



NFTs, DeFi, and other assets efficiency and volatility dynamics: An asymmetric multifractality analysis[☆]

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ABSTRACT

This paper examines the efficiency and asymmetric multifractal features of NFTs, DeFi, cryptocurrencies, and traditional assets using Asymmetric Multifractal Cross-Correlations Analysis covering the period from November 2017 to February 2022. Considering the full sample with a significant variation among asset classes, the study reveals DeFi-DigiByte is the most efficient while the cryptocurrency-Tether is the least efficient. However, S&P 500 showed high efficiency before COVID-19, and DeFi-Enjin Coin advanced as the most efficient asset during COVID-19. The volatility dynamics of NFTs, DeFi, and cryptocurrencies follow strong nonlinear cross-correlations, but evidence of weaker nonlinearity exists in traditional assets. Additionally, the sensitivity to smaller events in bull markets is high for NFTs and DeFi. The findings have significant implications for portfolio diversification when an investor's portfolio set includes traditional assets and cryptocurrency and relatively new blockchain-based assets like NFTs and DeFi.

1. Introduction

Since their inception, the blockchain-based digital asset classes have received immense interest from investors and portfolio managers as an alternative investment platform. Along with other established traditional cryptocurrencies such as Bitcoin, Litecoin, Ripple, and Ethereum, new blockchain asset classes such as Decentralized Finance (DeFi) and Non-Fungible Tokens (NFTs) have made a considerable contribution to the asset market's recent expansion (Aharon & Demir, 2021; Alam, Chowdhury, Abdullah, & Masih, 2023; Maouchi, Charfeddine, & el Montasser, 2021; Yousaf & Yarovaya, 2022). Fundamentally, NFTs and DeFi differ from traditional cryptocurrencies as they are not virtual currency. Where NFTs are non-transferable cryptographic digital assets

created by Ethereum smart contracts and can be sold and traded, the interchangeability of NFTs when comparing the other cryptocurrencies is very low (Karim, Lucey, Naeem, & Uddin, 2022; Q. Wang, Li, Wang, & Chen, 2021; Y. Wang, 2022).

On the other hand, DeFi is a new financial technology service based on distributed ledgers and smart contracts using blockchain, providing financial instruments without relying on intermediaries like brokers, exchanges, or banks (Karim et al., 2022; Yousaf, Nekhili, & Gubareva, 2022). The NFTs and DeFi are relatively contemporary and unexplored asset classes, but their market capitalization has grown substantially as risk minimizing assets, particularly during the COVID-19 period. In the NFT space, the pandemic has increased demand for digital art and collectibles as the shift to remote work, and online commerce fueled

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interest in digital assets (Alam et al., 2023). The interest has driven up prices for some NFTs and has contributed to a general increase in the popularity of NFTs. However, for the DeFi assets, the pandemic has had a mixed impact. The global economic downturn has also increased interest in DeFi tokens, yield farming platforms, and stablecoins to access financial services and investments that are not available through traditional channels (Karim et al., 2022; Yousaf & Yarovaya, 2022). However, the pandemic-led decrease in overall spending and investment caused a slowdown in the DeFi market. In addition, unlike the return on NFTs, which is considered uncorrelated with other assets, such as stocks, bonds, and commodities, because they are not tied to any underlying financial performance or revenue streams, increased return volatility in the broader financial markets has had a spillover effect on the DeFi market.

From 2014 through 2022, the NFTs and DeFi market and prices are influenced by various factors, including the development of blockchain technology, underlying protocols and products, the level of liquidity in the market, and shifts in investor sentiment toward the NFTs and DeFi ecosystem (Alam et al., 2023; Ko, Son, Lee, Jang, & Lee, 2022). NFTs gained widespread attention in the blockchain space in 2017 with the launch of the Ethereum network's ERC-721 standard (Wilson, Karg, & Ghaderi, 2021). In the early days of NFTs, the market was relatively small, with sales typically tens of thousands of dollars. However, the market for NFTs has seen tremendous growth since 2019, with some sales reaching millions of dollars in 2020. The first popular NFTs were CryptoKitties, a collectible game built on the Ethereum blockchain, and CryptoPunks, a set of 10,000 unique digital characters (Dowling, 2022a; Pinto-Gutiérrez, Gaitán, Jaramillo, & Velasquez, 2022). The concept of decentralized finance (DeFi) was first introduced in the blockchain space in the early- to mid-2010s, with the launch of the Ethereum network and the development of smart contracts (Karim et al., 2022). The rapid growth of decentralized exchanges and the popularity of yield farming and liquidity provision have driven the growth of a new asset class composed of various DeFi tokens and protocols. Since its inception, DeFi assets have generated substantial returns for investors in a short period, particularly during the yield farming craze in the summer of 2020, with the total value locked in DeFi protocols reaching all-time highs in billions of dollars in 2020 and 2021 (Maouchi et al., 2021).

Overall, the historical development of the NFTs and DeFi market has been one of rapid growth but with high volatility. As a result, these two markets can be highly volatile, with prices for some NFTs and DeFi assets changing dramatically quickly. This high volatility is partly due to the fact that the markets are still in their early stages, with a limited number of buyers and sellers.

