Theoretical Consideration of Axial Ratio Deterioration in CP Loop-Shaped Antennas

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***Abstract*— Conventionally, the deterioration in the axial ratio (AR) for a circularly polarized antenna based on a traveling wave current is explained using backward currents reflected from the antenna arm end. This letter reveals the existence of a new cause for AR deterioration in a situation of no backward currents. For this, a theoretical loop antenna model of one-guided wavelength circumference is constructed, where only a forward traveling wave current (a positive *s*'-directed current) flows with attenuation due to the radiation. The derived AR equation finds that the AR deteriorates due to the attenuation of the current, even when backward currents do not exist. Subsequently, a current that flows in the direction normal to the positive *s*'-directed current (a *t'*-directed current specified by perturbation weight *p*) is added to the loop. It is found that the AR in the presence of the attenuating current is improved by virtue of the positive *t'*-directed current.**

***Index Terms*— attenuation constant, axial ratio, circularly polarized radiation, forward current, perturbation length, traveling wave current.**

I. INTRODUCTION

The *axial ratio* (AR) is a major property used in evaluating circularly polarized (CP) antennas [1][2]. The AR is defined as $(|E_L|+|E_R|)/(|E_L|-|E_R|)$ for $|E_L| > |E_R|$ and $(|E_L|+|E_R|)/(|E_R|-|E_L|)$ for $|E_L| < |E_R|$, where E_L and E_R are the left-hand and right-hand CP components of the radiation field, respectively. Either E_L or E_R is designated as the principal (desired) radiation field component and the other is considered the cross-polarization component. An antenna with an AR of less than 3 dB is referred to as a circularly polarized (CP) antenna [3]-[6].

Fig. 1(a) shows a loop antenna, which is made of a conducting strip and excited from point *F*. The circumference of the loop, C_{ring}, is chosen to be one guided wavelength of the current on the loop arm: $C_{ring} = 1\lambda_g$. If the loop is constructed such that it supports a traveling wave current, the radiation field in the z-direction is CP. This is achieved by terminating loop end point *T* through a matched load to the ground plane.

Recently, the metaloop antenna shown in Fig. 1(b) has been realized [7]. The loop consists of a chain of subwavelength metaatoms [8][9]. It has been found that, when the loop circumference, C_{ring}, is $1\lambda_g$ and point *T* is terminated with a Bloch impedance [10], the radiation field in the z-direction is CP. The details of working principle and realization are described in [7][11][12][13][14].

It is clear that, when the loop antennas in Figs. 1(a) and (b) have currents that travel back toward the feed point, a cross-polarized radiation field is generated, and hence the axial ratio in the z-direction becomes larger than one, *i.e.*, the AR for CP radiation deteriorates. This is the conventional explanation.

A question arises as to whether the backward currents are the only cause of AR deterioration in the z-direction for loop-shaped antennas, as in Fig. 1(a) and (b); in other words, are there any factors (other than the backward currents) that cause the AR deterioration? This letter is devoted to answering this question, deriving the AR for a generalized loop antenna model where *only* a forward traveling wave current (a positive *s*'-directed current) flows with attenuation due to the radiation, *i.e.*, no backward currents (negative *s*'-directed currents) flow.

The present letter is composed of five sections. Section II shows the derivation process for the radiation field and AR(θ, ϕ), assuming that only a positive *s*'-directed current travels with attenuation due to the radiation. Based on the results in section II, section III reveals the behavior of the axial ratio in the z-direction, *i.e.*, AR($\theta = 0, \phi$) is derived. The

