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ORIGINAL RESEARCH

How is price explosivity triggered in the cryptocurrency markets?

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Abstract
We shed light on how the price explosivity characterising Bitcoin and other major cryptocurrencies is triggered, by employing the Quantile Self-Exciting Threshold Autoregressive (QSETAR) model. Our results for Bitcoin, Ripple, and Stellar reveal that the explosive behaviour originates from the extreme upper tails of the return distributions following a price increase in the preceding day. We do not find evidence of explositivity in the price of Litecoin.

Keywords Explosiveness · Cryptocurrencies · Bayesian methods · Quantile SETAR model

JEL Classification C21 · C51 · C53 · C58 · G12 · G15

1 Introduction

Much has been written about Bitcoin—and other cryptocurrencies—since its inception in 2008. As the related markets keep developing and maturing, it is important to delve more deeply into the dynamics characterising their functioning (Katsiampa et al. 2019a). For instance, despite the fact that Bitcoin and other cryptocurrencies have become increasingly popular, their price behaviour—still extremely volatile—does not seem to have been entirely comprehended as of yet (King and Koutmos 2021).

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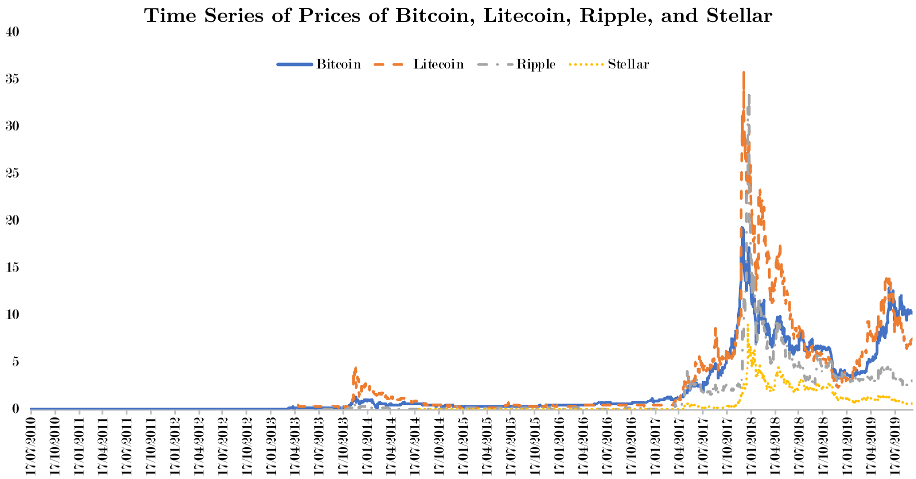

Fig. 1 Time series of prices of Bitcoin, Litecoin, Ripple, and Stellar. This figure plots the time series of prices of Bitcoin, Litecoin, Ripple, and Stellar. To ensure the readability and informativeness of the chart, we scale the prices, in a similar fashion to Borri (2019), as follows: we divide Bitcoin by 1,000, and Litecoin by 10; whereas we multiple both Ripple and Stellar by 10. The sample for each of the sampled cryptocurrencies starts from the first available daily observation (i.e., 17/7/2010, 28/4/2013, 4/8/2013, and 5/8/2014, for Bitcoin, Litecoin, Ripple, and Stellar, respectively), until 19/09/2019

During the past three years, cryptocurrency enthusiasts witnessed a number of strong rallies in the Bitcoin markets. For example, the first rally started sometime in the beginning of October 2017 when the price of Bitcoin per US dollar (BTC/USD) rose sharply from around \$4,000 to about \$20,000 in mid-December—an increase of approximately 400% in a space of two and a half months. After the spectacular rise, BTC/USD steadily marched downward to reach its lowest point of just above \$3,200. Around the end of March 2019, the currency unexpectedly staged a comeback. The BTC/USD soared to about \$12,000 around August from approximately \$4,000—an increase of roughly 200%. During the periods of these two powerful market rallies, the Bitcoin market saw no fewer than 10 days of very large positive price jumps: daily returns of 20.34% realised on 7th December 2017 and 16.72% observed on 2nd April 2019, for example.[1] The “bubble-like” and “explosive” behaviour of BTC/USD, clearly illustrated in Fig. 1, might be ascribed to the relative newness of the cryptocurrency markets and the consequent appeal that this generates on speculators (Bouri et al. 2019).

Originally designed to be an innovative and decentralised method of payment free from central banks’ intervention, the excitement which Bitcoin sparked triggered off debates on whether it should be considered as a speculative vehicle (Baur et al. 2018) or an asset class to hold for hedging or diversification purposes (Panagiotidis et al. 2019). Besides that, scholars have also devoted particular attention to research on a number of other themes regarding Bitcoin and cryptocurrencies in general, such as volatility (Katsiampa 2017; Baur and Dimpfl 2018), volatility spillovers (Ji et al. 2019; Koutmos 2018; Corbet et al. 2018b), price jump risk (Chaim and Laurini 2018; Scaillet et al. 2018; Gronwald 2019), tail risk (Gkillas and

[1] It is worth noting that our sample ends in September 2019, which is when we started working on this project. At the time of writing, we were aware of the most recent and still ongoing rallies in the Bitcoin markets. These rallies had started approximately in December 2020 and had pushed BTC/USD up to just over \$40,000 in January 2021, which is the all-time high. This period is not included in our analysis.

Katsiampa 2018; Borri 2019), technical trading (Hudson and Urquhart 2021), herding and feedback trading (King and Koutmos 2021), price efficiency (Urquhart 2016; Nadarajah and Chu 2017; Bariviera 2017; Khuntia and Pattanayak 2018) and, more broadly, the importance of information in market activity (Katsiampa et al. 2019b). While much has been written on the inefficiency and on the exuberance of the cryptocurrency markets, more investigations are needed to study how observed "bubble-like" behaviours are brought about.