Furthermore, asymmetric price movement and spike of short-term risk spillover were observed during the COVID-19 period among NFTs, DeFi, cryptocurrencies, and other assets (Karim et al., 2022), therefore investor had to pay attention to the selection of efficient assets in portfolio construction. Moreover, the interest in asset allocation based on asset efficiency criteria has gained new momentum as the extreme events of the COVID-19 lockdown (Abdullah, Wali Ullah, & Chowdhury, 2022), followed by the Russia-Ukraine war, have triggered severe stress in the global financial markets.

Extreme events in financial markets generated from outside the system, rather than those endogenous shocks, are often characterized as unexpected asset price volatility that makes investors uncertain about their investment decisions (Alam, 2022). The degree of uncertainty is primarily driven by unknown asymmetrical nonlinear dynamics, scaling patterns, asset class self-similarity, long memory, herding behavior, and, notably, the lack of conformity of assets with the efficient market hypothesis (EMH) (Fama, 1970). For example, recent studies (Bariviera, 2021; Frezza, Bianchi, & Pianese, 2021; Manahov & Urquhart, 2021; Urquhart, 2016) found that cryptocurrency's information efficiency is mainly violated during the overreaction of investors to extreme events. EMH assumes that all assets have the same information and price history, and prices follow a random walk. However, returns and volatility

are highly influenced by information asymmetry in a limited number of buyers and sellers for NFTs, DeFi, and Cryptocurrencies, where market participants have different levels of informational advantage over market maturity and on-random price movements. Also, the EMH is not well supported for other asset classes in the short run and during crisis periods because of typical streams of multifractality in the financial time series. Understandably, accurate measurement of volatility scaling patterns, fractality, and nonlinear dynamics is essential for investors since scaling patterns play a vital role in identifying and sorting efficient assets.

In the light of the significance of volatility scaling patterns, we aim to examine the efficiency and multifractality of NFTs and DeFi along with cryptocurrencies and other traditional assets using Asymmetric Multifractal Cross-Correlations Analysis (MF-ADCCA) (Cao, Cao, Xu, & He, 2014). The traditional financial assets include gold, crude oil, and S&P500. Our sample period ranges from November 2017 to February 2022, covering the starting date of three major NFTs and DeFi, volatility calm periods of major assets and commodities, and the Covid-19 pandemic crisis.

MF-ADCCA is a modified version of the symmetric Multifractal Detrended Fluctuation Analysis (MFDFA) technique that detects multifractality and assumes that the influence of a downward trend on price dynamics is the same as that of an upward trend. The phrase fractals and multifractality were initially proposed by Mandelbrot and Mandelbrot (1982) and utilized to illustrate geometric shapes containing self-similar structures randomly distributed on small scales. In early influential work in finance, Peters (1994) introduced the fractal market hypothesis (FMH) in investment decisions based on Mandelbrot's fractal framework. The argument was that the financial market follows a standard stochastic process with interactive and adaptive properties. However, as Dowling (2022b) shows, unlike traditional assets, the NFTs and DeFi respond differently during up and down trends and have varied impacts on their volatility and returns. Therefore, we use the MF-ADCCA framework, given its merit in capturing asymmetric upward and downward motions, identifying the scaling performance of assets, and its utility in sorting the most efficient assets. Furthermore, given the advantage of the MF-ADCCA approach and the fact of information asymmetry in the asset market, identification of the multifractality has significant implications for portfolio diversification and predicting asset price volatility when an investor's portfolio set includes not only traditional and cryptocurrency but also relatively new blockchain-based assets like NFTs and DeFi.

Our main findings show a considerable asymmetry in asset efficiency variation among the asset classes, where Digi Byte is the most efficient asset class and the cryptocurrency Tether is the least efficient. Relatively weak multifractality of all assets is evidenced during an uptrend compared to the downtrend market conditions. DeFi, NFTs, and Gold are more efficient than cryptocurrencies. Before COVID-19, traditional asset (S&P 500) was the most efficient asset class, while during the COVID-19 period, DeFi (Enjin Coin) ranked first, and Traditional Asset (S&P 500) ranked second in terms of efficiency. Considering the entire sample period, DeFi (Digi Byte) is the most efficient asset class, and Gold is second most efficient. The asymmetric spectrum result suggests NFTs and DeFi are more sensitive to smaller events, large fluctuations dominating bull markets, and small fluctuations dominating bear markets.

The findings of this paper contribute to the three areas of literature focusing on the efficiency and multifractality of financial and digital asset markets. Firstly, the empirical estimation of the MF-ADCCA adds to the literature that estimates the multifractal properties of NFTs and DeFi. It also ranks the assets in terms of their degree of efficiency. In addition, the multifractality provides evidence of nonlinear assets' cross-correlations, long-memory, and biased herding behavior. Secondly, the findings of our study provide new evidence on possible cross-assets asymmetries in price and volatility movements between pre-and during-COVID-19 periods. Finally, while EMH often fails to capture scaling patterns and herding behavior, the asymmetric herding behavior

presented in our findings can signal short- and long-term investors.

In addition, this study extends [Dowling \(2022a\)](#) rolling window Hurst Exponent analysis in the asymmetric MF-DFA framework by splitting the sample into the pre-and during-COVID-19 periods. As a result, it is possible to provide economic interpretations of the asymmetric MF-DFA coefficients and scaling exponent that represents the long-range power-law correlation properties. This economic interpretation can contribute to the development of fintech theory in which portfolio managers, investors, and policymakers can design strategies to choose efficient assets in extreme crises. For example, our approach provides evidence of the long-range power-law correlation in asset returns in which heterogeneous investors with diverse investment horizons interact. So, an underlying theoretical conjecture is that investors choose efficient asset in a dynamic multi-asset (NFTs, DeFi, Cryptocurrencies, and other assets) environment that is subject to accurate forecasting on how multifractality in asset price volatility will evolve in the future. Furthermore, the estimated values of the q -th order volatility functions in the MF-ADCCA method can predict how an asset allocation model should consider the various magnitudes of fluctuations over the short- and long-run investment horizon.