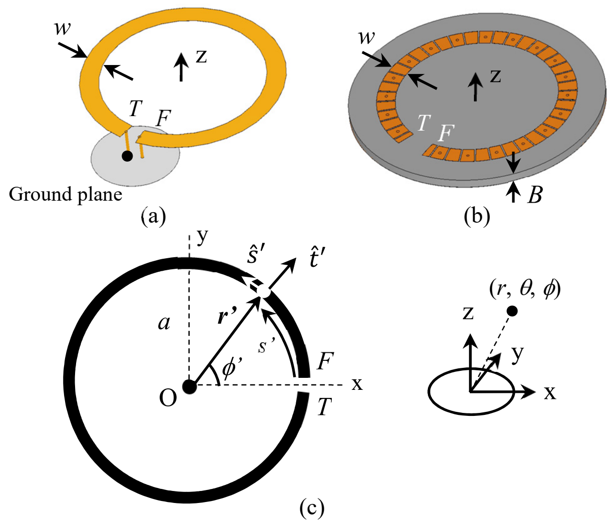

Fig. 1. Loop antennas. (a) Circularly polarized loop antenna. (b) Metaloop antenna. (c) Generalized antenna model.

Manuscript received xxxxx 2022; revised xxxxx; accepted xxxxx. Date of publication xxxxx; date of current version xxxxx.

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existence of a new cause in AR deterioration is clarified. Section IV presents another derivation for the AR(θ, ϕ) when a *t'*-directed current is added to the attenuating *s*'-directed current, where the *t'*-directed current is orthogonal to the *s'*-directed current. The improvement in the AR(θ= 0, ϕ) is discussed in the presence of an attenuating current. Section V summarizes the findings obtained in this study.

As abovementioned, this letter clarifies the cause of the AR deterioration when the current travels *with attenuation* due to the radiation. Therefore, the purpose, discussion process, and final results of this letter differ from those of the previous paper [12] that focusses on the antenna gain-balancing. The discussions in [12] have been performed assuming that the attenuation in the current is *absent* (the current of Eq. (1) in [12] does *not* have attenuation constant α, unlike the current in this letter). This has raised the question as to how the presence of the attenuating current influences the AR (constructive or destructive influence, or no influence?). To the best of authors' knowledge, no papers have discussed the AR in the presence of an attenuating current.

II. Theoretical Model and Axial Ratio

The current on a loop antenna of $1\lambda_g$-circumference plays a role in generating CP radiation in the normal direction (z-direction). Based on this fact, this paper investigates the axial ratio, AR(θ, ϕ), in the normal direction, *i.e.*, AR(θ= 0, ϕ) of the radiation field from a generalized loop model of $1\lambda_g$-circumference shown in Fig. 1(c). Note that the distance between feed point *F* and terminal point *T* is infinitesimally small and treated as zero in the following discussion.

We assume a current on the loop, $\boldsymbol{I}_s(s')$, as

$$\boldsymbol{I}_s(s') = I_0 e^{-(\alpha+j\beta)s'}\hat{s}' \quad (1)$$

where $\hat{s}'$ is the unit vector tangential to the loop at a source point that is specified by spherical coordinates $(r'=a, \theta'=\pi/2, \phi')$, with *a* being the loop radius; the arc arm length is $s' = a\phi'$; I_0 is a constant; and α and $\beta(=2\pi/\lambda_g)$ are the propagation attenuation and phase constants, respectively. It should be emphasized that the current is assumed to be a traveling wave current that flows only in the $\hat{s}'$-direction (forward direction (counterclockwise)), and is referred to as a *positive s'-directed current*. Eq. (1) does not have currents in the $(-\hat{s}')$-direction (backward direction (clockwise)), referred to as *negative s'-directed currents*, to reveal the effect of attenuation constant α.

The electric radiation field, $\boldsymbol{E_s}$, at an observation point of spherical coordinates (r, θ, ϕ) with unit vectors $(\hat{r}, \hat{\theta}, \hat{\phi})$ is given as [2]

$$\boldsymbol{E}_s(r,\theta,\phi) = \frac{-j\omega\mu}{4\pi}\frac{e^{-jk_0 r}}{r}\int_0^{2\pi a} \boldsymbol{I}_s(s')\, e^{jk_0\hat{r}\cdot\boldsymbol{r}'}\, ds' \quad (2)$$

where ω is the angular frequency: $\omega = 2\pi f$ with f being the operating frequency; μ is the permeability; $k_0 = 2\pi/\lambda_0$ with λ_0 being the free-space operating wavelength; and $\boldsymbol{r}'$ is a vector directed from the coordinate origin to a source point specified by s'.

