential of the proposed method in the usage of Smith predictor [34], simulations are also conducted. Consider the following system:

$$\dot{x}=-x+u(t-\tau), \tag{12}$$

with $\tau=20$. Suppose we only know that $\tau\in[1,100]$. We first use the proposed method to estimate its input delay and then use Simith predictor enhanced proportional-integral (PI) controller to drive the state of the system to converge to $x_{\text{d}}=10$. During the estimation process, we set $h(\dot{u},u,\mathbf{x},t)$ in equation (4) to $h(\dot{u},u,\mathbf{x},t)=0.2e(t)+0.2\int_0^t e(\chi)\text{d}\chi$, where $e(t)=x_{\text{d}}(t)-x(t)$ denotes the regulation error. To fulfill the requirement of Theorem 2, we inject zero-mean white noise with noise power 10×10^{-6} to the auxiliary system. The relevant parameters are set to $\kappa_1=1$, $\kappa_2=26$, and $d_0=0.2$. For the PI controller, the proportional parameter is set to 0.6 and the integral parameter is set to 2. For the Smith predictor, the time constant of the filter dynamics is set to 20. As seen from Fig. 3, with the same setting for the PI controller, when delay estimation obtained by the proposed method, i.e., $\hat{\tau}=20.14$ is adopted in the Smith predictor, the regulation error is larger than the case with a rough estimation of input delay, i.e., $\hat{\tau}=50$. The result shows the significance of the proposed method and validate the theoretical results.

TABLE I
VALUES OF PARAMETERS IN CSTR SYSTEM (13)

Parameter	Value	Parameter	Value
V	100 L	q	100 L$/min$
C_{Af}	1 mol/L	K_0	7.2×10^10 1/min
E/R	8750 K	ρ	1000 g/L
C	0.239 g/(LK)	T_{f}	350 K
$-\Delta H$	5.0×10^4 J/min	U	5.0×10^4 J/(min K)
τ	0.2 min		

V. APPLICATION TO INPUT DELAY ESTIMATION OF CSTR SYSTEM

To further show the effectiveness of the proposed method, in this section, we present an application to the estimation of input delay of a continuous stirred tank reactor (CSTR) system, where a first-order reaction A→B occurs. The dynamics of the system is described as follows [35]–[37]:

$$\begin{aligned}V\dot{C}_{\text{A}}(t)&=q(C_{\text{Af}}-C_{\text{A}}(t))-VK_0\exp\left(\frac{-E}{RT(t)}\right)C_{\text{A}}(t),\\ V\rho C\dot{T}(t)&=q\rho C(T_{\text{f}}-T(t))+V(-\Delta H)K_0\exp\left(\frac{-E}{RT(t)}\right)\\ &\quad\times C_{\text{A}}(t)-U(T(t)-T_{\text{c}}(t-\tau)),\end{aligned} \tag{13}$$

where V denotes the reactor volume; $C_{\text{A}}(t)$ and C_{Af} denote the effluent concentration and the feed concentration, respectively; q denotes the feed flow rate; K_0 denotes the reaction velocity constant; E/R denotes the ratio of Arrhenius activation energy to the gas constant; $T(t)$ and T_{f} denote the reactor temperature and the feed temperature, respectively; ρ denotes the density; C denotes the specific heat; $-\Delta H$ denotes the heat of reaction; U denotes heat transfer coefficient of the reactor surface area; $T_{\text{c}}(t)$ denotes the coolant temperature in the reactor cooling coil, which is the input variable of the CSTR system. The units and values of the parameters are shown in Table I [35], [36]. Evidently, CSTR system (13) can be described by affine dynamical system (1) with $\mathbf{x}=[C_{\text{A}},T]$ and $u=T_{\text{c}}$. The difference of this system from the system in the above simulation is that this is a physical system and the values of the variable cannot be extremely large or small.