The rest of the paper is outlined as follows: [Section 2](#) provides a brief literature review, [Section 3](#) discusses the methodology and data of this study, [Section 4](#) elaborates on the empirical findings, [Section 5](#) discusses the findings with policy implications and [Section 6](#) concludes the study.

2. Literature review

The efficient market hypothesis (EMH) is a foundational theory of modern finance ([Fama, 1970](#)); EMH categorizes market efficiency into three levels based on how much accessible information is represented in asset price: strong, semi-strong, and weak. Investing in a financial instrument is deemed efficient in the weak form if market prices completely represent the available information. However, NFTs, DeFi, and Cryptocurrencies have unique characteristics as these asset classes are relatively new and still evolving, where returns and volatility are highly influenced by asymmetric market sentiment in a limited number of buyers and sellers and the success or failure of underlying protocols and products. In addition, behavioral biases such as artist popularity, herding, overconfidence, and overreaction can further affect the prices of these assets in ways that are not reflected in the underlying EMH.

[Peters \(1994\)](#) introduced the fractal market hypothesis (FMH) based on Mandelbrot's fractal framework ([Mandelbrot & Mandelbrot, 1982](#)) and considered the financial market as a sophisticated stochastic process with interactive and adaptive properties. FMH has gained popularity in capital markets research due to its ability to explain numerous economic events that traditional efficient market theories cannot. The FMH is based on the idea that financial markets are characterized by self-similar patterns, or fractals, which can be used to predict future price movements. However, unlike the EMH, the literature in FMH suggests that financial markets are not perfectly efficient and that prices can be predicted based on the recognition of fractal patterns. Furthermore, the hypothesis argues that the efficiency of different asset classes may vary depending on the presence or absence of fractal patterns. In this context, the scope of our findings in explaining asset class efficiency refers to its ability to provide insight into the efficiency of underlying asset classes, such as DeFi, NFTs assets, and cryptocurrencies, in constructing efficient portfolios.

Fractal theories are classified into two types: monofractal theories and multifractal theories. As it can only characterize the microstructure outline of capital asset price movements, the monofractal theory could illustrate the internal structure and properties of asset price movements. The multifractal theory may offer comprehensive information on the levels of volatility in capital asset values across time ([Mandelbrot, 1997](#)). As a result, multifractal theory, as contrasted with monofractal theory, gives a more thorough explanation of the complicated nonlinear dynamics of capital markets. Two typical multifractal analysis

approaches are the rescaled range analysis (R/S) method and the multifractal detrended fluctuation analysis (MF-DFA) method based on the Hurst exponent ([Hurst, 1951](#)). The MF-DFA approach was developed by [Kantelhardt et al. \(2002\)](#) as a generic analytical method relying on the DFA method ([Peng et al., 1994](#)).

Recent literature utilized MF-DFA to analyze multifractal noise, market volatility, and portfolio selection in general, to improve price predictability for portfolio diversification and optimization. Closely related to our paper, several recent studies focus on the multifractal features of stock ([Chai, Chu, Zhang, Abedin, & Lucey, 2022](#); [Mensi, Lee, Vinh Vo, & Yoon, 2021](#); [Tiwari, Aye, & Gupta, 2019](#)), commodities ([Guo et al., 2021](#); [Mensi, Vo, & Kang, 2022](#)), foreign exchange ([Diniz-Maganini, Rasheed, & Sheng, 2021](#)) and cryptocurrencies ([Bariviera, 2021](#); [Cao & Xie, 2021](#); [Chowdhury, Abdullah, & Masih, 2022](#); [Kakinaka & Umeno, 2022](#)). [Tiwari et al. \(2019\)](#) examine the multifractality and efficiency of emerging and developed countries' financial markets. They provide time-varying evidence of multifractality where nearly all asset markets show higher efficiency in the long term. Finally, [Chai et al. \(2022\)](#) focus on Chinese emission trading schemes' multifractality and document a significant multifractal trait across Chinese provinces.

On the application of multifractality in digital currencies, [Kristjanpoller and Bouri \(2019\)](#) compare multifractal features of cryptocurrencies and traditional currencies. Their result suggests a robust multifractal feature of Bitcoin and Litecoin. Another study by [Mensi, Lee, Al-Yahyaee, Sensoy, and Yoon \(2019\)](#) investigates the multifractality characteristics of Bitcoin and Ethereum and shows Ethereum is more efficient than Bitcoin. [El Alaoui, Bouri, and Roubaud \(2019\)](#) also found similar findings for Bitcoin's low efficiency. Finally, using 84 cryptocurrencies, [Bariviera \(2021\)](#) documents heterogenous long-range dependence, some of which follow multifractality dynamics, and others follow monofractality. Concerning the efficiency property of assets, [Mensi et al. \(2021\)](#) analyze the efficiency of top crude oil-producing countries and consumer countries' stock markets. Their findings suggest a strong multifractality in a bull market and a decline in efficiency during the global financial crisis COVID-19. Using data from 20 exchange rates, [Diniz-Maganini, Diniz, and Rasheed \(2021\)](#) examine exchange rates' regime-specific efficiency and show that managed-float countries' currencies are less efficient than free-float currencies. They also analyze the influence of the 2008 global financial crisis and find that efficiency recovery is prolonged in free-float countries.