Introducing a new parameter of $\psi' \equiv \phi - \phi'$, Eq. (2) is transformed into

$$\boldsymbol{E}_s(r,\theta,\phi) = -CaI_0 e^{-(\alpha+j\beta)a\phi}\int_\phi^{\phi-2\pi} e^{(\alpha+j\beta)a\psi'}\, e^{jk_0 a\sin\theta\cos\psi'} \cdot [-\sin(\phi-\psi')\,\hat{x} + \cos(\phi-\psi')\hat{y}]d\psi' \quad (3)$$

where

$$\hat{s}' = -\sin\phi'\hat{x} + \cos\phi'\hat{y} \quad (4)$$

$$\boldsymbol{r}'\cdot\hat{r} = a\sin\theta\,\cos(\phi-\phi') \quad (5)$$

are used. Note that $(\hat{x}, \hat{y}, \hat{z})$ are unit vectors associated with rectangular coordinates (x, y, z), and coefficient *C* is defined as

$$C = \frac{-j\omega\mu}{4\pi}\frac{e^{-jk_0 r}}{r} \quad (6)$$

The loop circumference is set to be $2\pi a = 1\lambda_g$, and hence $\beta a = 1$. The radiation field components in the θ- and ϕ-directions are, respectively,

$$E_{s\theta}(r,\theta,\phi) = \boldsymbol{E}_s(r,\theta,\phi)\cdot\hat{\theta}$$
$$= -CaI_0 e^{-(\tilde{\alpha}+j)\phi}\cos\theta\int_\phi^{\phi-2\pi} e^{(\tilde{\alpha}+j)\psi'}\, e^{jk_0 a\sin\theta\cos\psi'}\cdot\sin\psi'\, d\psi'$$
$$= j\frac{1}{2}D\cos\theta\int_\phi^{\phi-2\pi}\{e^{(\tilde{\alpha}+2j)\psi'} - e^{\tilde{\alpha}\psi'}\}e^{jk_0 a\sin\theta\cos\psi'}\, d\psi' \quad (7)$$

$$E_{s\phi}(r,\theta,\phi) = \boldsymbol{E}_s(r,\theta,\phi)\cdot\hat{\phi}$$
$$= -CaI_0 e^{-(\tilde{\alpha}+j)\phi}\int_\phi^{\phi-2\pi} e^{(\tilde{\alpha}+j)\psi'}\, e^{jk_0 a\sin\theta\cos\psi'}\cdot\cos\psi'\, d\psi'$$
$$= -\frac{1}{2}D\int_\phi^{\phi-2\pi}\{e^{(\tilde{\alpha}+2j)\psi'} + e^{\tilde{\alpha}\psi'}\}e^{jk_0 a\sin\theta\cos\psi'}\, d\psi' \quad (8)$$

where

$$D = CaI_0 e^{-(\tilde{\alpha}+j)\phi} \quad (9)$$

and

$$\hat{\theta}\cdot\hat{s}' = \cos\theta\sin(\phi-\phi') \quad (10)$$

$$\hat{\phi}\cdot\hat{s}' = \cos(\phi-\phi') \quad (11)$$

are used.

Eqs. (7) and (8) are written as

$$E_{s\theta}(r,\theta,\phi) = +j\frac{1}{2}D(A-B)\cos\theta \quad (12)$$

$$E_{s\phi}(r,\theta,\phi) = -\tfrac{1}{2}D(A+B) \tag{13}$$

where A and B are defined as

$$A = \int_{\phi}^{\phi-2\pi}\{e^{(\tilde{a}+2j)\psi'}\}e^{jk_0 a\sin\theta\cos\psi'}d\psi' \tag{14}$$

$$B = \int_{\phi}^{\phi-2\pi} e^{\tilde{a}\psi'}e^{jk_0 a\sin\theta\cos\psi'}d\psi' \tag{15}$$

with a multiplied attenuation constant of

$$\alpha a \equiv \tilde{a} \tag{16}$$

We decompose radiation field $\boldsymbol{E}_s(r,\theta,\phi)$ into a left-hand CP wave component, E_L, and a right-hand CP wave component, E_R.

