International Journal of Systems Science

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/tsys20

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To cite this article: Yinyan Zhang, Gang Xiao & Shuai Li (2022): Adaptive quadratic optimisation with application to kinematic control of redundant robot manipulators, International Journal of Systems Science, DOI: 10.1080/00207721.2022.2141594

To link to this article: https://doi.org/10.1080/00207721.2022.2141594

Published online: 07 Nov 2022.

INTERNATIONAL JOURNAL OF SYSTEMS SCIENCE
https://doi.org/10.1080/00207721.2022.2141594

Adaptive quadratic optimisation with application to kinematic control of redundant robot manipulators

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ABSTRACT

The primal-dual gradient dynamics is a broadly investigated approach for handling optimisation problems. In this paper, we provide an extension of such dynamics under the adaptive updating framework for solving equality-constrained quadratic programmes. We show that the performance of the proposed method is theoretically guaranteed and it has asymptotic convergence to the solution of the optimisation problem and the minimum inter-event time is non-trivial. A numerical example and an application show the effectiveness and advantages of the proposed method.

ARTICLE HISTORY
Received 14 July 2022
Accepted 24 October 2022

KEYWORDS
Quadratic optimisation; primal-dual gradient dynamics; kinematic control; redundant robot manipulators

1. Introduction

In practice, many problems can be formulated as a quadratic programme problem. For example, it is shown in Zhang and Li (2017) that the receding horizon optimal consensus problem can be converted to a quadratic programme by performing the Taylor expansion. In the field of robotics, the inverse kinematics problem of redundant robot manipulators is also often described as a quadratic programme problem (Jin et al., 2017; Xie et al., 2022; Zhang et al., 2018). For the two problems, control actions are executed according to the real-time solution generated by the solver. To guarantee the performance, traditional numerical algorithms for solving convex optimisation problems are not used, while dynamical system-based methods such as dynamical neural networks are adopted (Cui et al., 2022; Zhang et al., 2021, 2022). For these methods, the updating frequency for the control actions is the same as the sampling frequency, which requires many computational resources, and thus they are time-triggered.

Time-triggered approaches are widely used to deal with both optimisation and control problems on digital platforms, for which the update of variables or control actions is performed in a periodic manner (Huang et al., 2021; Tan et al., 2021). For example, for control problems, in general, a controller is derived on the basis of continuous-time system dynamics and is implemented in the physical system through digital controllers with a fixed sampling period. A general requirement for this kind of implementation is that the sampling period is sufficiently small. Compared with time-triggered approaches, the event-triggered ones do not need to conduct periodic updating of state variables or control actions, while guaranteeing the performance (Qi et al., 2022). Intuitively, in the event-triggered framework, actions will only be executed only when certain events occur, i.e. when some condition is met. Such conditions are referred to as triggering conditions. By this way, the costs of computational resources and communications are significantly reduced (Wang et al., 2015). Additionally, the event-triggered approach preserves closed-loop stability.

Although great progress have been achieved in event-triggered control, such as Li et al. (2021), Liu et al. (2022), and Zhao and Hua (2021) and the references therein, existing results on event-triggered optimisation are relatively limited with a particular focus on distributed optimisation. For example, Chen and Ren (2016) proposed a novel algorithm to deal with the distributed consensus optimisation problem on a directed network, where a set of agents cooperatively minimise the sum of their utility

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functions, which is an unconstrained distributed optimisation problem. The problem was also investigated in Meinel et al. (2014). However, the results of Chen and Ren (2016), Li et al. (2021), Liu et al. (2022), Meinel et al. (2014), and Zhao and Hua (2021) do not apply to optimisation-based methods for the control of systems due to the existence of constraints in the problem formulation. Note that event-triggered dynamics are different from model predictive control (MPC) (Hadian et al., 2021). MPC is a time-triggered method, where the control is computed in a fixed sampling gap.

Inspired by the above observations, an event-triggered method for solving quadratic programmes is beneficial and demanded. As a preliminary work, we focus on an event-triggered optimisation problem with an equality constraint. The proposed method is based on some results reported on the primal-dual gradient dynamics for handling optimisation problems (Cherukuri et al., 2016; Feijer & Paganini, 2010; Qu & Li, 2019). Note that the existing results about the primal-dual gradient dynamics are derived under the time-triggering framework, and it is not trivial to extend them to the case with an event-triggering rule. Conventionally, the derivation procedure of a controller with an event-triggered property is often on the basis of a Lyapunov function with known system equilibrium. Compared with the problems addressed by existing event-triggered control methods, the equilibrium of the problem of interest in this paper is unknown (note that the equilibrium corresponds to the solution of the quadratic programme, which needs to be solved), which is also shown in Figure 1. Thus, it is not straightforward to use the existing event-triggered control methods to solve the problem here.

In this paper, we extend the primal-dual gradient dynamics to the event-triggered scenario, so as to inherit the convergence to the optimal solution as well as to have the advantages of event-triggered methods. Recently, Cherukuri et al. (2018) proposed a self-triggered approach for the primal-dual gradient dynamics, of which the analysis is based on a Lyapunov candidate function with the equilibrium explicitly included. Our work is different from Cherukuri et al. (2018) because (1) from the methodology perspective, our method falls into the event-triggered category, while Cherukuri et al. (2018) belong to the self-triggered one; (2) in terms of technical details, in our approach, the equilibrium of the primal-dual gradient dynamics is not explicitly included in the Lyapunov candidate function, and thus the theoretical analysis is different. Specifically, we derive the event-triggering condition and analyse the corresponding minimum inter-event time. We employ the Lyapunov approach to conduct an analysis on the convergence of the method and use a numerical example to validate the theoretical conclusions. This is our first work on even-triggered dynamics, which is different from our previous works on time-triggered optimisation and control (Zhang & Li, 2017; Zhang et al., 2021, 2018, 2022). The contributions of this paper include the following.

(1) For the first time, an event-triggered method for equality-constrained quadratic optimisation is proposed and illustrated on the basis of the primal-dual gradient dynamics.

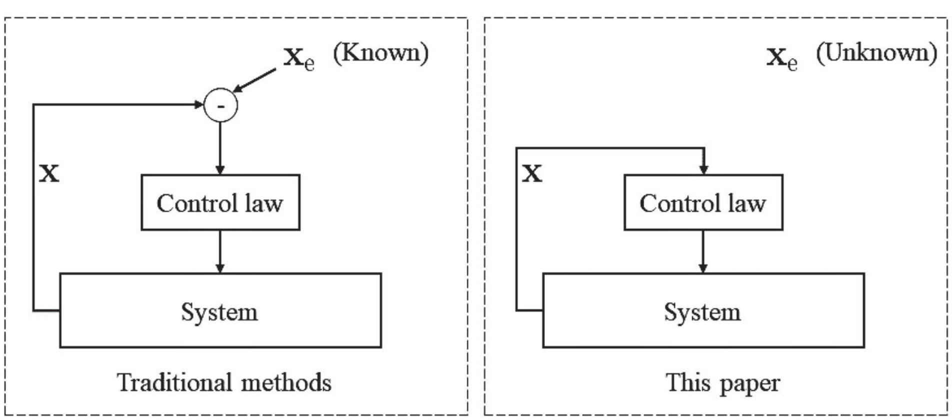

Figure 1. Comparison with existing event-triggered methods, where $\mathbf{x}_e$ denotes the system equilibrium, from the perspective of control.

(2) Theoretical results are presented to guarantee the performance of the proposed method, including the convergence to the optimal solution and the existence of the non-trivial minimum inter-event time.
(3) The proposed method is applied to the kinematic control problem of redundant robot manipulators.