Nevertheless, as the asset price movements and return volatility are asymmetrical between short- and long-run horizons and in bull and bear market conditions, [Lee, Song, Park, and Chang \(2017\)](#) propose an index-based MF-DFA method based on [Cao et al. \(2014\)](#)'s asymmetric MF-DFA. Recent development and an extension of the MF-DFA to cross-correlations are the MF-ADCCA. The MF-ADCCA shows an improvement in prediction, as it considers the asymmetric price movements, cross-correlations, multifractal scaling patterns, and dynamic non-linearity between two-time series. Several studies examine the asymmetric multifractality in BTC and other leading currencies using the MF-ADCCA approach. For example, [Kakinaka and Umeno \(2021\)](#) examine the cryptocurrency market's asymmetric price and volatility dynamics. They find significant cross-correlations concerning volatility and price of Ethereum, Bitcoin, Litecoin, and Ripple, where volatility responses are asymmetrical in bull and bear markets. [Kristjanpoller and Bouri \(2019\)](#) also examine the asymmetric multifractality of cryptocurrencies and traditional currencies. [Stavroyiannis, Babalos, Bekiros, Lahmiri, and Uddin \(2019\)](#) focus on the multifractal properties of BTC, and [Gajardo, Kristjanpoller, and Minutolo \(2018\)](#) examine price behaviors of BTC and world-leading currencies. Finally, for the COVID-19 periods, [Naem, Bouri, Peng, Shahzad, and Vo \(2021\)](#) and [Kakinaka and Umeno \(2022\)](#) examine cryptocurrencies' multifractality and efficiency properties. The findings suggest weak multifractal features in the long-term but strong in the short-term.

This paper is also associated with another stream of literature on NFTs and DeFi, and that literature is still growing. Several studies have

Table 1
List of selected assets.

Variables	Description	Asset class	Data source
BAT	Basic Attention Token	NFTs	CoinMarketCap
LINK	Chainlink	NFTs	CoinMarketCap
MANA	Decentraland	NFTs	CoinMarketCap
DGB	DigiByte	DeFi	CoinMarketCap
ENJ	Enjin Coin	DeFi	CoinMarketCap
MKR	Maker	DeFi	CoinMarketCap
BTC	Bitcoin	Cryptocurrency	CoinMarketCap
ETH	Ethereum	Cryptocurrency	CoinMarketCap
USDT	Tether	Cryptocurrency	CoinMarketCap
Gold	Gold	Traditional Asset	DataStream
WTI	Crude Oil	Traditional Asset	DataStream
S&P500	The Standard and Poor's 500	Traditional Asset	DataStream

been conducted after this topic received attention during COVID-19. Dowling (2022a) examined the Decentraland pricing and argues that because of its initial stage of growth, the NFTs market is still inefficient. A recent study by Ante (2021) demonstrates that Ethereum and Bitcoin drive NFTs price, while cryptocurrencies price are not affected by NFTs. Another study by Dowling (2022b) investigated the linkages between NFTs and cryptocurrency pricing and found that there is a minor volatility transmission between NFTs and cryptocurrencies and co-movement between NFTs and Ethereum. Karim et al. (2022) investigated the extreme risk transmission of blockchain markets using quantile connectedness methodology and discovered that, among other blockchain markets, NFTs provide greater diversification avenues with significant risk-bearing potential to protect investments and minimize extreme risks. Yousaf and Yarovaya (2022) examined the static and dynamic return and volatility spillovers between DeFi, NFTs, and traditional assets and found some DeFi and NFTs are net transmitters of volatility and return spillovers and connectedness became higher during COVID-19.

Moreover, another study by Yousaf, Nekhili and Gubareva (2022) examined the returns spillover between DeFi and traditional currencies and found DeFi return spillovers vary over time, and connectedness increased during the initial stage of COVID-19. Vidal-Tomás (2022) analyzed the dynamics and short/long-run performance of NFTs, and his findings suggest there are significant performance dynamics and lower co-movement with the cryptocurrency market. Ko et al. (2022) investigated the diversification benefit of using NFTs over traditional assets and found that NFTs are decoupled from traditional financial assets, and there is diversification benefit using NFTs. Another study by Maouchi et al. (2021) discussed digital bubbles in the context of the COVID-19 pandemic, showing specific DeFi and NFT bubbles in summer 2020, with bubbles occurring less frequently before the pandemic period. During the COVID-19 outbreak, Aharon and Demir (2021) investigated the spillover of NFTs across Ethereum stocks, gold, foreign exchange, and oil and discovered that overall connectedness significantly increased. In contrast, Umar, Gubareva, Teplova, and Tran (2022) recently demonstrated that NFTs connectedness holds only for horizons of less than two weeks in the short run.

Based on the above discussion of related literature, the efficiency and asymmetric multifractal features of newly developed blockchain based assets NFTs and DeFi has not yet been uncovered. Among these studies, the application of MF-ADCCA most similar to our study is Kakinaka and Umeno (2022). However, while they are mainly interested in two cryptocurrencies, we focus on the multifractality and efficiency measure in a comprehensive portfolio set that incorporates four heterogeneous asset classes. In addition, the sample period of our study differs from Kakinaka and Umeno (2022) in measuring asset market efficiency and identifying the asymmetric spectrum parameters.