$$E_{s\theta}(r,\theta,\phi)\,\hat{\theta} + E_{s\phi}(r,\theta,\phi)\hat{\phi}$$
$$= E_L(\hat{\theta}+j\hat{\phi}) + E_R(\hat{\theta}-j\hat{\phi}) \tag{17}$$

From Eq. (17),

$$2E_L = j\tfrac{1}{2}D(Ac_+ + Bc_-) \tag{18}$$

$$2E_R = -j\tfrac{1}{2}D(Ac_- + Bc_+) \tag{19}$$

where

$$c_{\mp} = 1 \mp \cos\theta \tag{20}$$

Using Eqs. (18) and (19), the axial ratio is written as

$$\mathrm{AR}(\theta,\phi) = \frac{|E_R|+|E_L|}{|\{|E_R|-|E_L|\}|}$$
$$= \frac{|Ac_-+Bc_+|+|Ac_++Bc_-|}{|\{|Ac_-+Bc_+|-|Ac_++Bc_-|\}|} \tag{21}$$

III. Axial Ratio in the Normal Direction

Our interest is in the axial ratio for the normal direction (z-direction): $\theta = 0$. Eqs. (14) and (15) for $\theta = 0$ are reduced to

$$A(\theta=0,\phi) = \int_{\phi}^{\phi-2\pi}\{e^{(\tilde{a}+j2)\psi'}\}d\psi'$$
$$= e^{(\tilde{a}+j2)\phi}\frac{e^{-2\pi\tilde{a}}-1}{(\tilde{a}+j2)} \equiv A_0 \tag{22}$$

$$B(\theta=0,\phi) = \int_{\phi}^{\phi-2\pi}\{e^{\tilde{a}\psi'}\}d\psi'$$
$$= e^{\tilde{a}\phi}\frac{e^{-2\pi\tilde{a}}-1}{\tilde{a}} \equiv B_0 \tag{23}$$

Because $c_- = 0$ and $c_+ = 2$ for $\theta = 0$, Eq. (21) yields,

$$\mathrm{AR}(\theta=0,\phi) = \frac{|B_0|+|A_0|}{|\{|B_0|-|A_0|\}|}$$
$$= \frac{\sqrt{\tilde{a}^2+4}+\tilde{a}}{\sqrt{\tilde{a}^2+4}-\tilde{a}}$$
$$\equiv \mathrm{AR_s}(0,\phi) \tag{24}$$

Fig. 2 shows $\mathrm{AR_s}$ as a function of multiplied attenuation constant $\tilde{a}$ $(=a\alpha)$. The desired case is $\mathrm{AR_s}(0,\phi) = 1$ with $\tilde{a} = 0$, *i.e.*, perfect circular polarization. This cannot happen due to the fact that the traveling current attenuates due to the radiation. Eq. (24) concludes that the $\mathrm{AR_s}(0,\phi)$ deteriorates as $\tilde{a}$ increases, even though the current travels only in the forward direction (positive $\hat{s}'$-direction) with no backward currents in the $-\hat{s}'$ direction. This is the first goal of this letter, based on the theoretical result of Eq. (24). To the best of the authors' knowledge, this has not been revealed up to now.

IV. Mitigation of Axial Ratio Deterioration in the Normal Direction

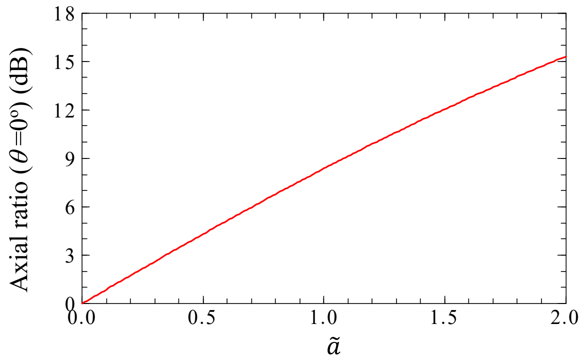

Fig. 2. Axial ratio as a function of multiplied attenuation constant $\tilde{a}$.