We use the proposed method to perform delay estimation for the CSTR system. The initial state of the CSTR system is

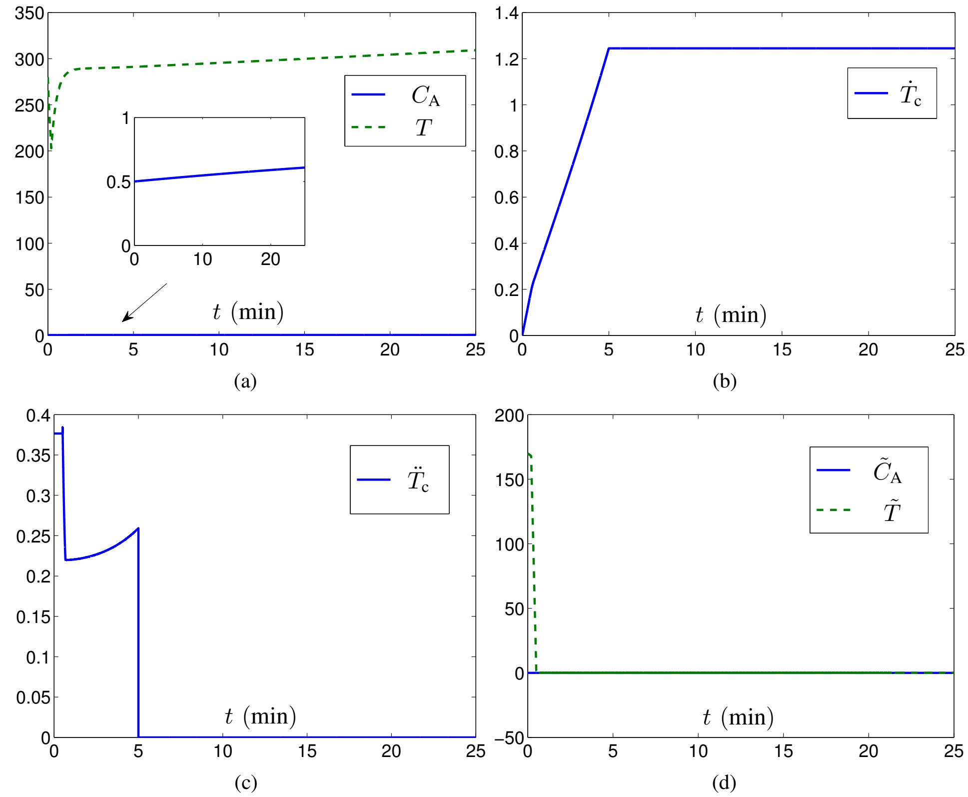

Fig. 4. Data profiles when auxiliary system (14) is used for input delay estimation of CSTR system (13). (a) Profile of effluent concentration $C_{\mathrm{A}}(t)$ and reactor temperature $T(t)$ of CSTR system (10). (b) Profile of the first-order time derivative of coolant temperature in the reactor cooling coil T_{c}, i.e., $\dot{T}_{\mathrm{c}}(t)$. (c) Profile of the second-order derivative of coolant temperature in the reactor cooling coil T_{c}, i.e., $\ddot{T}_{\mathrm{c}}(t)$. (c) Profiles of the state variable difference between CSTR system (13) and auxiliary system (14).

set to $C_{\mathrm{A}}(0) = 0.5$ mol/L and $T(0) = 280$ K. The value of d_0 is set to 0.1. The initial input is set to $T_{\mathrm{c}}(0) = 260$ K. For the CSTR system, the corresponding auxiliary system (6) can be rewritten as follows:

$$\begin{aligned} V\dot{\hat{C}}_{\mathrm{A}}(t) &= q(C_{\mathrm{Af}} - C_{\mathrm{A}}(t)) - VK_0 \exp\left(\frac{-E}{RT(t)}\right) C_{\mathrm{A}}(t) \\ &\quad - \kappa_1 \mathrm{sgn}(\hat{C}_{\mathrm{A}}(t) - C_{\mathrm{A}}(t)), \\ V\rho C\dot{\hat{T}}(t) &= q\rho C(T_{\mathrm{f}} - T(t)) + V(-\Delta H)K_0 \exp\left(\frac{-E}{RT(t)}\right) \\ &\quad \times C_{\mathrm{A}}(t) - U(T(t) - T_{\mathrm{c}}(t - \hat{\tau}(t))) \\ &\quad - \kappa_1 \mathrm{sgn}(\hat{T}(t) - T(t)), \\ \dot{\hat{\tau}}(t) &= \kappa_2 \dot{T}_{\mathrm{c}}(t - \hat{\tau}(t)) \frac{U}{V\rho C}(\hat{T}(t) - T(t)). \end{aligned} \tag{14}$$