3. Data and methodology

3.1. Data

We choose twelve assets from four heterogeneous asset classes, where NFTs and DeFi are selected based on higher market capitalization. As the data of Enjin Coin, one of the most important NFT with significant market capitalization, is available from November 21, 2017, the sample period of this study covers from November 21, 2017, to February 8, 2022. The description of variables and their data sources are presented in Table 1. To examine the sensitivity of efficiency across major economic events, which is COVID-19 in the study sample period, the dataset was split into three panels: Panel A: Full Sample (2017/11/21 to 2022/02/08), Panel B: Before COVID-19 (2017/11/21 to 2019/12/31), and Panel C: During COVID-19 (2020/01/01 to 2022/02/08). NFTs, DeFi, and cryptocurrencies data are collected from CoinMarketCap, while the data of traditional assets (Gold, WTI, and S&P500) are sourced from DataStream.

3.2. Methodology

Asymmetric Multifractal Detrended Fluctuation Analysis (A-MFDFA) is a technique that is often used to examine the efficiency of a financial time series. It also detects the random walk features of the asset using generalized Hurst exponents. A generalized Hurst exponent investigates long-term memory in a financial time series. The Hurst exponent calculates the comparative propensity of a time series either to regress intensely to the average or to cluster in a direction. If the time series has a greater level of long-term autocorrelation, there are evident inefficiencies (Kantelhardt et al., 2002). This study applies the index-based Multifractal Asymmetric Detrended Cross-correlation Analysis (MF-ADCCA) proposed by Cao et al. (2014) and Lee et al. (2017), which is a modified version of the symmetric MFDFA technique that detects multifractality and assumes that the influence of a downward trend on price dynamics is the same as that of an upward trend. As previously mentioned, the NFTs and DeFi respond differently during up and down trends (Dowling, 2022b), which has a varied impact on NFTs or DeFi volatilities and returns. Consequently, MF-ADCCA allows us to capture asymmetric upward and downward trends, as well as asymmetry in scaling performance of NFTs and DeFi.

As previously mentioned index-based MF-ADCCA was employed, which requires the return and volatility of assets. Therefore, the daily return of each asset is calculated using the following logarithmic equation (Kakinaka & Umeno, 2021):

$$r_{i,t} = \ln(p_{i,t}) - \ln(p_{i,t-1}) \quad (1)$$

where $r_{i,t}$ = return, i = asset, p_i denotes the price of the asset at time t .

Next, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model proposed by Bollerslev (1986) is applied to estimate the volatility for daily returns. Following the simple constant expected return (CER) model, the continuously compounded daily return, R_t on an asset can be expressed as,

$$R_t = \mu + \varepsilon_t, t = 1, \dots, T$$

$$\varepsilon_t = \sigma Z_t$$

$$Z_t \sim iid N(0, 1). \quad (2)$$

here, ε_t is the unexpected return, σ is the unconditional volatility of the unexpected return and assumed to be constant, and Z is the standardized unexpected return ($Z_t = \varepsilon_t/\sigma$). Then, following Glosten, Jagannathan, and Runkle (1993) and Zakoian (1994) we define the univariate threshold generalized autoregressive conditional heteroskedasticity (TGARCH) (1,1) model to estimate the daily return volatility as,

$$\sigma_{i,t}^2 = \alpha_1 + (\alpha_2 + \gamma D_{t-1})\epsilon_{t-1}^2 + \beta\sigma_{i,t-1}^2 \quad (3)$$

where α' s, γ , and β are nonnegative parameters satisfying standard assumptions of a GARCH model, and D is an indicator variable,

$$D_{t-1} = \begin{cases} 1 & \text{if } \epsilon_{t-1} < 0 \\ 0 & \text{if } \epsilon_{t-1} \geq 0 \end{cases} \quad (4)$$

The TGARCH model in Eq. (3) considers the impact of a past positive and a negative shock separately and uses zero as the threshold level volatility value (Alam, 2022). Finally, we calculate the change in volatility as follows (Kakinaka & Umeno, 2022):

$$v_{i,t} = \ln(\sqrt{\sigma_{i,t}^2}) - \ln(\sqrt{\sigma_{i,t-1}^2}) \quad (5)$$

where v presents the volatility of asset i at time t .

Afterward, based on asymmetric cross-correlations across return and volatility, MF-ADCCA approach evaluates if the aggregated index has a positive or negative fluctuation $\{x_t : t = 1, \dots, N\}$ and $\{y_t : t = 1, \dots, N\}$. First, the outlines from the data set are created.

$$X(k) = \sum_{t=1}^k (x_t - \bar{x}), t = 1, \dots, N, \quad (6)$$

$$Y(k) = \sum_{t=1}^k (y_t - \bar{y}), t = 1, \dots, N, \quad (7)$$

where \bar{x} and \bar{y} are the averages of the entire return series. The proxy series index is also computed at $I(k) = I(k-1) \exp(x_k)$ for $k = 1, \dots, N$ with $I(0) = 1$, which evaluates if the directions of the index series are negative and positive. The $X(k)$, $Y(k)$, and proxy index $I(k)$ profiles are then allocated into $N_s = [N/s]$ non-overlapping sections of length s . If N is not a multiple of s , just a small portion of the profile remains. To analyze the entire profile, the division is repeated beginning at the opposite end of the data set. As a result, each series has a total of $2N_s$ segments.