The remainder of this paper is organised as such. In Section 2, the problem formulation and the preliminary are shown. Then, the proposed event-triggered method is presented in Section 3. A numerical example is shown in Section 4 to validate the theoretical conclusions and illustrate the performance and advantages of the method. An application of the proposed method to the kinematic control of redundant robot manipulators is shown in Section 5, followed by conclusions and final remarks given in Section 6.

2. Problem formulation and preliminary

In this section, we describe the problem investigated in the paper and give some useful preliminaries.

2.1. Problem formulation

In this paper, we are concerned with the following quadratic programme with an equality constraint:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{x}^{\mathrm{T}}\mathbf{H}\mathbf{x}/2, \\ \text{s.t.} \quad & \mathbf{A}\mathbf{x} = \mathbf{b}, \end{aligned} \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the decision variable; $\mathbf{H} \in \mathbb{R}^{n\times n}$ is a symmetric and positive definite matrix; matrix $\mathbf{A} \in \mathbb{R}^{m\times n}$ is full row-rank; and $\mathbf{b} \in \mathbb{R}^m$ is a vector.

As in existing results of primal-dual gradient dynamics, we adopt the following assumption (Qu & Li, 2019).

Assumption 2.1: *There exists $\kappa_1 > 0 \in \mathbb{R}$ and $\kappa_2 > 0 \in \mathbb{R}$ such that $\mathbf{A}\mathbf{A}^{\mathrm{T}} - \kappa_1\mathbf{I}_m \succeq 0$ and $\kappa_2\mathbf{I}_m - \mathbf{A}\mathbf{A}^{\mathrm{T}} \succeq 0$ (i.e. they are positive semidefinite), where $\mathbf{I}_m \in \mathbb{R}^{m\times m}$ denotes the m-by-m identity matrix.*

In this paper, we are interested in designing an event-triggered method to solve the optimisation problem (1).

2.2. Time-triggered primal-dual gradient dynamics

To lay a basis for latter comparison and discussion, the time-triggered primal-dual gradient dynamics for solving equality-constrained quadratic programme (1) is given as follows (Cherukuri et al., 2016; Feijer & Paganini, 2010; Qu & Li, 2019):

$$\begin{aligned} \rho\dot{\mathbf{x}} &= -\nabla_{\mathbf{x}}L(\mathbf{x},\lambda) = -\mathbf{H}\mathbf{x} - \mathbf{A}^{\mathrm{T}}\lambda, \\ \rho\dot{\lambda} &= \eta\nabla_{\lambda}L(\mathbf{x},\lambda) = \eta(\mathbf{A}\mathbf{x} - \mathbf{b}), \end{aligned} \tag{2}$$

where the Lagrangian function is defined as

$$L(\mathbf{x},\lambda) = \mathbf{x}^{\mathrm{T}}\mathbf{H}\mathbf{x}/2 + \lambda^{\mathrm{T}}(\mathbf{A}x - \mathbf{b}),$$

with $\lambda \in \mathbb{R}^m$ being the Lagrangian multiplier; $\eta > 0 \in \mathbb{R}$ and $\rho > 0 \in \mathbb{R}$ are design parameters scaling the feedback strength. Let vector $\mathbf{z} = [\mathbf{x}^{\mathrm{T}}, \lambda^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{m+n}$. The time-triggered primal-dual gradient dynamics can be rewritten as

$$\rho\dot{\mathbf{z}} = f(\mathbf{z}) \tag{3}$$

with

$$f(\mathbf{z}) = \mathbf{Q}\mathbf{z} - \begin{bmatrix} \mathbf{0} \\ \eta\mathbf{b} \end{bmatrix}, \tag{4}$$

where matrix $\mathbf{Q}$ is given as follows:

$$\mathbf{Q} = \begin{bmatrix} -\mathbf{H} & -\mathbf{A}^{\mathrm{T}} \\ \eta\mathbf{A} & \mathbf{O} \end{bmatrix}.$$

From the control perspective (Jin et al., 2017), the primal-dual gradient dynamics can be viewed as a first-order system as follows:

$$\rho\dot{\mathbf{z}} = \mathbf{u},$$

with the input $\mathbf{u} = f(\mathbf{z})$, which drives the state variable $\mathbf{z}$ to converge to the theoretical solution of (1). Note that (Jin et al., 2017) deals with the problem of nonlinear equations by using a time-triggered approach, while our method proposed in this paper deals with the quadratic programme problem by using an even-triggered approach. Thus, the two works are totally different.

About the time-triggered primal-dual gradient dynamics (2), we have the following lemma.

Lemma 2.1: *If Assumption 2.1 holds, the equilibrium of (2) is identical to the solution of (1) and it is globally exponentially stable.*

Proof: The proof can be readily generalised from the proof of Theorem 1 in Qu and Li (2019). ■

The analysis on the robustness of the primal-dual gradient dynamics can be found in Nguyen et al. (2018). As seen from Equation (2), when directly using primal-dual gradient dynamics to solve equality-constrained quadratic programme (1), the system states need to be monitored all the time and the term $f(\mathbf{z})$ need to be updated at each sampling instant. If the solution of the quadratic programme is used for a control system with $\dot{\mathbf{x}} = \mathbf{u}$ with $\mathbf{u}$ denoting the input, then we need to update the system input at each sampling instant. Such examples include the kinematic control problem of robot manipulators. Obviously, if the actuator state is frequently changed, the life time of it can be shortened, which is not desirable. Thus, an event-triggered approach can be beneficial, for which we still keep on monitoring the states of the system while changing the actuator state only when a well-defined event is detected.

It should be noted that the extension of the time-triggered primal-dual gradient dynamics to the event-triggered one is not trivial since the equilibrium of the dynamical system is not explicitly given. In particular, the condition $\|f(\mathbf{z})\|_2 \le \chi\|\mathbf{z}\|_2$ with $\chi > 0 \in \mathbb{R}$ is not satisfied, which is widely adopted in literature about event-triggered control when analysing the minimum inter-event time, such as Tabuada (2007) and Zhu et al. (2017). Meanwhile, the derivation of the triggering condition should get rid of the Zeno behaviour (Tabuada, 2007; Zhu et al., 2017).

The difference between the time-triggered approach and the even-triggered approach for the primal-dual gradient dynamics lies in the updating for $\dot{\mathbf{z}}$. For the time-triggered approach, $\dot{\mathbf{z}}$ is updated in each sampling instant, while $\dot{\mathbf{z}}$ is only updated when an event occurs in our event-triggered approach. The advantage of the event-triggered approach lies in the reduction of the updating frequency of $\dot{\mathbf{z}}$ without significantly sacrificing the solution performance.

3. Event-triggered primal-dual gradient dynamics

In this section, the event-triggered primal-dual gradient dynamics is derived to solve the problem described in the previous section. For event-triggered method, there are two major issues to be addressed, i.e. the triggering condition and the minimum inter-event time. Specifically, we need a well-defined condition with respect to the state of the system to check whether we need to modify the input to the system, and we need to guarantee that the minimum difference between two consecutive event times is strictly larger than 0.