In this application, the auxiliary system (14) adopts the following setting: $\kappa_1 = 10$, $\kappa_2 = 1000$, $\hat{C}_A(0) = 0.5$ mol/L, and $\hat{T}(0) = 450$ K. Besides, $h(\dot{u}, u, \mathbf{x}, t)$ in equation (4) is set as follows:

$$h(\dot{u}, u, \mathbf{x}, t) = \begin{cases} 50 \text{ K/min}^2, \ 0 \le t \le 5 \text{ min}, \\ 0, \ t > 5 \text{ min}. \end{cases} \tag{15}$$

The upper bound and lower bound of input delay τ is set to $\tau_{\min} = 0.01$ min and $\tau_{\max} = 1$ min.

The performance of the proposed method in this application is tested in Simulink with the data profiles shown in Fig. 4 and Fig. 5. The units of the variables in the figures follow the corresponding ones in Table I. As seen from Fig. 4, in the delay estimation process, the values of the effluent concentration $C_{\mathrm{A}}(t)$ and reactor temperature $T(t)$ of the CSTR system (13) are in a reasonable region. Meanwhile, the state differences between the variables of CSTR system (13) and corresponding ones of auxiliary system (14) converges to zero. In addition, the first-order time derivative of coolant temperature in the reactor cooling coil T_{c}, i.e., $\dot{T}_{\mathrm{c}}(t)$, converges to a constant value, and the second-order derivative of coolant temperature in the reactor cooling coil T_{c}, i.e., $\ddot{T}_{\mathrm{c}}(t)$ converges to zero, which satisfy the conditions given in Theorem 2. As stated in Theorem 2, when the two conditions are satisfied, the proposed method can be used to estimate input delay. As

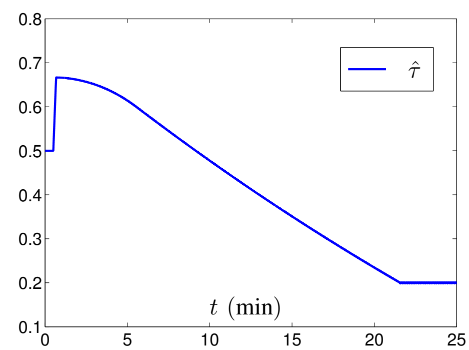

Fig. 5. Data profile of input delay estimation $\hat{\tau}(t)$ when auxiliary system (14) is used for input delay estimation of CSTR system (13).

seen from Fig. 5, $\hat{\tau}(t)$ of auxiliary system (14) converges to 0.2 min, which is input delay τ of CSTR system (13). The above results demonstrate the potential and effectiveness of the proposed method in the application.

VI. Conclusions

In this paper, a novel method based on Taylor expansion and an auxiliary system has been proposed for affine nonlinear dynamical systems. Theoretical analysis has shown that the proposed method can guarantee asymptotic convergence of the difference between the state of the dynamical system and the corresponding state of the auxiliary to zero. Besides, it has also been shown that, under mild conditions, the proposed method can theoretically guarantee the asymptotic convergence of input delay estimation error to zero. In addition, simulative examples have validated the theoretical results and the effectiveness of the proposed method. The performance of the proposed method has also been validated in a continuous stirred tank reactor system. It should be pointed out that due to the above two properties of the proposed method, it can be used for two purposes, i.e., a tool to facilitate the usage of controllers designed for corresponding systems with known input delay and an input delay estimator.

References

[1] X. Li and S. Song, "Stabilization of delay systems: Delay-dependent impulsive control," *IEEE Trans. Autom. Control*, vol. 62, no. 1, pp. 406–411, Jan. 2017.

[2] X. Li, "Global exponential stability of impulsive delay systems with flexible impulse frequency," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be pulished, doi: 10.1109/TSMC.2017.2766260.