The series is then detrended, and the local trends of the profiles are measured by fitting a degree-2 polynomial least-square \bar{X}_v and \bar{Y}_v to detrend $X(k)$ and $Y(k)$, respectively, for each section $v = 1, \dots, 2N_s$ of interval s . Simultaneously, the linear least-square fit is evaluated, $\bar{I}_v(i) = a_{i_v} + b_{i_v} i$ ($i = 1, \dots, s$) for $X(k)$ and $Y(k)$, in order to determine the index series' local asymmetric direction. The slope b_{i_v} determines whether the trend is positive (upward) or negative (downward). Then, for each of the $2N_s$ segments, the detrended covariance is calculated as follows:

$$f^2(s, v) = \frac{1}{s} \sum_{i=1}^s |X((v-1)s+i) - \bar{X}_v(i)| |Y((v-1)s+i) - \bar{Y}_v(i)| \quad (8)$$

For $v = 1, \dots, N_s$ and

$$f^2(s, v) = \frac{1}{s} \sum_{i=1}^s |X(N - (v - N_s)s + i) - \bar{X}_v(i)| |Y(N - (v - N_s)s + i) - \bar{Y}_v(i)| \quad (9)$$

For $v = N_s + 1, \dots, 2N_s$. The downward and upward q th order volatility functions are measured by considering the mean of all sections and dividing it by the number of sections:

$$F_q^+(s) = \left\{ \frac{1}{M^+} \sum_{v=1}^{2N_s} \frac{1 + \text{sgn}(b_{i_v})}{2} [f^2(s, v)]^{q/2} \right\}^{1/q} \quad (10)$$

$$F_q^-(s) = \left\{ \frac{1}{M^-} \sum_{v=1}^{2N_s} \frac{1 - \text{sgn}(b_{i_v})}{2} [f^2(s, v)]^{q/2} \right\}^{1/q}, \quad (11)$$

For any real value $q \neq 0$, and

$$F_0^+(s) = \exp \left\{ \frac{1}{2M^+} \sum_{v=1}^{2N_s} \frac{1 + \text{sgn}(b_{i_v})}{2} \ln [f^2(s, v)] \right\}, \quad (12)$$

$$F_0^-(s) = \exp \left\{ \frac{1}{2M^-} \sum_{v=1}^{2N_s} \frac{1 - \text{sgn}(b_{i_v})}{2} \ln [f^2(s, v)] \right\}, \quad (13)$$

when $q = 0$, $M^- = \sum_{v=1}^{2N_s} \frac{1 - \text{sgn}(b_{i_v})}{2}$ and $M^+ = \sum_{v=1}^{2N_s} \frac{1 + \text{sgn}(b_{i_v})}{2}$ denote the number of sections with negative and positive trends, respectively, under the assumption that $b_{i_v} \neq 0$ for each $v = 1, \dots, 2N_s$, therefore $M^+ + M^- = 2N_s$. The overall trend's q th order fluctuation functions correspond to the MF-DCCA method, which is demonstrated as:

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{v=1}^{2N_s} [f^2(s, v)]^{q/2} \right\}^{1/q} \quad (14)$$

For $q = 0$, and $q \neq 0$

$$F_0(s) = \exp \left\{ \frac{1}{4N_s} \sum_{v=1}^{2N_s} \ln [f^2(s, v)] \right\} \quad (15)$$

The q th order volatility functions emulate a power-law of the functions $F_q^+(s) \sim s^{h_{xy}^+(q)}$, $F_q^-(s) \sim s^{h_{xy}^-(q)}$, and $F_q(s) \sim s^{h_{xy}(q)}$ if the financial time series x_k and y_k are power-law and long-range cross-correlated. The scaling exponent, also known as the generalized Hurst exponent, is used to illustrate the long-range power-law correlation features. A log-log linear regression is used to calculate the scaling exponent. On the other hand, the regression's performance is largely determined by the range of scales used. A scale ranging from $s_{min} = \max(20, N/100)$ to $s_{max} = \min(20s_{min}, N/10)$ and 100 points in the regression, as suggested by Thompson and Wilson (2016), is used to avoid biases.

In the absence of cross-correlations, $h_{xy}(q) = 0.5$ is sufficient. The cross-correlations among the financial time series are perseverance with long-memory if $h_{xy}(q) > 0.5$. $h_{xy}(q) < 0.5$, on the other hand, implies that the financial time series' cross-correlations are anti-perseverance with a short memory. For $h_{xy}^+(q)$ and $h_{xy}^-(q)$, the same explanation applies, but the cross-correlations scaling exponents are calculated individually for negative and positive fluctuations.

The order of q indicates how the various magnitudes of fluctuations should be weighed. The behavior of larger fluctuations is reflected by scaling exponents for $q > 0$, where the volatility function $F_q(s)$ is dominated by extensive movements. Because small movements influence the volatility function, scaling exponents for $q < 0$ represents the dynamics of smaller movements. The scaling dynamics of the detrended covariance $f^2(s, v)$ is different for each section if $h_{xy}(q)$ is not influenced by q , in this case the series' cross-correlation is monofractal. On the other hand, if the value varies with q , and small and large movements have distinct scaling characteristics, then the cross-correlations of the series are multifractal. Moreover, it is worth noting that when $q = 2$, the Hurst exponent equates to $h_{xy}(q)$.

