TOPOLOGICAL INTERFACE STATES IN ACOUSTIC METAMATERIALS

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Abstract:

This paper studies the topological interface modes at the interface of one-dimensional acoustic metamaterial chain in different unit cell configurations. The chain is composed of two sublattices that are inverted copies of each other and connected at the interface. The corresponding governing equations for the bulk band and frequency response function and eigenvalues of the finite chain are given and solved. The obtained results reveal the existence of interface modes within the bandgaps, showing the potential of the presented system to create sub-wavelength interface modes. The effect of stronger stiffness at the interface will be presented to show its influence on interface modes.

Key words: topological interface modes, acoustic metamaterials, bulk band, Floquet-Bloch theorem

1. Introduction

Advances in the physics of topological wave insulators and topological classification of phases of matter [1] initiated a research on toplogically protected edge modes in phononic crystals and metamaterials in their mechanical setup. The most intriguing feature of edge modes is their robustness to dectects and disorder induced backscattering. If one observes the mechanical wave analogues of the quantum valley and quantum spin Hall effetcs that can be achived by breaking the inversion symmetry, then edge modes can be achived in classical mechanical systems [2]. Here, we will investigate topological interface states in the one-dimensional acoustic metamaterial chain, where interface modes are localized at the interface of two sub-latticies that are inverted copies of each other (see Fig.1b).

2. Problem formulation and numerical results

The motion equations of the interface mass and the resonator of the acoustic metamaterial (see Figs. 1a and 1b) are given as

$$m_a u_{1a}^0 + k_1(2u_{1a}^0 - u_{2a}^0 - u_{2a}^{-1}) + k_b(u_{1a}^0 - u_{1b}^0) = 0, \tag{1}$$

$$m_b u_{1b}^0 + k_b(u_{1b}^0 - u_{1a}^0) = 0, \tag{2}$$

where m_a is the connecting chain mass, $k_1 = k(1+\gamma)$ and $k_2 = k(1-\gamma)$ are alternating stiffnesses with k being the mean stiffness and γ is the dimensionless stiffness parameter, while k_b and m_b denotes the stiffness and mass of the local resonators, respectively. For the sub-lattice I on the left side of the interface the equations are given as

$$m_a u_{1a}^p + k_2(u_{1a}^p - u_{2a}^p) + k_1(u_{1a}^p - u_{2a}^{p-1}) + k_b(u_{1a}^p - u_{1b}^p) = 0, \tag{3}$$

$$m_b u_{1b}^p + k_b(u_{1b}^p - u_{1a}^p) = 0, \tag{4}$$

$$m_a u_{2a}^p + k_2(u_{2a}^p - u_{1a}^p) + k_1(u_{2a}^p - u_{1a}^{p+1}) + k_b(u_{2a}^p - u_{2b}^p) = 0, \tag{5}$$

$$m_b u_{2b}^p + k_b(u_{2b}^p - u_{2a}^p) = 0, \tag{6}$$

The equations for the sub-lattice II on the right side of the interface are similar to those given in Eqs. (3)-(6) where stifnesses k_1 and k_2 change their places.

2.1 Numerical results

For brevity, only a two-mass chain with resonators will be considered. In numerical calculations the system with $n = 30$ unit cells on each side of the interface is considered, $m_a = 1$, $m_b = 0.5$ and $k_b = 0.8$. Each unit cells contains two identical masses with two local resonators connected to them and two different connection springs while springs of the resonators remain the same. The proposed system natural frequencies and frequency response function (FRF) of the central mass u_{2a}^0 of the chain are obtained from the eigenvalue problem defined by the above given governing equations and presented in Fig. 1c. The results show the existence of interface modes, where both symmetric and anti-symmetric modes are obtained at the lower and higher frequency bandgaps. The trivial case when $\gamma < 0$ is omitted for the brevity. The bulk band and Zak phase analysis will be used to reveal the bulk band properties of the proposed unit cell configuration. Band inversion effect will be demonstrated based on bulk band analysis.

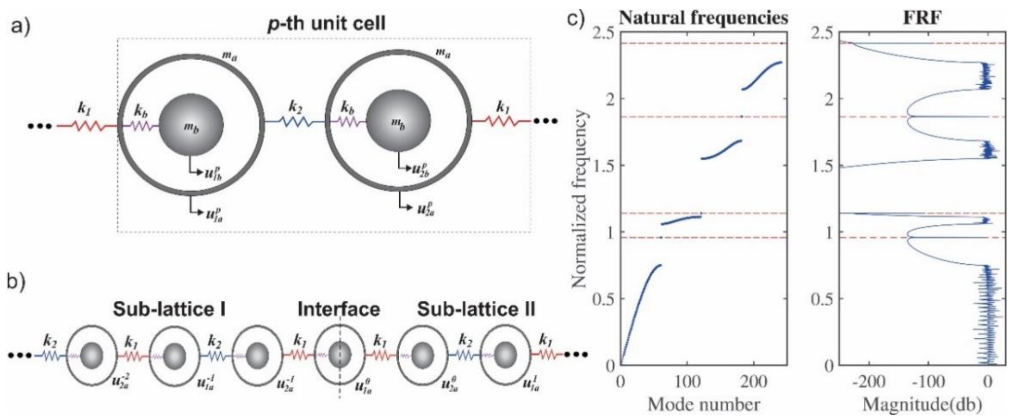

Fig. 1. Illustration of the acoustic metamaterial a) two-mass unit cell with resonators b) mass-spring-resonator chain with the interface c) interface modes (red dashed lines) for $\gamma = 0.5$.

3. Conclusions

The presented results for the locally resonant acoustic metamaterial chain with the mirror-like interface confirms the existence of interface modes based on frequency response function and eigenvalue plots. It is demonstrated that interface modes exist at both, lower and higher frequency band gaps. The obtained results and methodology will be used to investigate the trivial case, where anti-symmetric modes vanish and migrate into the bulk, band inversion effect and different polyatomic unit cell configurations.

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