A number of scholars have looked into the explosivity theme characterizing the exuberance in cryptocurrency prices. More specifically, Corbet et al. (2018a) analyse the underline fundamentals (i.e., blockchain position, hashrate, and liquidity) that can significantly thrive price growth, and then employ such measures to detect and datestamp Bitcoin and Ethereum bubbles. The authors observe the existence of short periods in which each fundamental clearly impacts on the price formation, and nevertheless highlight that such effects vanish rapidly. Fry (2018) corroborates the existence of bubbles in the Bitcoin and Ethereum markets, and warns that booms and bust can occur even in the absence of a clear bubble. The presence of bubbles in the Bitcoin market is further supported by Cretarola and Figà-Talamanca (2021) who attribute their existence to the correlation between market attention (as proxied by the Google Search Volume Index) and Bitcoin returns. Cagli (2019) widens his sample and includes some more altcoins in order to detect potential price explosivity and pairwise co-movements in their explosive behaviour. He finds all cryptocurrencies, except one, to show explosivity, as well as a number of co-explosive relationships in some altcoins pairs. Bouri et al. (2019) take a step further and observe that the explosivity in one cryptocurrency is likely due to the existence of explosive dynamics in other cryptocurrencies. They demonstrate that co-explosivity is not necessarily driven by the size of the cryptocurrencies. Finally, a recent paper by Gronwald (2019) reveals overwhelming evidence that cryptocurrency prices are characterised by (temporary) explosiveness and confirms that some cryptocurrency prices are explosive even if expressed in terms of Bitcoin and not, as conventionally, in USD. Overall, while in general scholars and experts confirm evidence of price explosivity (Corbet et al. 2018a; Fry 2018; Bouri et al. 2019; Cagli 2019), none of them however shed light on how the behaviour is brought about.

In this article, using the Quantile Self-Exciting Threshold Autoregressive (QSETAR) model, we characterise the bubble-like behaviour in the prices of Bitcoin and three other major cryptocurrencies: Litecoin, Ripple,[2] and Stellar.[3] The QSETAR model of Cai and Stander (2008) specifies the cryptocurrency return at the different quantiles of the return distribution to follow different autoregressive processes according to the pre-defined thresholds. Specifically, our framework examines how the cryptocurrency markets react to past price information when returns are located at the different quantiles with added flexibility that allows the reaction to vary depending on the state in which the markets were previously. The technique captures a more realistic trading behaviour: cryptocurrency traders enter the markets after the market direction has become clear. Evidence of price explosivity uncovered in this paper goes beyond the findings documented in Corbet et al. (2018a), Fry (2018), Cagli (2019).

Our empirical results for Bitcoin, Ripple, and Stellar show that the explosive behaviour originates only from the extreme right tail of the return distribution following a price increase in the preceding day. Taking Bitcoin as an example, we find that when the return is located at the 95th percentile after the return on Bitcoin turns positive on the preceding day, a rise of

[2] In this paper, for ease of readability, we use the term 'Ripple' (i.e., the name of a technology company) to refer to the cryptocurrency named 'XRP'. Indeed, 'XRP' is Ripple's cryptocurrency.

[3] The term 'Stellar' is commonly used to identify Stellar's cryptocurrency, whose symbol is 'XLM'.

1% in the return is estimated to push the price up by a further 1.30%. We do not find evidence of explosivity in the price of Litecoin, however.

In the following section, we present an exposition of the QSETAR model of Cai and Stander (2008). Sections 3 and 4 discuss our sample and the empirical results, respectively. Section 5 concludes.

2 Methodology

A general quantile self-exciting threshold autogressive (QSETAR) time series model (Cai and Stander 2008) is defined by

$$q^{\tau}_{r_t|\boldsymbol{r}_{t-1}} = \sum_{i=1}^{m+1} (\beta^{\tau}_{i0} + \beta^{\tau}_{i1} r_{t-1} + \ldots + \beta^{\tau}_{ip} r_{t-p}) I_{[r_{t-d^{\tau}} \in \Omega_i]} \tag{1}$$

where $\Omega_i = (w_{i-1}, w_i]$, $i = 1, \ldots, m$, and $\Omega_{m+1} = (w_m, w_{m+1})$, where $m \geq 0$ and $-\infty = w_0 < w_1 < \cdots < w_{m+1} = \infty$ are thresholds, p is the order of the model, $\boldsymbol{r}_{t-1} = (r_{t-1}, r_{t-2}, \ldots, r_0)^T$, β^{τ}_{ij} for $j = 0, 1, \ldots, p$ and d^{τ} are model parameters depending on τ. Moreover, $\tau \in (0, 1)$ is the probability such that $P(r_t < q^{\tau}_{r_t|\boldsymbol{r}_{t-1}} | \boldsymbol{r}_{t-1}) = \tau$ and d^{τ} is the delay parameter of the model. Therefore, $q^{\tau}_{r_t|\boldsymbol{r}_{t-1}}$ is the τth conditional quantile of the distribution of r_t.