Additionally, multifractality characteristics can be further investigated using the following function:

$$\tau_{xy}(q) = qh_{xy}(q) - 1 \quad (16)$$

The cross-correlation of the financial time series is monofractal if $\tau_{xy}(q)$ is a linear function of q ; otherwise, it is multifractal. The singularity spectrum is measured using the following function:

$$\alpha = h_{xy}(q) + h'_{xy}(q), \quad (17)$$

$$f_{xy}(\alpha) = q(\alpha - h_{xy}(q)) + 1, \quad (18)$$

where α is the bivariate series' singularity. The level of multifractality of the bivariate time series is represented by $\Delta\alpha = \alpha_{max} - \alpha_{min}$, where α_{max} and α_{min} are the α values at the max and min of $f_{xy}(\alpha)$, respectively. The singularity spectrum in the situation of monofractality is theoretically nothing more than a point. The multifractal features of asymmetric generalized Hurst exponents $h_{xy}^+(q)$ and $h_{xy}^-(q)$ were investigated using the prior discussions. Therefore, the singularity spectra, $h_{xy}^+(q)$ and

Table 2
Return series descriptive statistics and unit-root test results of panel A: Full sample.

	BAT	LINK	MANA	DGB	ENJ	MKR	BTC	ETH	USDT	Gold	WTI	S&P500
N	1062	1062	1062	1062	1062	1062	1062	1062	1062	1062	1062	1062
Mean	0.002	0.004	0.005	0.001	0.004	0.002	0.002	0.002	0.000	0.000	0.000	0.001
SD	0.081	0.085	0.099	0.087	0.102	0.078	0.049	0.063	0.005	0.009	0.057	0.013
Min	-0.514	-0.615	-0.641	-0.537	-0.721	-0.818	-0.465	-0.551	-0.053	-0.051	-1.324	-0.128
Max	0.518	0.481	0.923	0.6	0.743	0.458	0.225	0.344	0.057	0.058	0.723	0.09
Skw	0.126	-0.1	1.117	0.194	0.892	-0.467	-0.787	-0.642	0.704	-0.284	-9.688	-1.048
Kurt	6.059	5.897	15.73	7.496	13.409	15.665	9.98	7.957	38.512	6.025	293.306	18.198
ADF	-9.11**	-9.53**	-10.41**	-8.55**	-8.31**	-9.62**	-9.36**	-8.72**	-13.34**	-10.61**	-7.94**	-9.14**
PP	-1176.22**	-1162.80**	-1098.33**	-1297.88**	-1224.92**	-1206.80**	-1198.01**	-1192.97**	-1153.42**	-968.56**	-1404.50**	-1410.93**

Note: Min = Minimum, SD = Standard deviation, Max = Maximum, Skw = Skewness, Kurt = Kurtosis, ADF = Augmented Dickey-Fuller test, PP = Phillips-Perron test. **, ***, * denote significance at 1%, 5% and 10% significance level.

Table 3
Return series descriptive statistics and unit-root test results of panel b: Before COVID-19.

	BAT	LINK	MANA	DGB	ENJ	MKR	BTC	ETH	USDT	Gold	WTI	S&P500
N	531	531	531	531	531	531	531	531	531	531	531	531
Mean	0.000	0.005	0.002	-0.001	0.002	0.001	0.000	-0.002	0.000	0.000	0.000	0.000
SD	0.086	0.091	0.100	0.088	0.109	0.075	0.049	0.061	0.006	0.007	0.020	0.009
Min	-0.428	-0.370	-0.641	-0.537	-0.721	-0.335	-0.239	-0.272	-0.048	-0.023	-0.082	-0.042
Max	0.518	0.481	0.923	0.600	0.743	0.458	0.225	0.247	0.057	0.035	0.137	0.048
Skw	0.467	0.725	1.506	0.496	1.288	0.677	0.065	0.002	0.757	0.173	-0.026	-0.636
Kurt	5.716	3.754	18.414	9.725	15.147	6.765	3.723	2.641	21.482	2.492	5.696	3.948
ADF	-7.708**	-6.914**	-8.631**	-7.699**	-6.127**	-7.701**	-7.356**	-7.385**	-9.361**	-8.295**	-7.11**	-7.829**
PP	-591.37**	-571.61**	-592.14**	-660.60**	-623.82**	-511.33**	-571.92**	-546.05**	-575.73**	-557.05**	-606.56**	-530.43**

Note: Min = Minimum, SD = Standard deviation, Max = Maximum, Skw = Skewness, Kurt = Kurtosis, ADF = Augmented Dickey-Fuller test, PP = Phillips-Perron test. **, ***, * denote significance at 1%, 5% and 10% significance level.

Table 4
Return series descriptive statistics and unit-root test results of panel C: During COVID-19.