Let $\{t_k\}_{k=0}^{\infty}$ with $t_{k+1} > t_k$ denote the time when an event happens, where $k \in \{0, 1, 2, \ldots\}$. The event-triggered primal-dual gradient dynamics can is described as

$$\rho\dot{\mathbf{z}}(t) = \mathbf{u}(t) = f(\mathbf{z}(t_k)), \quad \forall\, t \in [t_k, t_{k+1}). \tag{5}$$

We assume that the first event occurs at time $t_0 = 0$. If at time $t \in (t_k, +\infty)$, an event occurs, the term $\mathbf{u}(t)$ is updated by using the latest sampled state. An event occurs when a triggering condition described by an inequality is satisfied. Note that, in some literature, such a condition is also defined by an equality. In practice, an inequality condition is more robust than an equality one. For the sake of presentation, we define the state measurement error as follows:

$$\varepsilon(t) = \mathbf{z}(t_k) - \mathbf{z}(t), \quad \forall\, t \in [t_k, t_{k+1}).$$

As stated above, the triggering condition is directly related to the state measurement error.

3.1. Triggering condition

In this subsection, we derive the triggering condition for the primal-dual gradient dynamics (5). Note that, for the convenience of presentation, the argument t is omitted somewhere.

Let $\mathbf{z}_k = \mathbf{z}(t_k)$. According to Equation (4), we have

$$f(\mathbf{z}_k) = \mathbf{Q}\mathbf{z}_k + \begin{bmatrix} \mathbf{0} \\ -\eta\mathbf{b} \end{bmatrix}.$$

Let

$$\mathbf{P} = \begin{bmatrix} \eta c\mathbf{H} & \eta\mathbf{A}^{\mathrm{T}} \\ \eta\mathbf{A} & c\mathbf{I} \end{bmatrix},$$

where $c = 4\max\{l, \eta\kappa_2/\nu\}$ with $l = \lambda_{\max}(\mathbf{H}) > 0$ and $\nu = \lambda_{\min}(\mathbf{H}) > 0$. Evidently, there exist $\bar{p}$ and $\bar{q}$ such that $\|\mathbf{P}\|_{\mathrm{F}} \le \bar{p}$ and $\|\mathbf{Q}\|_{\mathrm{F}} \le \bar{q}$ due to the boundedness of the elements in the two matrices. Let $\mathbf{M} = -\mathbf{Q}^{\mathrm{T}}\mathbf{P} - \mathbf{P}\mathbf{Q}$.

Note that, for matrix $\mathbf{M}$ we have the following lemma.

Lemma 3.1 (Qu & Li, 2019): *Let* $\tau = \min\{\frac{\eta\kappa_1}{4l}, \frac{\kappa_1\nu}{4\kappa_2}\}$. *Then,* $\mathbf{M} \succeq \tau\mathbf{P}$.

Based on Lemmas 2.1 and 3.1, we have the following theorem to show the triggering condition, which guarantees the convergence of the corresponding event-triggered primal-dual gradient dynamics.

Theorem 3.1: *If Assumption* 2.1 *holds, the optimisation problem* (1) *is globally asymptotically solved by event-triggered primal-dual gradient dynamics* (5) *with the triggering condition given as follows:*

$$\|\varepsilon\|_2 \geq \frac{(1-\alpha)\underline{\lambda}\|f(\mathbf{z})\|_2}{2\bar{p}\bar{q}^2}. \tag{6}$$

Proof: Consider the Lyapunov candidate function

$$V = \rho f^{\mathrm{T}}(\mathbf{z})\mathbf{P}f(\mathbf{z}).$$

Because matrix $\mathbf{P}$ is symmetric and positive-definite (Qu & Li, 2019), V is nonnegative. The derivative of V along the state trajectory of event-triggered primal-dual gradient dynamics (5) is then calculated as follows:

$$\begin{aligned}
\dot{V} &= \rho\left(\frac{\partial f(\mathbf{z})}{\partial \mathbf{z}}\dot{\mathbf{z}}\right)^{\mathrm{T}}\mathbf{P}f(\mathbf{z}) + \rho f^{\mathrm{T}}(\mathbf{z})\mathbf{P}\frac{\partial f(\mathbf{z})}{\partial \mathbf{z}}\dot{\mathbf{z}} \\
&= (\mathbf{Q}f(\mathbf{z}_k))^{\mathrm{T}}\mathbf{P}f(\mathbf{z}) + f^{\mathrm{T}}(\mathbf{z})\mathbf{P}\mathbf{Q}f(\mathbf{z}_k) \\
&= f^{\mathrm{T}}(\mathbf{z}_k)\mathbf{Q}^{\mathrm{T}}\mathbf{P}f(\mathbf{z}) + f^{\mathrm{T}}(\mathbf{z})\mathbf{P}\mathbf{Q}f(\mathbf{z}_k) \\
&= (f(\mathbf{z}) + f(\mathbf{z}_k) - f(\mathbf{z}))^{\mathrm{T}}\mathbf{Q}^{\mathrm{T}}\mathbf{P}f(\mathbf{z}) + f^{\mathrm{T}}(\mathbf{z})\mathbf{P}\mathbf{Q}(f(\mathbf{z}) \\
&\quad + f(\mathbf{z}_k) - f(\mathbf{z})) \\
&= f^{\mathrm{T}}(\mathbf{z})\mathbf{Q}^{\mathrm{T}}\mathbf{P}f(\mathbf{z}) + f^{\mathrm{T}}(\mathbf{z})\mathbf{P}\mathbf{Q}f(\mathbf{z}) + 2f^{\mathrm{T}}(\mathbf{z})\mathbf{P}\mathbf{Q}\mathbf{Q}\varepsilon \\
&= -f^{\mathrm{T}}(\mathbf{z})\mathbf{M}f(\mathbf{z}) + 2f^{\mathrm{T}}(\mathbf{z})\mathbf{P}\mathbf{Q}\mathbf{Q}\varepsilon \\
&\leq -f^{\mathrm{T}}(\mathbf{z})\mathbf{M}f(\mathbf{z}) + 2\|f\|_2\|\mathbf{P}\|_{\mathrm{F}}\|\mathbf{Q}\|_{\mathrm{F}}^2\|\varepsilon\|_2.
\end{aligned}$$

To guarantee the asymptotic convergence of the system, the triggering condition needs to make the following inequality hold:

$$\dot{V} \leq -\alpha f^{\mathrm{T}}(\mathbf{z})\mathbf{M}f(\mathbf{z})$$

with $0 < \alpha < 1$ being a design parameter to scale the sacrifice of convergence rate due to the introduction of the event-triggering mechanism, i.e.

$$\begin{aligned}
&-f^{\mathrm{T}}(\mathbf{z})\mathbf{M}f(\mathbf{z}) + 2\|f\|_2\|\mathbf{P}\|_{\mathrm{F}}\|\mathbf{Q}\|_{\mathrm{F}}^2\|\varepsilon\|_2 \\
&\quad \leq -\alpha f^{\mathrm{T}}(\mathbf{z})\mathbf{M}f(\mathbf{z}).
\end{aligned} \tag{7}$$

Based on Lemma 3.1, there exists $\underline{\lambda} > 0 \in \mathbb{R}$ such that, for all $\mathbf{z}$, the following inequality holds:

$$f^{\mathrm{T}}(\mathbf{z})\mathbf{M}f(\mathbf{z}) \geq \underline{\lambda}\|f(\mathbf{z})\|_2^2.$$

Together with

$$2\|f(\mathbf{z})\|_2\|\mathbf{P}\|_{\mathrm{F}}\|\mathbf{Q}\|_{\mathrm{F}}^2\|\varepsilon\|_2 \leq 2\|f(\mathbf{z})\|_2\bar{p}\bar{q}^2\|\varepsilon\|_2,$$

inequality (7) can be guaranteed by making

$$(1-\alpha)\underline{\lambda}\|f(\mathbf{z})\|_2^2 \geq 2\|f(\mathbf{z})\|_2\bar{p}\bar{q}^2\|\varepsilon\|_2,$$

i.e.