Let $\beta^{\tau} = (\beta^{\tau}_{10}, \ldots, \beta^{\tau}_{1p}, \ldots, \beta^{\tau}_{m+10}, \ldots, \beta^{\tau}_{m+1p}, d^{\tau})^T$. Then β^{τ} may be estimated by solving the following minimisation problem (Koenker 2005):

$$\min_{\beta^{\tau}} \sum_{k+1}^{n} \rho_{\tau}(u_t) \tag{2}$$

where $\rho_{\tau}(u_t) = u_t(\tau - I_{[u_t<0]})$, and

$$u_t = r_t - q^{\tau}_{r_t|r_{t-1}} = r_t - \sum_{i=1}^{m+1} \left(\beta^{\tau}_{i0} + \beta^{\tau}_{i1} r_{t-1} + \cdots + \beta^{\tau}_{ip} r_{t-p} \right) I_{[r_{t-d^{\tau}} \in \Omega_i]} ,$$

in which $k = \max\{p, d_{max}\}$ and d_{max} is the maximum value of the delay parameter that one would like to consider. It is worth noting that Yu et al. (2010) suggest that for threshold GARCH models, $d_{max} = 3$ would be a reasonable choice. In this paper, we let $d_{max} = 5$ to cover a wider range of possible values of the delay parameter.

Since the delay parameter d^{τ} is an integer, it is not straightforward to solve the minimisation problem on the parameter space. However, it is worth noting that minimizing Eq. (2) is equivalent to maximising

$$\ell(r_{k+1}, \ldots, r_n \mid \boldsymbol{\beta}^{\tau}, \mathbf{r}_k) = \tau^{n-k}(1-\tau)^{n-k} e^{-\sum_{t=k+1}^{n} \rho_{\tau}(u_t)} . \tag{3}$$

This expression can be viewed as the likelihood function of $r_{k+1}, r_{k+2}, \ldots, r_n$ given $\mathbf{r}_k = (r_k, r_{k-1}, \ldots, r_0)^{\top}$ if we assume that r_t follows the threshold model

$$r_t = \sum_{i=1}^{m+1} \left(\beta^{\tau}_{i0} + \beta^{\tau}_{i1} r_{t-1} + \cdots + \beta^{\tau}_{ip} r_{t-p} + \epsilon^{\tau}_t \right) I_{[r_{t-d^{\tau}} \in \Omega_i]} ,$$

where ϵ^{τ}_t are iid asymmetric Laplace random variables with density function $f(\epsilon) = \tau(1-\tau)e^{-\rho_{\tau}(\epsilon)}$. Therefore, the parameter estimates obtained by maximising the likelihood function, shown in Eq. (3), are the same as those obtained by minimising Eq. (2).

Note that the involvement of the delay parameter d^τ will also make it difficult to solve the maximisation problem on the parameter space straightforward. We therefore adopt the Bayesian approach of Cai and Stander (2008) for the parameter estimation, whereby the posterior density function is given by

$$\pi(\boldsymbol{\beta}^\tau|\mathbf{r}) \propto \ell(r_{k+1}, \ldots, r_n|\boldsymbol{\beta}^\tau, \mathbf{r}_k)\pi(\boldsymbol{\beta}^\tau) = \tau^{n-k}(1-\tau)^{n-k}e^{-\sum_{t=k+1}^{n}\rho_\tau(u_t)}\pi(\boldsymbol{\beta}^\tau)$$

where $\pi(\boldsymbol{\beta}^\tau)$ is the prior density function.

The prior density function $\pi(\boldsymbol{\beta}^\tau)$ allows us to make use of possible prior information about the parameters when estimating parameters. Cai and Stander (2008) showed that the posterior density function is well defined on the parameter space for any prior density function. This is important because it guarantees that a Markov chain Monte Carlo (MCMC) method can be used for parameter estimation.

Following the work of Cai and Stander (2008), we employ a flat prior for the parameters and hence no prior information about the parameters is used for the parameter estimation. This is in fact one of the advantages of their method as in reality we usually do not have any information on the parameters. The basic idea of this method is as follows. First, given τ and the current parameter value $\boldsymbol{\beta}^\tau$, the next possible parameter value, denoted by $\boldsymbol{\beta}'$, is proposed in which the delay parameter d' is simulated from a uniform distribution on $\{1, \ldots, d_{max}\}$, and any other element in $\boldsymbol{\beta}'$ is simulated from a normal distribution, centered at its current value given in $\boldsymbol{\beta}^\tau$. Then, the proposed value $\boldsymbol{\beta}'$ is accepted as the next parameter value with a probability given by $\min\left\{\frac{\pi(\boldsymbol{\beta}'|\boldsymbol{r})}{\pi(\boldsymbol{\beta}^\tau|\boldsymbol{r})}, 1\right\}$. By repeating these steps multiple times, a sequence of model parameters can be generated. The Markov chain theory guarantees that the equilibrium distribution of the Markov chain is the posterior distribution defined by $\pi(\boldsymbol{\beta}^\tau|\boldsymbol{r})$.[4]

3 Data

This article focuses on four cryptocurrencies, currently actively traded in the markets: Bitcoin, Litecoin, Ripple, and Stellar. In selecting the cryptocurrencies to be included in our investigation, we adopt the following criteria. First, based on the data on coinmarketcap.com as of 19/9/2019, we focus on the top ten most capitalised crytpocurrencies which cover about 89% of the entire crytpocurrency markets. Second, to ensure a sufficiently long time series, we choose only the cryptocurrencies that have been actively traded during the last five years. After the filtering process, we end up with four cryptocurrencies: Bitcoin, Litecoin, Ripple, and Stellar.