[3] W. Sun, Y. Zhao, J. Li, L. Zhang, and H. Gao, "Active suspension control with frequency band constraints and actuator input delay," *IEEE Trans. Ind. Electron.*, vol. 59, no. 1, pp. 530–537, Jan. 2012.

[4] X. Liang, J. Xu, and H. Zhang, "Optimal control and stabilization for networked control systems with packet dropout and input delay," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 64, no. 9, pp. 1087–1091, Sep. 2017.

[5] Z. Zhang and J. Cao, "Novel finite-time synchronization criteria for inertial neural networks with time delays via integral inequality method," *IEEE Trans. Neural Netw. Learn. Syst.*, to be published, doi: 10.1109/TNNLS.2018.2868800.

[6] H. Wang, W. Sun, and P. X. Liu, "Adaptive intelligent control of nonaffine nonlinear time-delay systems with dynamic uncertainties," *IEEE Trans. Syst. Man, Cybern., Syst.*, vol. 47, no. 7, pp. 1474–1485, Jul. 2017.

[7] A. A. Pyrkin and A. A. Bobtsov, "Adaptive controller for linear system with input delay and output disturbance," *IEEE Trans. Autom. Control*, vol. 61, no. 12, pp. 4229–4234, Dec. 2016.

[8] M. Krstic, *Delay Compensation for Nonlinear, Adaptive, and PDE systems*. Birkhäser Boston, 2009.

[9] O. J. M. Smith, "A controller to overcome dead time," *ISA J.*, vol. 6, pp. 28–33, 1959.

[10] R. Sanz, P. García, Q. Zhong, and P. Albertos, "Predictor-based control of a class of time-delay systems and its application to quadrotors," *IEEE Trans. Ind. Electron.*, vol. 64, no. 1, pp. 459–469, Jan. 2017.

[11] J. Na, X. Ren, C. Shang, and Y. Guo, "Adaptive neural network predictive control for nonlinear pure feedback systems with input delay," *J. Process Control*, vol. 22, pp. 194–206, 2012.

[12] I. Furtat, E. Fridman, and A. Fradkov, "Disturbance compensation with finite spectrum assignment for plants with input delay," *IEEE Trans. Autom. Control*, vol. 63, no. 1, pp. 298–305, Jan. 2018.

[13] J. Lei and H. K. Khalil, "Feedback linearization for nonlinear systems with time-varying input and output delays by using high-gain predictors," *IEEE Trans. Autom. Control*, vol. 61, no. 8, pp. 2262–2268, Aug. 2016.

[14] C. Wang, Z. Zuo, Z. Lin, and Z. Ding, "Consensus control of a class of Lipschitz nonlinear systems with input delay," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 62, no. 11, pp. 2730–2738, Nov. 2015.

[15] V. Léchappé, S. Rouquet, A. González, F. Plestan, J. D. León, E. Moulay, and A. Glumineau, "Delay estimation and predictive control of uncertain systems with input delay: Application to a DC motor," *IEEE Trans. Ind. Electron.*, vol. 63, no. 9, pp. 5849–5857, Sep. 2016.

[16] D. Bresch-Pietri, J. Chauvin, and N. Petit, "Adaptive backstepping controller for uncertain systems with unknown input time-delay. Application to SI engines," in *Proc. 49th IEEE CDC*, Atlanta, GA, USA, Dec. 2010, pp. 3680–3687.

[17] H. Li, L. Wang, H. Du, and A. Boulkroune, "Adaptive fuzzy backstepping tracking control for strict-feedback systems with input delay," *IEEE Trans. Fuzzy Syst.*, vol. 25, no. 3, pp. 642–652, Jun. 2017.

[18] D. Li, Y. Liu, S. Tong, C. L. P. Chen, and D. Li, "Neural networks-based adaptive control for nonlinear state constrained systems with input delay," *IEEE Trans. Cybern.*, to be published, doi: 10.1109/TCYB.2018.2799683.

[19] D. Bresch-Pietri, J. Chauvin, and N. Petit, "Adaptive control scheme for uncertain time-delay systems," *Automatica*, vol. 48, no. 8, pp. 1536–1552, Aug. 2012.