$$\|\varepsilon\|_2 \leq \frac{(1-\alpha)\underline{\lambda}\|f(\mathbf{z})\|_2}{2\bar{p}\bar{q}^2},$$

which is guaranteed by the triggering condition (6). As a result, we have

$$\dot{V} \leq -\alpha f^{\mathrm{T}}(\mathbf{z})\mathbf{M}f(\mathbf{z}),$$

which together with Lemma 3.1, gives

$$\dot{V} \leq -\alpha\tau f^{\mathrm{T}}(\mathbf{z})\mathbf{P}f(\mathbf{z}) = -\alpha\tau V/\rho,$$

by which V exponentially converges to zero with the decay rate being $\alpha\tau/\rho$ and, consequently, $f(\mathbf{z})$ exponentially converges to zero. According to Lemma 2.1, the solution to (1) is identical to the solution of $f(\mathbf{z}) = 0$. Thus, it is concluded that the state variable of the event-triggered primal-dual gradient dynamics asymptotically converges to the solution to optimisation problem (1). Together with the fact that the Lyapunov candidate function is radially unbounded, we further concluded that state variable of the event-triggered primal-dual gradient dynamics globally asymptotically converges to the solution to optimisation problem (1). The proof is complete. ■

The proposed event-triggered primal-dual gradient dynamics consists of (5) and the triggering condition (6). If (6) is satisfied, then the term $\mathbf{u}(t)$ in (5) is updated with $\mathbf{u}(t) = f(\mathbf{z}(t))$; Otherwise, the value of $\mathbf{u}(t)$ is not changed. Such dynamics is suitable for hardware implementation.

Regarding the Lyapunov candidate function in Theorem 3.1, we have the following remark.

Remark 3.1: In the adopted Lyapunov candidate function, the solution of the equality-constrained quadratic programme is not explicitly included.

Instead, the right-hand side of the original time-triggered primal-dual gradient dynamics, i.e. $f(\mathbf{z})$, is used. As the equilibrium of the time-triggered primal-dual gradient dynamics corresponds to the solution of the equality-constrained quadratic programme, the right-hand side of it actually provides a measure of the difference between the current state value and the theoretical solution. In fact, we cannot include the theoretical solution of the equality-constrained quadratic programme into the Lyapunov candidate function as it will lead to a triggering condition with respect to the theoretical solution, which is infeasible in practice.

Regarding the intuition of the event-triggering primal-dual dynamics, we have the following remark.

Remark 3.2: As seen from the proof of Theorem 3.1, essentially, the event-triggering primal-dual dynamics is achieved via sacrificing the convergence rate of the corresponding Lyapunov candidate function, which is scaled by the coefficient α, while guaranteeing the negativeness of the derivative of it along the state trajectory. As a result, although the convergence can be a bit slower, the global and asymptotic convergence properties are preserved. The benefit of it is that during a certain time region, the input to the system does not need to be changed, i.e. control actions do not need to be updated. It can be expected that, if the value of α is set smaller, the number of triggered events would also be smaller.

3.2. Minimum inter-event time

In this subsection, we analyse the minimum inter-event time of the event-triggered primal-dual dynamics (3) with the triggering condition given in (6). The analysis for the minimum inter-event time is necessary to guarantee that the event-triggered method is feasible in practice.

The minimum time interval between two consecutive event times is defined as follows:

$$\Delta_{\min} = \min_{k=0,1,2,\ldots} \{t_{k+1} - t_k\},$$

which is often referred to as the minimum inter-event time in the event-triggered control community (Berneburg & Nowzari, 2021). To make event-triggered method implementable in practice, it is necessary to avoid the existence of the Zeno behaviour (Chen & Ling, 2021; Chen et al., 2021; Yu & Chen, 2021), i.e. an infinite number of events are generated in finite time. As a result, it is necessary to theoretically guarantee that the minimum inter-event time is not trivial, i.e. strictly larger than zero.

For the proposed event-triggered primal-dual dynamics (3) with the triggering condition given in (6), we have the following theorem to guarantee that its minimum inter-event time is non-trivial.

Theorem 3.2: *The minimum inter-event time $\Delta_{\min}$ of the event-triggered primal-dual gradient dynamics* (5) *with the triggering condition given in* (6) *is strictly larger than zero.*

Proof: Suppose that at time t_k the triggering condition is satisfied. Then, according to the triggering condition (6), the next event time is determined by

$$\arg\min_{t>t_k}\left\{\|\varepsilon(t)\|_2 \geq \frac{(1-\alpha)\underline{\lambda}\|f(\mathbf{z}(t))\|_2}{2\bar{p}\bar{q}^2}\right\}.$$

Note that, when an event is invoked, we have $\varepsilon(t) = 0$. Thus, the inter-event time is determined by the evolution of $\|\varepsilon(t)\|_2/\|f(\mathbf{z}(t))\|_2$ from 0 to $1/C$ with

$$\frac{1}{C} = \frac{(1-\alpha)\underline{\lambda}}{2\bar{p}\bar{q}^2} > 0.$$

Note that we have

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\frac{\|\varepsilon\|_2}{\|f(\mathbf{z})\|_2} \\
&= \frac{\sqrt{\varepsilon^{\mathrm{T}}\varepsilon}}{\sqrt{f^{\mathrm{T}}(\mathbf{z})f(\mathbf{z})}} \\
&= \frac{\frac{1}{\sqrt{\varepsilon^{\mathrm{T}}\varepsilon}}\varepsilon^{\mathrm{T}}\dot{\varepsilon}\sqrt{f^{\mathrm{T}}(\mathbf{z})f(\mathbf{z})} - \frac{1}{\sqrt{f^{\mathrm{T}}(\mathbf{z})f(\mathbf{z})}}f^{\mathrm{T}}(\mathbf{z})\frac{\mathrm{d}f(\mathbf{z})}{\mathrm{d}t}\sqrt{\varepsilon^{\mathrm{T}}\varepsilon}}{f^{\mathrm{T}}(\mathbf{z})f(\mathbf{z})} \\
&= \frac{\frac{1}{\sqrt{\varepsilon^{\mathrm{T}}\varepsilon}}\varepsilon^{\mathrm{T}}\dot{\varepsilon}\sqrt{f^{\mathrm{T}}(\mathbf{z})f(\mathbf{z})} - \frac{1}{\sqrt{f^{\mathrm{T}}(\mathbf{z})f(\mathbf{z})}}f^{\mathrm{T}}(\mathbf{z})\mathbf{Q}\dot{\mathbf{z}}\sqrt{\varepsilon^{\mathrm{T}}\varepsilon}}{f^{\mathrm{T}}(\mathbf{z})f(\mathbf{z})} \\
&= \frac{\frac{1}{\sqrt{\varepsilon^{\mathrm{T}}\varepsilon}}\varepsilon^{\mathrm{T}}\dot{\varepsilon}\sqrt{f^{\mathrm{T}}(\mathbf{z})f(\mathbf{z})} - \frac{1}{\sqrt{f^{\mathrm{T}}(\mathbf{z})f(\mathbf{z})}}f^{\mathrm{T}}(\mathbf{z})\mathbf{Q}f(\mathbf{z})\sqrt{\varepsilon^{\mathrm{T}}\varepsilon}/\rho}{f^{\mathrm{T}}(\mathbf{z})f(\mathbf{z})} \\
&= \frac{-\varepsilon^{\mathrm{T}}f(\mathbf{z})/\rho}{\|\varepsilon\|_2\|f(\mathbf{z})\|_2} - \frac{f^{\mathrm{T}}(\mathbf{z})\mathbf{Q}f(\mathbf{z})\|\varepsilon\|_2/\rho}{\|f(\mathbf{z})\|_2^3} \\
&\leq \left(\frac{\|\varepsilon\|_2\|f(\mathbf{z})\|_2}{\|\varepsilon\|_2\|f(\mathbf{z})\|_2} + \frac{\bar{q}\|f(\mathbf{z})\|_2^2\|\varepsilon\|_2}{\|f(\mathbf{z})\|_2^3}\right)/\rho \\
&= \frac{1}{\rho}\left(1 + \frac{\bar{q}\|\varepsilon\|_2}{\|f(\mathbf{z})\|_2}\right),
\end{aligned}$$