Daily dollar returns on the cryptocurrencies are computed using price data collected either from coinmarketcap.com or www.CryptoCompare.com. Specifically, we rely on coinmarketcap.com to gather data for Litecoin, Ripple, and Stellar because this website provides cryptocurrency prices as the volume weighted average of hundreds cross currency pairs converted to USD (Wei 2018). The downside of it, however, is that prices are only available from April 2013, which is not ideal for Bitcoin provided that this pioneer cryptocurrency started being traded early in 2010. Therefore, limited to this case, we rely on www.CryptoCompare.com as the website offers daily information starting from 17/7/2010—that is when Bitcoin was first traded. The sample for each of the cryptocurrencies thus starts from the first available daily observation and ends on 19/9/2019. More specifically, our samples for Bitcoin, Litecoin, Ripple, and Stellar start from 17/7/2010, 28/4/2013, 4/8/2013, and 5/8/2014, respectively.

[4] See Brooks (1998), for example.

Table 1 Summary statistics of cryptocurrency returns

	Bitcoin	Litecoin	Ripple	Stellar
N	3351	2335	2237	1871
Mean	0.0037	0.0012	0.0018	0.0019
Median	0.0019	−0.0003	−0.0027	−0.0035
Std. deviation	0.0665	0.0653	0.0740	0.0769
Skewness	2.9480	1.7170	2.0510	1.9870
Std. error of skewness	0.0420	0.0510	0.0520	0.0570
Kurtosis	94.3830	25.2090	29.0410	16.0710
Std. error of Kurtosis	0.0850	0.1010	0.1030	0.1130
Minimum	−0.8488	−0.5139	−0.6163	−0.3664
Maximum	1.4744	0.8290	1.0274	0.7231

This table reports summary statistics for Bitcoin, Litecoin, Ripple, and Stellar. Our samples for Bitcoin, Litecoin, Ripple, and Stellar start from 17/7/2010, 28/4/2013, 4/8/2013, and 5/8/2014, respectively. The time series for all the cryptocurrencies in the sample ends on 19/9/2019

Table 2 Quantiles of cryptocurrency returns

τ	Bitcoin	Litecoin	Ripple	Stellar
0.05	−0.0770	−0.0839	−0.0923	−0.1024
0.25	−0.0136	−0.0206	−0.0230	−0.0310
0.50	0.0019	−0.0003	−0.0027	−0.0035
0.75	0.0218	0.0194	0.0203	0.0267
0.95	0.0871	0.0912	0.1043	0.1166

This table reports the distributions of returns on Bitcoin, Litecoin, Ripple, and Stellar. We report the return values at the 5th, 25th, 50th, 75th, and 95th quantiles. Our samples for Bitcoin, Litecoin, Ripple, and Stellar start from 17/7/2010, 28/4/2013, 4/8/2013, and 5/8/2014, respectively. The time series for all the cryptocurrencies in the sample ends on 19/9/2019

Table 1 shows the number of observations available for each of the sampled cryptocurrencies along with some commonly used summary statistics. According to the reported statistics, Bitcoin has the highest daily mean return of around 0.4% and is the only cryptocurrency in the sample with positive median. The skewness and the kurtosis measures for all the cryptocurrencies in the sample point to the return distributions that are right-skewed and fat-tailed. Bitcoin, once again, has the highest values of both skewness and kurtosis as well as the largest maximum and the smallest minimum values. Complementing the descriptive statistics in Table 1, Table 2 reports the empirical distributions of returns, for each cryptocurrency, at different quantiles.

4 Empirical analysis and results

We estimate Eq. (1), setting $m = 1, p = 1, \ldots, 7, d_{max} = 5, \tau = 0.05, 0.25, 0.50, 0.75, 0.95$. We fix $m = 1$ because we are only interested in the conditional quantile function of the cryptocurrency return process when, subject to the delay parameter, the past return is located either above or below a pre-defined threshold. For each cryptocurrency, we employ two threshold

Table 3 AIC statistics when the threshold is $w_1 = 0$

Bitcoin					
$p\backslash\tau$	0.05	0.25	0.50	0.75	0.95
1	6.502108	7.629882	7.567755	7.982147	7.212924
2	8.516421	9.367684	9.568465	9.864459	9.198823
3	10.739352	11.322411	11.569194	11.858959	11.324587
4	12.599169	13.288983	13.588904	13.699591	13.119390
5	14.579397	15.343937	15.574193	15.739112	15.371966
6	16.498389	17.269221	17.575322	17.725333	17.965440
7	18.643327	19.343978	19.667224	19.781388	19.299285
Litecoin					
$p\backslash\tau$	0.05	0.25	0.50	0.75	0.95
1	6.309759	7.177583	7.400427	7.570019	7.029870
2	8.554905	9.103804	9.420087	9.561237	9.343828
3	10.343398	11.113302	11.380647	11.572815	11.497163
4	12.357591	13.088285	13.391061	13.562340	13.110089
5	14.312543	15.135724	15.383583	15.479943	15.243180
6	16.395870	17.121087	17.395170	17.521574	17.428193
7	18.336617	19.099428	19.396694	19.622497	18.999818
Ripple					
$p\backslash\tau$	0.05	0.25	0.50	0.75	0.95
1	6.265001	7.203146	7.404664	7.265546	6.544424
2	8.464208	9.069515	9.474161	9.556994	8.581126
3	10.204962	11.175128	11.490708	11.265592	10.597363
4	12.345551	13.077693	13.416451	13.805275	12.715281
5	14.159573	15.114419	15.572170	15.636419	15.634413
6	16.255371	17.217220	17.392986	17.523048	17.328143
7	18.858845	19.083811	19.387955	19.563025	19.472317
Stellar					
$p\backslash\tau$	0.05	0.25	0.50	0.75	0.95
1	6.183000	7.056618	7.401042	7.527477	6.734905
2	8.616877	9.045981	9.405205	9.512064	8.667104
3	10.722696	11.076436	11.437971	11.750134	11.187539
4	12.044138	13.079619	13.430794	13.514415	13.465772
5	14.067955	15.092446	15.612555	16.075160	15.087909
6	16.089324	17.158386	17.434216	17.486799	17.358809
7	18.406601	19.038180	19.400287	19.580555	19.544746