Next, a different situation is considered, where the current on the loop is assumed to be

$$\boldsymbol{I}_t(s') = I_0 e^{-(\alpha+j\beta)s'}\,\hat{t}' \tag{25}$$

where $\hat{t}'$ is a unit vector perpendicular to $\hat{s}'$, *i.e.*, $\hat{t}'\cdot\hat{s}' = 0$.

$$\hat{t}' = \cos\phi'\hat{x} + \sin\phi'\hat{y} \tag{26}$$

The current in Eq. (25) is referred to as the *t'-directed current.* The radiation field generated by this current, $\boldsymbol{E}_t$, is

$$\boldsymbol{E}_t(r,\theta,\phi) = CI_0\int_0^{2\pi a} e^{-(\alpha+j\beta)s'}e^{jk_0 a\sin\theta\cos(\phi-\phi')}(\hat{t}')ds' \tag{27}$$

The θ- and ϕ-components of Eq. (27) are expressed by the following Eqs. (28) and (29), respectively, using Eqs. (9), (14), and (15).

$$E_{t\theta}(r,\theta,\phi) = \cos\theta\, E_{s\phi}(r,\theta,\phi)$$
$$= -\tfrac{1}{2}D(A+B)\cos\theta \tag{28}$$

$$E_{t\phi}(r,\theta,\phi) = -\frac{1}{\cos\theta}E_{s\theta}(r,\theta,\phi)$$

$$= -j\frac{1}{2}D(A-B) \quad (29)$$

If current $jp\boldsymbol{I}_{\rm t}(s')$ is distributed on the ring region, Eqs. (28) and (29) become weighted by jp, where p is referred to as the *perturbation weight*. We superimpose these weighted field components onto the field components of Eqs. (12) and (13) generated by current $\boldsymbol{I}_{\rm s}(s')$. Then, the θ-component of the total radiation field becomes

$$E_{\rm s\theta}(r,\theta,\phi) + j\,pE_{\rm t\theta}(r,\theta,\phi) = j\frac{1}{2}D(Ap_- - Bp_+)\cos\theta \quad (30)$$

where

$$p_\mp = 1 \mp p \quad (31)$$

The ϕ-component of the total radiation field becomes

$$E_{\rm s\phi}(r,\theta,\phi) + jp\,E_{\rm t\phi}(r,\theta,\phi) = -\frac{1}{2}D(Ap_- + Bp_+) \quad (32)$$

Therefore, the total radiation field due to the positive *s'*-directed current superimposed with the weighted *t'*-directed current, $\boldsymbol{E}_{\rm st}(r,\theta,\phi)$, is

$$\boldsymbol{E}_{\rm st}(r,\theta,\phi) = j[\tfrac{1}{2}D(Ap_- - Bp_+)\cos\theta]\,\hat{\theta} + [-\tfrac{1}{2}D(Ap_- + Bp_+)]\,\hat{\phi} \quad (33)$$

The left-hand and right-hand CP components of the radiation field of Eq. (33) are

$$2E_{\rm L} = j\frac{1}{2}DAp_-c_+ + j\frac{1}{2}DBp_+c_- \quad (34)$$

$$2E_{\rm R} = -j\frac{1}{2}DAp_-c_- - j\frac{1}{2}D\,Bp_+c_+ \quad (35)$$

Hence, the axial ratio is expressed as

$$\mathrm{AR}(\theta,\phi) = \frac{|Ap_-c_- + Bp_+c_+| + |Ap_-c_+ + Bp_+c_-|}{|\{|Ap_-c_- + Bp_+c_+| - |Ap_-c_+ + Bp_+c_-|\}|} \quad (36)$$