where $\bar{q} \geq \|\mathbf{Q}\|_F > 0$. Let

$$y = \frac{\|\varepsilon\|_2}{\|f(\mathbf{z})\|_2}.$$

By the above analysis, $\dot{y} \leq (1 + \bar{q}y)/\rho$, and we conclude that $y(t) \leq \varphi(t)$, where $\varphi(t)$ is the solution of the following equation:

$$\dot{\varphi} = \frac{1}{\rho}(1 + \bar{q}\varphi)$$

with $\varphi(0) = 0$. Then, the inter-execution time is lower bounded by the solution $t_{\min}$ of $\varphi(t_{\min}) = 1/C$. By solving the differential equation with $\varphi(0) = 0$, we have

$$\varphi(t) = \frac{1}{\bar{q}}\left(\exp\left(\frac{\bar{q}}{\rho}t\right) - 1\right),$$

where $\bar{q} > 0$ and $\rho > 0$. It is obvious that $\varphi(t)$ is strictly increasing on $[0, +\infty)$ because

$$\dot{\varphi}(t) = \frac{1}{\rho}\exp(\bar{q}t) > 0, \quad \forall\, t \in [0, +\infty).$$

Together with $\varphi(0) = 0$ and $\varphi(t_{\min}) = 1/C > 0$, it is concluded that $t_{\min} > 0$. Therefore, the minimum inter-event time $\Delta_{\min}$ of the event-triggered primal-dual gradient dynamics (5) with the triggering condition given in (6) is strictly larger than zero. The proof is complete. ■

The above theorem shows that the minimum inter-event time is strictly larger than zero, and thus events will not be triggered all the time. Consequently, compared with time-triggered approaches, such as Cherukuri et al. (2016), Feijer and Paganini (2010), and Qu and Li (2019), for solving the optimisation problem, where events are actually triggered all the time, the proposed method does not require to update the system input at each sampling instant. Thus, it can save computational resources and potentially enhance the life time of associated actuators in practice. With Theorems 3.1 and 3.2, we show that even-triggered primal-dual gradient dynamics is theoretically feasible, and the convergence performance is still satisfactory. This makes it possible to use such event-triggered dynamics to deal with control problems. For example, as demonstrated in the robot manipulator application, the event-triggering mechanism allows more flexible updating of control inputs to the robot manipulator.

4. Numerical example

In this section, a numerical example is presented to verify the theoretical results and substantiate the effectiveness of the proposed event-triggered primal-dual gradient dynamics for solving equality-constrained quadratic programmes.

We verify the theoretical results by a numerical example as (1) with $\mathbf{H} = [1, 0; 0, 1]$, $\mathbf{x} = [x_1, x_2]^{\mathrm{T}}$, $\mathbf{A} = [1, 2]$, and $\mathbf{b} = [3]$, for which the theoretical solution is $\mathbf{x}^* = [0.6, 1.2]^{\mathrm{T}}$. This example is adopted because it is easy to verify the performance of the proposed event-triggered primal-dual gradient dynamics for this example, such as the solution accuracy. We set $\eta = 1$ and $\rho = 1$. Then, based on the aforementioned definitions, for this example, we have

$$\mathbf{Q} = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -2 \\ 1 & 2 & 0 \end{bmatrix}, \quad \mathbf{A}\mathbf{A}^{\mathrm{T}} = 5.$$

Thus, $\|\mathbf{Q}\|_F \approx 3.46$. As a result, $\bar{q}$ is set to 3.5, and κ_2 is set to 6. Because $\mathbf{H}$ is an identity matrix, we have $\nu = \lambda_{\min}(\mathbf{H}) = 1$ and $l = \lambda_{\max}(\mathbf{H}) = 1$, by which $c = 4\max\{l, \eta\kappa_2/\nu\} = 24$. Then, matrix $\mathbf{P}$ is obtained as follows:

$$\mathbf{P} = \begin{bmatrix} 24 & 0 & 1 \\ 0 & 24 & 2 \\ 1 & 2 & 24 \end{bmatrix}.$$

Thus, $\|\mathbf{P}\|_F \leq \bar{p} = 42$. So, $\bar{p}$ is set to 42. Then, matrix $\mathbf{M} = -\mathbf{Q}^{\mathrm{T}}\mathbf{P} - \mathbf{P}\mathbf{Q}$ is

$$\mathbf{M} = \begin{bmatrix} 46 & -4 & 1 \\ -4 & 40 & 2 \\ 1 & 2 & 10 \end{bmatrix},$$

and we have $\lambda_{\min}(\mathbf{M}) \approx 41.37$. Thus, we set $\underline{\lambda} = 41.36$. In addition, we set $\alpha = 0.1$. The simulation is conducted on Matlab by using the Euler difference rule with the step size being 1.0×10^{-4}. As seen from Figure 2(a), the state variables converges to the theoretical solution of the optimisation problem, which is also indicated by Figure 2(c) where the Lyapunov candidate function $V(t)$ asymptotically converges to zero. The convergence of the Lagrangian multiplier is shown in Figure 2(b). The above results verify Theorem 3.1. As seen from Figure 2(d), there are less than 600 triggered events among the $20/(1 \times 10^{-4}) = 2.0 \times 10^5$ sample instants, which shows the efficiency of the proposed event-triggered method compared with