This table shows the AIC statistics—for each quantile τ and $p = 1, ..., 7$—when the threshold value is $w_1 = 0$

Table 4 AIC statistics when the threshold is the median return

Bitcoin					
$p\backslash\tau$	0.05	0.25	0.50	0.75	0.95
1	6.558470	7.401306	7.616606	7.811130	7.237105
2	8.712559	9.377810	9.610050	9.769683	9.469767
3	10.616625	11.374364	11.609519	11.836909	11.562212
4	12.759651	13.365491	13.637782	13.894548	13.662482
5	14.537339	15.315875	15.617206	15.774277	15.434152
6	16.534188	17.319596	17.637874	17.830242	17.402648
7	18.694055	19.332562	19.608678	19.787000	19.311141
Litecoin					
$p\backslash\tau$	0.05	0.25	0.50	0.75	0.95
1	6.287696	7.068629	7.438560	7.557180	6.984493
2	8.493792	9.070666	9.356818	9.613444	9.924253
3	10.146881	11.050838	11.356131	11.752736	11.710226
4	12.154898	13.143162	13.355349	13.527987	13.620087
5	14.151685	15.123064	15.401824	15.480751	15.616061
6	16.095184	17.138353	17.359025	17.520704	17.356802
7	18.546042	19.076425	19.369570	19.526664	19.432512
Ripple					
$p\backslash\tau$	0.05	0.25	0.50	0.75	0.95
1	6.112647	7.021669	7.350633	7.645379	6.497065
2	8.174058	9.047064	9.397760	9.506547	9.251517
3	10.278312	11.019051	11.348291	11.513425	11.406681
4	12.830842	13.078399	13.350471	13.505912	13.777331
5	14.187123	15.006143	15.373361	15.517410	15.169858
6	16.114265	17.008317	17.344003	17.534641	17.712271
7	18.230105	19.025180	19.526577	19.517259	19.315860
Stellar					
$p\backslash\tau$	0.05	0.25	0.50	0.75	0.95
1	6.079600	7.056956	7.356917	7.557868	6.925788
2	8.107468	9.048540	9.409459	9.485658	8.623965
3	9.911982	10.981165	11.357618	11.588873	11.567617
4	11.984077	13.016580	13.357567	13.706924	13.672293
5	14.085852	15.004719	15.436294	15.524615	15.305201
6	16.028472	16.988928	17.352968	17.619626	17.016013
7	18.137357	18.978120	19.447552	19.498815	20.011219

This table shows the AIC statistics—for each quantile τ and $p = 1, ..., 7$—when the threshold value w_1 is the median return values in our analysis: $w_1 = 0$ and the cryptocurrency's median return. The threshold of zero allows the empirical analysis to mimic the real-world trading behaviour where cryptocurrency traders pay attention to whether or not the markets were positive or negative in the previous day.[5] In addition, we also experiment with the cryptocurrency's median return as a threshold because it is conventional for researchers employing the quantile regression technique to estimate the response of the dependent variable to the covariates when the dependent variable is located at the median.

For each value of p, and each quantile level τ, where $\tau = 0.05, 0.25, 0.50, 0.75, 0.95$, we run—for each cryptocurrency—the MCMC estimation method from which we obtain, in total, 140 estimated models. For each estimated model, we collect 250 posterior samples for each parameter after a burn-in period. Then, for the delay parameter, we estimate its value by using the mode of its posterior samples and, for other parameters, we estimate their values by using the mean of the respective posterior samples. We also calculate the corresponding 95% credible interval using the 2.5% and 97.5% quantiles of these posterior samples. The credible interval is an interval within which the model parameter falls with a 0.95 probability. Hence, if the interval contains number 0, then we can say that the parameter is not significant. Moreover, for each estimated model, we also obtain the AIC statistics, according to which we find that the best fitted models—for both thresholds and for all the cryptocurrencies investigated—are the ones with $p = 1$ (see, in this regard, Tables 3 and 4). This preliminary check, therefore, leads us to formalize the following model specification:

$$q^{\tau}_{r_t|r_{t-1}} = \sum_{i=1}^{2} \left(\beta^{\tau}_{i0} + \beta^{\tau}_{i1} r_{t-1}\right) I_{[r_{t-d^{\tau}} \in \Omega_i]}. \tag{4}$$

We examine the model fit by employing the following procedures. Using the fitted models, we obtained the estimated quantiles at levels $\tau = 0.05, 0.25, 0.5, 0.75, 0.95$. In Fig. 2, we present the observed return of the cryptocurrency, along with the estimated quantiles at the 5% and the 95% levels in black, green, and yellow, respectively. It can be seen that these quantile curves reflect the evolution of each cryptocurrency in our sample. In particular, Fig. 2 demonstrates that 5% of the return realisation for each cryptocurrency in the sample, as represented by the black line, are expected to be higher than the yellow line and 5% of them are expected to be lower than the green line.

To test this, we calculate the percentages of the observed cryptocurrency that are under each of the five estimated quantile curves. We then calculate the mean squared errors (MSE) to measure the differences between these percentages and the actual τ value. We find that the MSE values are given by 0.00036, 0.00094, 0.00148 and 0.00265 for Bitcoin, Litecoin, Ripple and Stellar, respectively—suggesting that these models explain the data well.