[20] S. Obuz, J. R. Klotz, R. Kamalapurkar, and W. Dixon, "Unknown time-varying input delay compensation for uncertain nonlinear systems," *Automatica*, vol. 76, pp. 222–229, Feb. 2017.

[21] Y. Zhang, S. Li, and X. Jiang, "Near-optimal control without solving HJB equations and its applications," *IEEE Trans. Ind. Electron.*, vol. 65, no. 9, pp. 7173–7184, Sep. 2018.

[22] Y. Zhang and S. Li, "Time-scale expansion-based approximated optimal control for underactuated systems using projection neural networks," *IEEE Trans. Syst. Man, Cybern., Syst.*, vol. 48, no. 11, pp. 1957–1967, Nov. 2018.

[23] Y. Zhang and S. Li, "Adaptive near-optimal consensus of high-order nonlinear multi-agent systems with heterogeneity," *Automatica*, vol. 85, pp. 426–432, Nov. 2017.

[24] C. Hwang and C. Kuo, "A stable adaptive fuzzy sliding-mode control for affine nonlinear systems with application to four-bar linkage systems," *IEEE Trans. Fuzzy Syst.*, vol. 9, no. 2, pp. 238–252, Apr. 2001.

[25] N. Sun, Y. Wu, Y. Fang, H. Chen, and B. Lu, "Nonlinear continuous global stabilization control for underactuated RTAC systems: Design, analysis, and experimentation," *IEEE/ASME Trans. Mechatron.*, vol. 22, no. 2, pp. 1104–1115, Apr. 2017.

[26] M. Hamdy and I. Hamdan, "Robust fuzzy output feedback controller for affine nonlinear systems via T-S fuzzy bilinear model: CSTR benchmark," *ISA Trans.*, vol. 57, pp. 85–92, 2015.

[27] J. Lee, P. H. Chang, and M. Jin, "Adaptive integral sliding mode control with time-delay estimation for robot manipulators," *IEEE Trans. Ind. Electron.*, vol. 64, no. 8, pp. 6796–6804, Aug. 2017.

[28] B. Chen, X. Liu, S. Tong, and C. Lin, "Observer-based stabilization of T-S fuzzy systems with input delay," *IEEE Trans. Fuzzy Syst.*, vol. 16, no. 3, pp. 652–663, Jun. 2008.

[29] G. H. Golub and C. F. V. Loan, *Matrix Computations*. 3rd ed. Baltimore, USA: The Johns Hopkins University Press, 2012.

[30] Y. Zhang, S. Li, and X. Liu, "Adaptive near-optimal control of uncertain systems with application to underactuated surface vessels," *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 4, pp. 1204–1218, Jul. 2018.

[31] Y. Zhang, S. Li, and X. Liu, "Neural network-based model-free adaptive near-optimal tracking control for a class of nonlinear systems," *IEEE Trans. Neural Netw. Learn. Syst.*, to be published, doi: 10.1109/TNNLS.2018.2828114.

[32] L. Liu, Z. Chen, and J. Huang, "Parameter convergence and minimal internal model with an adaptive output regulation problem," *Automatica*, vol. 45, no. 5, pp. 1306–1311, May 2009.

[33] H. K. Khalil, *Nonlinear Systems*. 3rd ed. Upper Saddle River, NJ, USA: Prentice-Hall, 2002.

[34] A. Ingimundarson and T. Hagglund, "Robust tuning procedures of dead-time compensating controllers," *Control Eng. Practice*, vol. 9, no. 11, pp. 1195–1208, Nov. 2001.

[35] X.M. Ren and A.B. Rad, "Adaptive non-linear compensation control based on neural networks for non-linear systems with time delay," *Int. J. Syst. Sci.*, vol. 40, no. 12, pp. 1283–1292, 2009.

[36] J. Na, X. Ren, C. Shang, and Y. Guo, "Adaptive neural network predictive control for nonlinear pure feedback systems with input delay," *J. Process Control*, vol. 22, no. 1, pp. 194–206, 2012.

[37] Y. Cao and P. M. Frank, "Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach," *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 2, pp. 200–211, Apr. 2000.