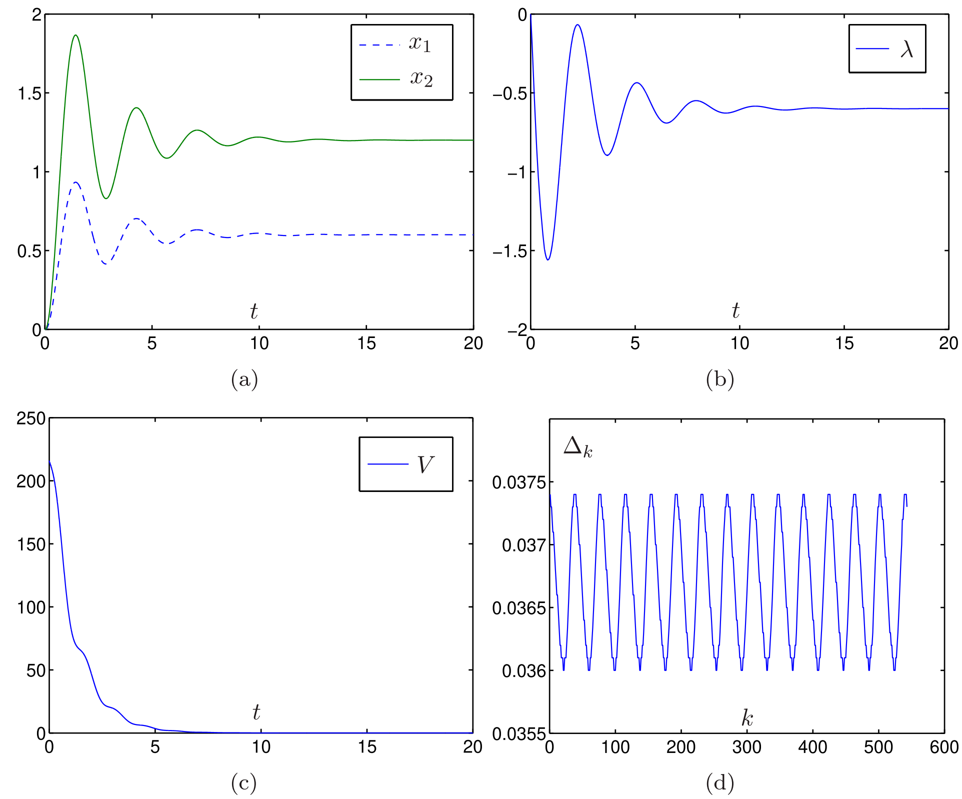

Figure 2. Data profiles for the simulation when the event-triggered primal-dual gradient dynamics is employed to solve the optimisation problem with initial state $\mathbf{x}(0) = [0, 0]^{\mathrm{T}}$. (a) Profiles of $\mathbf{x}$. (b) Profile of λ. (c) Profile of the Lyapunov candidate function V. (d) Profile of inter-event time $\Delta_k = t_{k+1} - t_k$ with k denoting the event index.

time-triggered ones. The average inter-event time is 3.67×10^{-2}, which is much larger than the step size 1.0×10^{-4}. Besides, as seen from Figure 2(d), the minimum inter-event time is not trivial (larger than 0.035), which verifies Theorem 3.2. Note that the triggered instants can be readily found from Figure 2(d) in view of $\Delta_k = t_{k+1} - t_k$ in which t_k is the triggered instant and $t_0 = 0$.

Owing to the fact that our work is on developing an event-triggered model for the primal-dual gradient dynamics, we only compare the model with the time-triggered primal-dual gradient dynamics. To our knowledge, there is no other methods for solving the equality-constrained quadratic optimisation problem via an event-triggered approach. To compare the performance of the proposed event-triggered primal-dual gradient dynamics with the time-triggered primal-dual gradient dynamics, the simulation results under the same setting when the latter is employed is shown in Figure 3. As seen from Figures 2 and 3, the convergence rate of the event-triggered primal-dual gradient dynamics is similar to the time-triggered primal-dual gradient dynamics, and the differences about the profiles of $\mathbf{x}$, λ, and V is quite small. This is consistent with the statement that the event-triggered primal-dual gradient dynamics can achieve almost the same solution performance with the time-triggered primal-dual gradient dynamics. The proposed method share similar advantages of event-triggered methods over time-triggered methods, i.e. the updating frequency for $\dot{\mathbf{z}}$ is reduced. For the time-triggered primal-dual gradient dynamics, $\dot{\mathbf{z}}$ should be updated in each sampling instant, while $\dot{\mathbf{z}}$ is only updated when an event occurs in our method without significantly sacrificing the solution performance.

To show whether the initial values affect the convergence of the proposed event-triggered primal-dual gradient dynamics, i.e. the global convergence property, simulation results for other two sets of initial values for the same problem are also presented. In the simulations, the parameters are set the same as the above, except that we use different initial values for $\mathbf{x}(0)$ and α is set to 0.5. As seen from Figure 4, with initial state $\mathbf{x}(0) = [100, 100]^{\mathrm{T}}$, the proposed event-triggered primal-dual gradient dynamics can still guarantee the convergence to the optimal solution

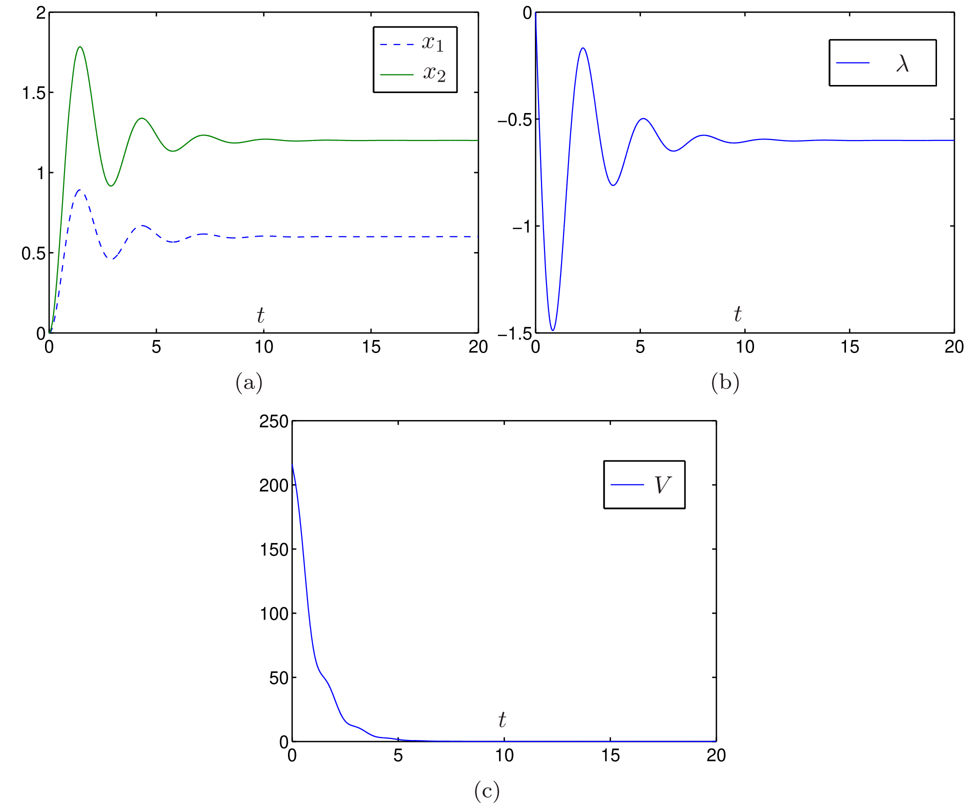

Figure 3. Data profiles for the simulation when time-triggered primal-dual gradient dynamics (2) is employed to solve the optimisation problem with initial state $\mathbf{x}(0) = [0, 0]^{\mathrm{T}}$. (a) Profiles of $\mathbf{x}$. (b) Profile of λ. (c) Profile of the Lyapunov candidate function V.

to the equality-constrained quadratic programme with the Lyapunov function asymptotically converging to zero. Note that, compared with the previous simulation shown in Figure 2, due to the much larger initial error, the state variable takes more time to converge to the theoretical solution. Besides, as seen from Figure 4(c), during the solution process, less than 1000 events are triggered, among the 2.0×10^5 sample instants, and the minimum inter-event time is larger than 0.02. As seen from Figure 5, for the case with $\mathbf{x}(0) = [-100, -100]^{\mathrm{T}}$, the above results also hold. Thus, the proposed event-triggered primal-dual gradient dynamics is globally convergent.

5. Application to kinematic control of redundant robot manipulators

To show the practical value of the proposed method, let us consider the kinematic control problem of a redundant robot manipulator (Li & Li, 2022; Ouyang et al., 2006). Let $\mathbf{r} \in \mathbb{R}^m$ denote the end-effector coordinate of a serial robot manipulator and $\theta \in \mathbb{R}^n$ denote its joint angle vector. Let $\omega = \dot{\theta} = \mathrm{d}\theta/\mathrm{d}t$ denote the joint velocity command, which is the input to the robot manipulator. The forward kinematics $\mathbf{r} = \phi(\theta)$ of a serial robot manipulator can be theoretically derived by following the steps of the D-H convention (Zhang & Jin, 2018).