We report our estimation results in Tables 5, 6, and 7. For each τ value, the means of the posterior samples of β_{11} and β_{21} are reported in the top row, and the associated 95% credible intervals are reported in the corresponding second and third rows. According to the results reported in Tables 5 and 6, we find no evidence of the explosive price behaviour in the Litecoin market while the evidence exists in the markets of the other cryptocurrencies, depending on the value of w_1. When $w_1 = 0$, the means of the posterior distributions of β_{11} and β_{21} for Bitcoin and Ripple, seen in Table 5, indicate that the explosive price behaviour is triggered when the rally, following the previous day's price increase, is strong enough for the returns to reach the 95th quantile. As an example, the value of β_{21} of 1.2916 for Bitcoin at

[5] This approach has earlier been adopted by Cai and Stander (2008) who investigate the time series property of the US GNP using the QSETAR model, also setting $m = 1$. In their application, the US economy is assumed to behave in a different manner after contraction and expansion.

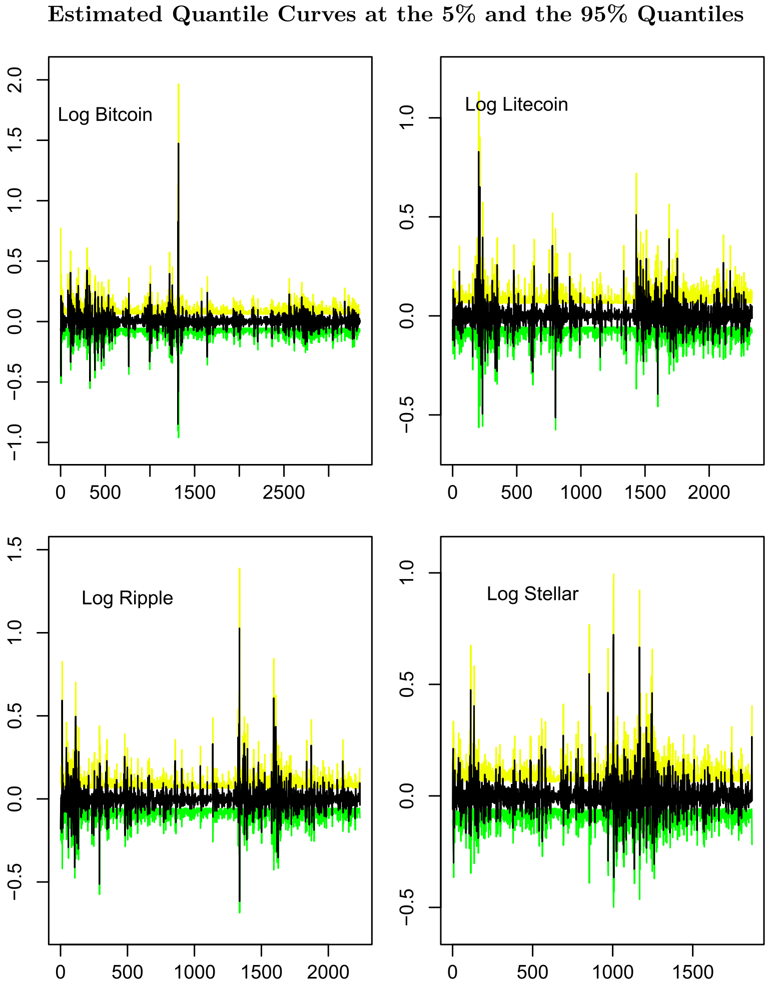

Fig. 2 Estimated quantile curves at the 5% and the 95% Quantiles. Estimated quantile curves at the 5% (green) and 95% (yellow) levels for each cryptocurrency, where the black curves are the observed cryptocurrency —5% of which are expected to be above the yellow line, and 5% of which are expected to be below the green line

$\tau = 0.95$, reported in Table 5, indicates that one day after the return on Bitcoin turns positive, an increase of 1% in the return—when the return is located at the 95th percentile—results in a further increase in return of around 1.30%.[6] When the threshold is the median return, the findings reported in Table 6 suggest that only Ripple and Stellar exhibit the explosive price behaviour at the 95th quantile one day after the previous day's return crosses the median return from below. We find no evidence of explosivity in the Bitcoin nor the Litecoin markets when the value of w_1 is set to be the cryptocurrency's median return.

Our empirical analysis complements and confirms previous research on price explosivity and co-explosivity by Bouri et al. (2019), Cagli (2019), and Gronwald (2021). While

[6] The estimated value of β_{21} is statistically significant because the credible interval for β_{21} does not contain the value of 0. In other words, the probability for β_{21} taking positive values is 95%.

Table 5 The QSETAR estimation results

τ	Bitcoin		Litecoin		Ripple		Stellar	
	β_{11}	β_{21}	β_{11}	β_{21}	β_{11}	β_{21}	β_{11}	β_{21}
0.05	0.9855	−0.6140	0.4085	−0.4334	0.4761	−0.7138	0.2939	−0.5502
	−0.3503	−1.8599	−0.8130	−1.6651	−0.4759	−2.4586	−0.8247	−2.8612
	2.7027	0.3126	2.2672	0.4692	2.5605	0.3373	2.2639	0.4234
0.25	0.2623	−0.1867	0.0700	−0.1081	0.0869	−0.1634	−0.0943	−0.1713
	−0.3371	−0.7970	−0.5166	−0.5637	−0.4961	−0.6801	−0.6292	−0.7942
	1.0691	0.2819	0.7971	0.3213	1.0846	0.3200	0.4969	0.3547
0.50	0.0064	0.0141	−0.0611	−0.0710	−0.0395	0.0087	−0.1284	−0.0594
	−0.3996	−0.3174	−0.4470	−0.5079	−0.4890	−0.3332	−0.5955	−0.5000
	0.4250	0.4661	0.3938	0.3846	0.3624	0.4690	0.3423	0.4446
0.75	−0.2447	0.3145	−0.2442	0.4009	−0.3373	0.6761	−0.1883	0.3748
	−0.9714	−0.2530	−0.8415	−0.3302	−0.9310	−0.1877	−0.7581	−0.3782
	0.2869	1.0992	0.4006	1.3946	0.2096	1.7699	0.3187	1.4293
0.95	−0.7315	**1.2916**	−0.6520	1.0852	−0.8664	**1.7142**	−0.5724	1.6340
	−2.2899	0.0161	−2.2344	−0.3370	−2.5596	0.1047	−2.5536	−0.0525
	0.1806	3.4924	0.6113	2.8898	0.4952	3.5153	0.8000	3.7254