Since $c_-= 0$ for $\theta = 0$, the axial ratio in the normal direction is

$$\mathrm{AR}(\theta=0,\phi) = \frac{|B_0p_+| + |A_0p_-|}{|\{|B_0p_+| - |A_0p_-|\}|} = \frac{(1+p)\sqrt{\tilde{a}^2+4} + (1-p)\tilde{a}}{(1+p)\sqrt{\tilde{a}^2+4} - (1-p)\tilde{a}} \equiv \mathrm{AR}_{\rm st}(0,\phi) \quad (37)$$

As seen from Eq. (37), a perfectly CP wave (where $\mathrm{AR}_{\rm st} = 1$) is expected at either $\tilde{a} = 0$ or $p = 1$ at least. Note that the former condition, *i.e.*, the absence of the attenuation [12] ($\alpha = 0$ and hence $\tilde{a} = \alpha a = 0$) is not realistic because the radiation causes attenuation in the current, *i.e.*, $\tilde{a}$ is not zero: $\tilde{a} \neq 0$. Hence, the value of $p = 1$ should be used for perfect CP radiation.

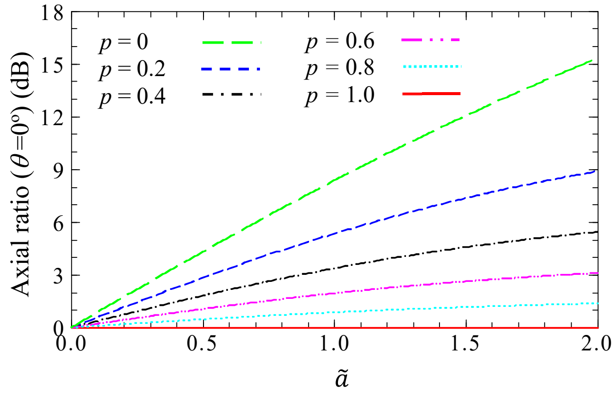

Fig. 3. Axial ratio as a function of multiplied attenuation constant $\tilde{a}$ with perturbation weight p as a parameter.

Fig. 3 illustrates the axial ratio of Eq. (37) as a function of $\tilde{a}$ with p as a parameter. At any $\tilde{a} \neq 0$, the $\mathrm{AR}_{\rm st}$ for $p \neq 0$ is smaller/better than the $\mathrm{AR}_{\rm st}$ for $p = 0$ (where the *t'*-directed current is zero). Note that the $\mathrm{AR}_{\rm st}$ for $p = 0$ equals the $\mathrm{AR}_{\rm s}$ in Eq. (24). Thus, the deterioration of the axial ratio *in the presence of attenuation in the current* ($a\alpha = \tilde{a} \neq 0$) is mitigated by adding the weighted *t'*-directed current. This is the second goal, based on the theoretical result of Eq. (37).

Four comments are made here. C1: The *t'*-directed current (across an infinitesimally small region of $\Delta s'$(*i.e.*, *ds'*) relative to the operating free-space wavelength) can be approximately realized using stub techniques in [7][9][11][13]. C2: As the arm width, *w*, for the loop antennas in Figs. 1(a) and (b) becomes wider, the attenuation constant, α, increases. As the antenna thickness, *B*, in Fig. 1(b) becomes thicker, α increases. C3: The AR becomes larger, as an observation point moves away from the z-axis. If the AR in the direction away from the z-axis is needed, use Eq. (36) after direct/numerical integration of Eqs. (14) and (15). C4: It is possible to make the relation between AR and attenuation for differently shaped CP leaky wave antennas, using a similar process mentioned in this letter.

V. Conclusion

Conventionally, backward currents are used to explain the AR deterioration in loop-shaped antennas that have a traveling-wave current. This letter has found a new cause of AR deterioration by distributing only an attenuating positive *s*'-directed current on a one-guided wavelength loop, *i.e.,* not distributing a negative *s*'-directed current. The important finding is that the AR deteriorates due to the attenuation in the current even when negative *s*'-directed currents do not exist. This has not been revealed until now. Subsequently, mitigation of the deteriorated AR has been investigated by superimposing a weighted *t'*-directed current onto the *s*'-directed current. The second finding is that the superimposition is effective for improving the deteriorated AR in the presence of the attenuating current.

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