According to Zhang and Jin (2018), for the end-effector regulation problem, the minimum-velocity-norm kinematic control scheme can be formulated as

$$\min_{\omega} \quad \frac{1}{2}\omega^{\mathrm{T}}\omega, \tag{8a}$$

$$\text{s.t.} \quad J(\theta)\omega = -\gamma(\phi(\theta) - \mathbf{r}_{\mathrm{d}}), \tag{8b}$$

where $J(\theta) = \partial\phi(\theta)/\partial\theta$ is called the Jacobian matrix of the forward kinematics, $\mathbf{r}_{\mathrm{d}} \in \mathbb{R}^m$ describes the desired end-effector coordinate, and $\gamma > 0 \in \mathbb{R}$ is a gain parameter. In the optimal kinematic control, it is desired to minimise the joint velocity norm so as to save energy while complete the given task. Thus, in the problem formulation of the kinematic control, the performance index is chosen as the joint velocity norm, and the equality constraint is an equivalent velocity-level description for the end-effector tracking task. Compared with non-optimal control approaches, the optimal kinematic control can help

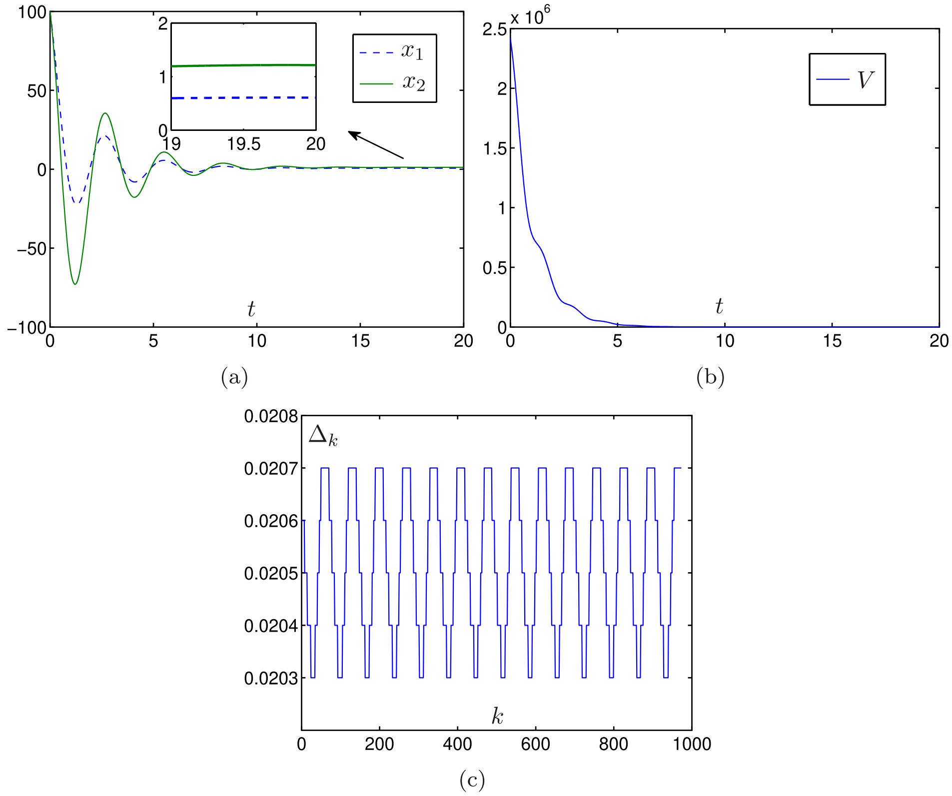

Figure 4. Data profiles for the simulation when the event-triggered primal-dual gradient dynamics is employed to solve the optimisation problem with initial state $\mathbf{x}(0) = [100, 100]^{\mathrm{T}}$. (a) Profiles of $\mathbf{x}$. (b) Profile of the Lyapunov candidate function V. (c) Profile of inter-event time $\Delta_k = t_{k+1} - t_k$ with k denoting the event index.

optimise a performance index of interest, such as the velocity norm considered in the simulation. The problem (8a) can be viewed as quadratic programme (1) in this paper with $\mathbf{x} = \omega$, $H = I$, $A = J(\theta)$, and $\mathbf{b} = -\gamma(\phi(\theta) - \mathbf{r}_{\mathrm{d}})$. Thus, the proposed event-triggered optimisation can be applied to the kinematic control problem. Firstly, in this scenario, with the proposed event-triggered optimisation method, we only need to update the input (i.e. the joint velocity command) to the robot manipulator when the triggering condition is met. This could be help to enlarge the lifetime of joint actuators (Houtzager et al., 2013). Secondly, the controller may not be located in the vicinity of the robot manipulator. Therefore, a communication network is needed for the interaction between the controller and the manipulator. In this case, event-triggered control could help save communication burden (Kumar, 2019).

To adapt to this application, the triggering condition is relaxed to

$$\|\varepsilon(t)\|_2 \geq \frac{(1-\alpha)\underline{\lambda}\,|\,\|f(\mathbf{z}_k)\|_2 - \bar{q}\|\varepsilon(t)\|_2|}{2\chi}, \quad \forall\, t \in (t_k, t_{k+1}], \tag{9}$$

with constant $\chi \geq \|PQQ\|_F$, where $\varepsilon(t) = \mathbf{z}(t_k) - \mathbf{z}(t), \forall t \in [t_k, t_{k+1})$. Clearly, for the triggering condition (9), the terms α, $\underline{\lambda}$, χ, and $\bar{q}$ are constants, while $\|f(\mathbf{z}_k)\|_2$ only needs to be calculated for once during two successive triggering instants. We consider a PUMA560 robot manipulator (Zhang & Jin, 2018), for which $n = 6$ and $m = 3$. We use the proposed event-triggered primal-dual gradient dynamics to solve the optimal kinematic control problem (8), and the obtained real-time solution ω is used as the control input for the manipulator. In the application, the relevant parameters are set to $\alpha = 0.1$, $\underline{\lambda} = 0.01$, $\bar{q} = 4$, $\gamma = 2$, $\rho = 10^{-3}$, $\alpha = 0.8$, and $\chi = 20$. The simulation is performed using Euler difference rule in Matlab with the step size being 10^{-6} s. The initial joint angle is $\theta(0) = [\pi/6, \pi/6, \pi/6, \pi/6, \pi/6, \pi/6]^{\mathrm{T}}$ rad and the desired end-effector coordinate is set to $\mathbf{r}_{\mathrm{d}} = [0.46, 0.48, 0.1]^{\mathrm{T}}$ m.