This table reports the estimated parameter values for the QSETAR model shown in Eq. (4) where $p = 1$, $d = 1$, and $w_1 = 0$. For each value of τ, where $\tau = 0.05, 0.25, 0.50, 0.75, 0.95$, we report the means of the posterior distributions of parameters β_{11} and β_{21} (first row) along with the credible intervals which are shown below the corresponding mean estimates (second and third rows). Bold font is used to highlight statistical significance

Table 6 The QSETAR estimation results

τ	Bitcoin		Litecoin		Ripple		Stellar	
	β_{11}	β_{21}	β_{11}	β_{21}	β_{11}	β_{21}	β_{11}	β_{21}
0.05	0.8701	−0.5122	0.3414	−0.5104	0.4652	−0.5437	0.2950	−0.4006
	−0.2267	−1.5665	−0.9416	−1.9766	−0.7190	−1.6411	−0.8194	−1.5886
	2.3418	0.3224	1.9459	0.5003	2.0469	0.2439	2.8161	0.2792
0.25	0.1585	−0.1348	0.0752	−0.1505	0.0370	−0.1550	−0.0831	−0.1703
	−0.4028	−0.5866	−0.4801	−0.7362	−0.5007	−0.6360	−0.6592	−0.7836
	0.9300	0.2508	0.9163	0.2512	0.7249	0.3089	0.4746	0.3413
0.50	0.0030	−0.0180	−0.0552	−0.0520	−0.0260	−0.0282	−0.1206	−0.0901
	−0.4085	−0.4304	−0.4654	−0.5033	−0.4120	−0.4120	−0.6023	−0.5318
	0.4114	0.4456	0.3995	0.4097	0.3969	0.3854	0.4409	0.3946
0.75	−0.2447	0.3123	−0.2498	0.3718	−0.2208	0.4323	−0.2096	0.3782
	−0.8362	−0.2455	−0.9202	−0.3446	−0.8281	−0.2272	−0.9726	−0.2552
	0.3237	0.7988	0.3299	1.1334	0.3664	1.4876	0.3977	1.5732
0.95	−0.8230	1.1721	−0.8410	1.2731	−0.8513	**1.7145**	−0.8104	**1.6691**
	−2.3601	−0.0422	−3.6114	−0.1570	−2.5309	0.3175	−2.6314	0.1740
	0.2966	3.2214	0.4896	3.4577	0.3208	3.6196	0.7097	3.7097

This table reports the estimated parameter values for the QSETAR model shown in Eq. (4) where $p = 1$, $d = 1$, and w_1 is the median of the cryptocurrency return distribution. For each value of τ, where $\tau = 0.05, 0.25, 0.50, 0.75, 0.95$, we report the means of the posterior distributions of parameters β_{11} and β_{21} (first row) along with the corresponding credible intervals which are shown below the corresponding mean estimates (second and third rows). Bold font is used to highlight statistical significance both Bouri et al. (2019) and Cagli (2019) report the existence of price explosivity and co-explovisivity among major cryptocurrencies, their papers stop short of examining the trigger behind the explosive behaviour. Our QSETAR estimation results show that the explosive behaviour is prompted by strong conviction by the market participants. Following previous day's price increase, the prices of Bitcoin and Ripple become explosive when their returns reach 8.71% and 10.43%, respectively. As for Stellar, the explosive behaviour is triggered only when the return reaches 11.66% and the previous day's return cross the median value of around −0.35% from below.

The evidence shows that the explosive price behaviour was triggered in a number of occasions. To provide a number of examples, we can report that Bitcoin experienced a burst of explosivity on 22nd and 23rd November 2011, that is one day after the previous day's return crossed the threshold of zero (i.e., moving from negative to positive) from −4.8% on 20th November to 5.8% on 21st November. During the two days between 22nd and 23rd November 2011, the returns on Bitcoin jumped by around 34%, having reached 15.2% on 22nd November and then rallied a further 18.7% on 23rd November. Similarly, Ripple showed explosive behaviour from 24th November 2013, namely one day after the previous day's return crossed the $w_1 = 0$ threshold, gaining approximately 112% during the three-day period between 24th and 26th November 2013. The explosive behaviour of Stellar was triggered on 4th May 2017 when a sharp market rally occurred one day after the previous day's return crossed the median return from below, leading to four consecutive days of price advance which resulted in the holding-period return of over 200%. All of the explosive episodes described share a common trait: it is triggered when the return reaches the 95th quantile of the return distribution one day after the return crosses the threshold from below.