The simulation results are shown in Figures 6 and 7, from which it is found that, with the aid of the event-triggered primal-dual gradient dynamics (5) with triggering condition (9), the end-effector of the robot manipulator is successfully regulated to the desired position. From the simulation data, it is found that

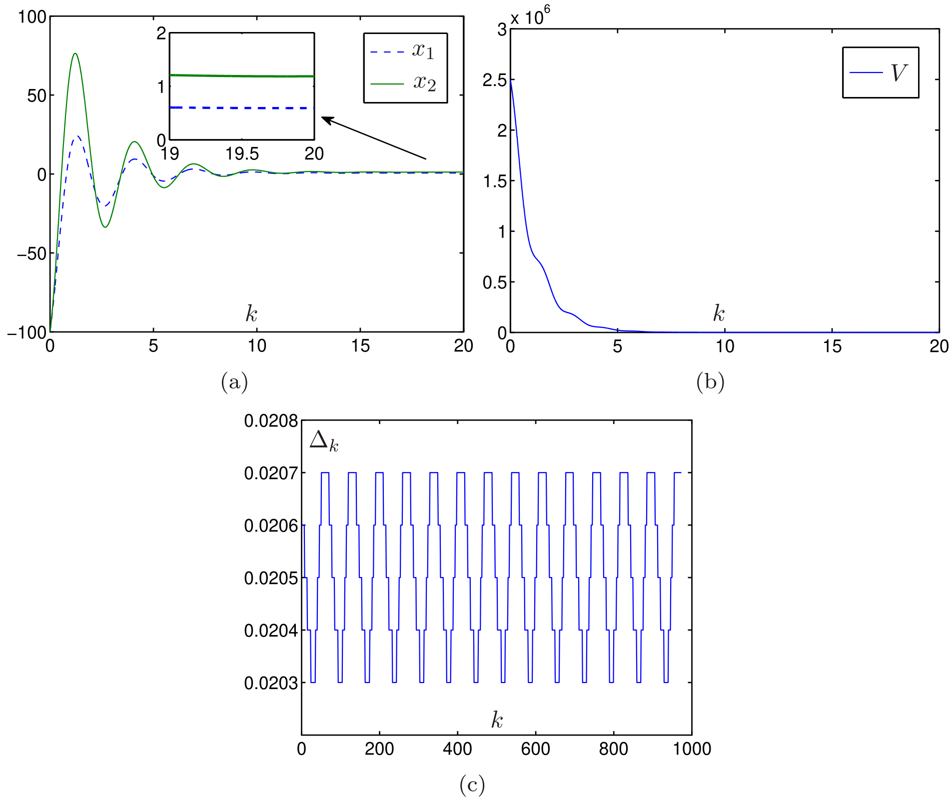

Figure 5. Data profiles for the simulation when the event-triggered primal-dual gradient dynamics is employed to solve the optimisation problem with initial state $\mathbf{x}(0) = [-100, -100]^{\mathrm{T}}$. (a) Profiles of $\mathbf{x}$. (c) Profile of the Lyapunov candidate function V. (d) Profile of inter-event time $\Delta_k = t_{k+1} - t_k$ with k denoting the event index.

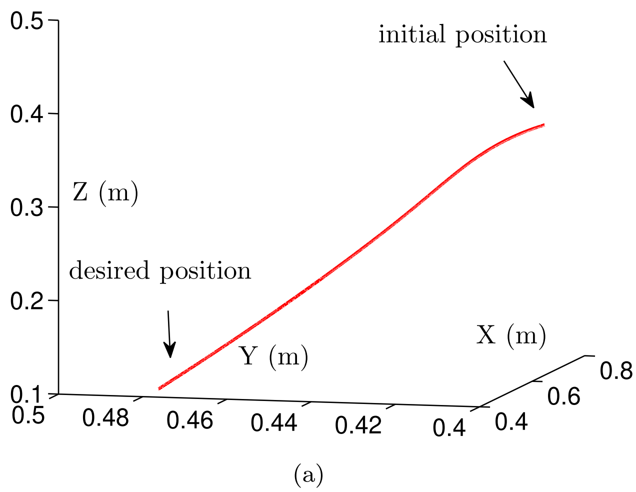

Figure 6. End-effector trajectory of the PUMA 560 robot manipulator during the end-effector regulation process with the aid of the event-triggered primal-dual gradient dynamics (5) with triggering condition (9).

there are totally 5×10^6 sampling instants, for which there are only 22,223 triggering events (meaning that the term $\|f(\mathbf{z}_k)\|_2$ only needs to be calculated in less than 5 thousandth of the sampling instants), and the minimum inter-event time is 2.25×10^{-4} s. The results substantiate the effectiveness of the proposed method for the robot manipulator application.

6. Conclusions

In this paper, the primal-dual gradient dynamics has been investigated under the event-triggering framework for solving the equality-constrained quadratic programmes. Theoretical results have shown that the proposed method guarantees the convergence to the solution of the problem and the minimum inter-event time is non-trivial. The theoretical results have been validated by a simulative example. The potential application of the proposed method has also been demonstrated. We would like to summarise the novelty of the paper as follow. This is the first work of event-triggered primal-dual gradient dynamics for solving equality-constrained quadratic optimisation problems with theoretically guaranteed asymptotic convergence and non-trivial minimum inter-event time. The extension of the current method to distributed optimisation (Cheng, 2011) will be our future work. The integration

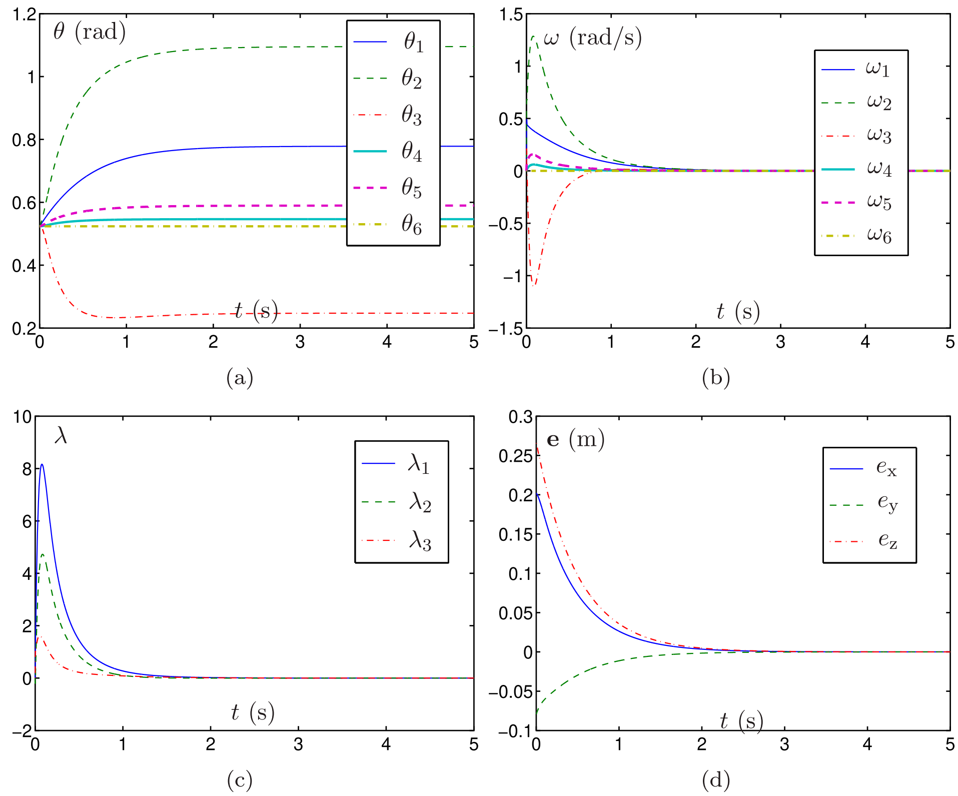

Figure 7. Data profiles for the application of the event-triggered primal-dual gradient dynamics (5) with triggering condition (9) to the end-effector regulation of the PUMA560 robot manipulator. (a) Joint angle θ. (b) Joint velocity command ω. (c) λ. (d) End-effector error $\mathbf{e} = \mathbf{r} - \mathbf{r}_\mathrm{d}$.

of the proposed method with reinforcement learning (Zamfirache et al., 2022) would also be interesting.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

This work was supported in part by the National Natural Science Foundation of China [grant number 62206109], and the Natural Science Foundation of Guangdong Province [grant number 2022A1515010976].

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