Table 7 shows that the delay parameter varies according to the quantile level. Our results highlight, as well, that the delay parameters are almost identical either when the threshold is $w_1 = 0$ or the median return. Moreover, when $\tau = 0.95$, we find $d^\tau = 1$ for all returns, which suggests the following: it takes one day after the cryptocurrency return crosses the threshold (i.e., 0 and the median return, depending on the specification) for the autoregressive coefficient (i.e,. the market reaction to the previous day's price information) to switch from β_{11} to β_{21} if the return crosses the threshold from below and from β_{21} to β_{11} if the return crosses the threshold from above.

To demonstrate the superior performance of the QSETAR model, we compare its coverage probabilities of the estimated quantiles with those calculated using the conventional SETAR model whose focus is on the mean return. A good model is expected to deliver the estimated coverage probabilities of the quantile estimates that are closer to the actual quantile level τ. We estimate the SETAR model with the same specification as the QSETAR in Eq. (4) as follows:

$$r_t = \sum_{i=1}^{2} (\beta_{i0} + \beta_{i1} r_{t-1} + h_i \varepsilon_t)\, I_{[r_{t-1} \in \Omega_i]} \tag{5}$$

where $\varepsilon_t \sim N(0, 1)$ and h_i^2 is the conditional variance of r_t in the ith regime.

We report the coverage probabilities of the estimated quantiles for both the QSETAR and the SETAR models in Table 8.[7] Our results reveal that the QSETAR model performs better than the SETAR model according to the the coverage probabilities—especially at the 25th

[7] The coverage probability is estimated as follows. Let $q^{\tau_i}_{r_t|\mathbf{r}_{t-1}}$ be the τ_ith conditional quantile of r_t, where $t = 1, ..., n$. Let $n_1^{\tau_i} = \sum_{t=1}^{n} I[r_t \leq q^{\tau_i}_{r_t|\mathbf{r}_{t-1}}]$, where $I[.]$ is the indicator function. Then the coverage probability of the τ_ith quantile estimates is estimated by $n_1^{\tau_i}/n$.

Table 7 Estimated values of the delay parameter d

$w_1 = 0$				
τ	Bitcoin	Litecoin	Ripple	Stellar
0.05	1	2	2	2
0.25	2	3	2	3
0.50	3	3	3	3
0.75	2	2	1	2
0.95	1	1	1	1
w_1 = median return				
τ	Bitcoin	Litecoin	Ripple	Stellar
0.05	1	2	2	2
0.25	2	3	2	3
0.50	3	3	3	3
0.75	2	2	2	2
0.95	1	1	1	1

This table shows the estimated values of d—when the threshold value is either $w_1 = 0$ (top panel) or the median return (bottom panel)—for the best estimated models according to the AIC

Table 8 Coverage probabilities for the quantile estimates from QSETAR and SETAR

τ	0.05	0.25	0.50	0.75	0.95	RMSE
QSETAR						
Bitcoin	0.0358	0.2181	0.5127	0.7660	0.9624	0.0423
Litecoin	0.0313	0.2266	0.5032	0.7807	0.9670	0.0463
Ripple	0.0304	0.2226	0.5150	0.7836	0.9660	0.0524
Stellar	0.0283	0.2261	0.5131	0.7686	0.9679	0.0434
SETAR						
Bitcoin	0.0355	0.1197	0.5066	0.8821	0.9660	0.1869
Litecoin	0.0343	0.1422	0.5107	0.8732	0.9649	0.1654
Ripple	0.0322	0.1346	0.5407	0.8703	0.9566	0.1726
Stellar	0.0380	0.1626	0.5257	0.8455	0.9535	0.1326

This table reports the coverage probabilities for the quantile estimates from QSETAR and SETAR, where the coverage probability is estimated as follows. Let $q^{\tau_i}_{r_t|\mathbf{r}_{t-1}}$ be the τ_ith conditional quantile of r_t, where $t = 1, ..., n$. Let $n^{\tau_i}_1 = \sum_{t=1}^{n} I[r_t \leq q^{\tau_i}_{r_t|\mathbf{r}_{t-1}}]$, where $I[.]$ is the indicator function. Then the coverage probability of the τ_ith quantile estimates is estimated by $n^{\tau_i}_1/n$

and the 75th quantiles—and the root-mean-square error (RMSE). Specifically, the RMSE results also suggest that the QSETAR model outperforms the SETAR model and therefore better suits the examination of price explosivity in the cryptocurrency markets.

5 Concluding remarks

In this paper, we characterise the bubble-like behaviour of prices of four major cryptocurrencies using the QSETAR model of Cai and Stander (2008). The technique is capable of characterising the behaviour of nonstationary time series with very large, but not necessarily symmetric, variations. We experiment with two threshold values: the cryptocurrencies' median returns and zero. Models for each cryptocurrency in the sample are subject to rigorous diagnostic tests. The results show that the QSETAR models fit the data very well and that they outperform the conventional SETAR model.

Our empirical analysis reveals that the explosive behaviour in the markets for Bitcoin, Ripple, and Stellar originates from the extreme upper tails of the return distributions. Specifically, our results suggest that one day following a price increase, explosivity is triggered when the market rallies are sufficiently strong, putting the cryptocurrency returns in the 95th quantile of the return distributions. We find no evidence of price explosivity in the markets for Litecoin.

The findings in this article echo concerns raised by researchers such as Gandal et al. (2018) that cryptocurrency markets appear to be prone to excessive speculation and price manipulation. The extreme price moves frequently observed in the markets seem to suggest that market participants behave irrationally, reacting strongly to unanticipated information, thereby causing unwarranted, sustained periods of strong market rallies. The existence of price explosivity is perhaps the alluring quality of cryptocurrencies which have attracted investors' attention. As rising prices become evident, more cryptocurrency traders joins the rallies, putting upward pressure on prices, leading to frothy markets.

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