	BAT	LINK	MANA	DGB	ENJ	MKR	BTC	ETH	USDT	Gold	WTI	S&P500
N	531	531	531	531	531	531	531	531	531	531	531	531
Mean	0.003	0.004	0.009	0.003	0.006	0.003	0.003	0.006	0.000	0.000	0.001	0.001
SD	0.076	0.080	0.097	0.086	0.095	0.080	0.048	0.065	0.004	0.012	0.079	0.016
Min	-0.514	-0.615	-0.630	-0.536	-0.624	-0.818	-0.465	-0.551	-0.053	-0.051	-1.324	-0.128
Max	0.302	0.289	0.779	0.448	0.640	0.341	0.192	0.344	0.053	0.058	0.723	0.090
Skw	-0.361	-1.318	0.698	-0.123	0.307	-1.406	-1.687	-1.191	0.462	-0.356	-7.560	-1.023
Kurt	6.420	9.190	12.980	5.246	9.992	22.553	17.120	12.301	85.791	4.630	166.212	14.854
ADF	-7.897**	-7.941**	-6.995**	-6.296**	-7.082**	-7.01**	-7.262**	-7.382**	-12.614**	-8.935**	-7.26**	-6.46**
PP	-579.42**	-590.58**	-502.38**	-609.03**	-583.93**	-674.42**	-608.42**	-622.13**	-590.67**	-474.67**	-701.22**	-754.10**

Note: Min = Minimum, SD = Standard deviation, Max = Maximum, Skw = Skewness, Kurt = Kurtosis, ADF = Augmented Dickey-Fuller test, PP = Phillips-Perron test. **, ***, * denote significance at 1%, 5% and 10% significance level.

Table 5
Returns Volatility series descriptive statistics and unit-root test results of panel A: Full sample.

	BAT	LINK	MANA	DGB	ENJ	MKR	BTC	ETH	USDT	Gold	WTI	SP500
N	1062	1062	1062	1062	1062	1062	1062	1062	1062	1062	1062	1062
Mean	0.000	0.000	0.001	0.000	0.001	0.001	0.001	0.001	-0.002	0.001	0.000	0.000
SD	0.070	0.068	0.181	0.085	0.116	0.111	0.067	0.088	0.240	0.045	0.116	0.135
Min	-0.097	-0.096	-0.472	-0.107	-0.169	-0.190	-0.077	-0.201	-0.297	-0.054	-0.190	-0.222
Max	0.612	0.641	1.047	0.765	0.924	1.196	0.711	0.797	1.843	0.339	1.109	0.854
Skw	2.638	2.729	1.638	2.694	2.918	3.002	3.405	2.296	1.726	2.608	3.335	2.117
Kurt	11.829	13.427	4.927	11.513	12.811	17.850	20.367	11.011	5.350	9.702	19.059	6.759
ADF	-11.11**	-11.85**	-12.7**	-11.86**	-10.81**	-12.32**	-10.42**	-13.25**	-12.24**	-10.12**	-10.47**	-10.59**
PP	-1048.47**	-1073.9**	-875.73**	-1059.58**	-948.43**	-952.31**	-1080.49**	-991.96**	-786.7**	-1103.51**	-849.48**	-1106.72**

Note: Min = Minimum, SD = Standard deviation, Max = Maximum, Skw = Skewness, Kurt = Kurtosis, ADF = Augmented Dickey-Fuller test, PP = Phillips-Perron test. **, ***, * denote significance at 1%, 5% and 10% significance level.

Table 6
Returns volatility series descriptive statistics and unit-root test results of panel B: Before COVID-19.

	BAT	LINK	MANA	DGB	ENJ	MKR	BTC	ETH	USDT	Gold	WTI	SP500
N	531	531	531	531	531	531	531	531	531	531	531	531
Mean	0.001	0.000	0.001	0.000	0.002	0.001	0.001	0.001	0.002	0.001	0.000	-0.001
SD	0.069	0.069	0.173	0.081	0.122	0.107	0.067	0.085	0.253	0.038	0.101	0.124
Min	-0.097	-0.096	-0.472	-0.107	-0.169	-0.189	-0.077	-0.164	-0.297	-0.034	-0.138	-0.184
Max	0.483	0.500	0.976	0.514	0.924	0.571	0.516	0.538	1.287	0.274	0.837	0.854
Skw	2.270	2.316	1.588	2.459	2.993	2.027	2.984	1.720	1.395	2.660	3.027	2.243
Kurt	8.248	8.224	5.508	8.519	13.628	5.460	13.236	5.323	2.546	9.368	15.341	8.017
ADF	-7.74**	-9.22**	-9.26**	-9.1**	-8.67**	-10.11**	-8.01**	-9.76**	-9.07**	-8.85**	-8.72**	-8**
PP	-536.25**	-538.87**	-483.66**	-506.32**	-460.2**	-474.75**	-516.88**	-464.5**	-427.13**	-561.31**	-502.09**	-526.15**

Note: Min = Minimum, SD = Standard deviation, Max = Maximum, Skw = Skewness, Kurt = Kurtosis, ADF = Augmented Dickey-Fuller test, PP = Phillips-Perron test. **, ***, * denote significance at 1%, 5% and 10% significance level.

Table 7
Returns Volatility series descriptive statistics and unit-root test results of panel C: During COVID-19.

	BAT	LINK	MANA	DGB	ENJ	MKR	BTC	ETH	USDT	Gold	WTI	SP500
N	531	531	531	531	531	531	531	531	531	531	531	531
Mean	0.000	0.000	0.000	0.001	0.001	0.000	0.000	0.000	-0.005	0.000	0.000	0.002
SD	0.072	0.066	0.188	0.088	0.109	0.116	0.066	0.091	0.225	0.051	0.129	0.145
Min	-0.096	-0.094	-0.469	-0.095	-0.157	-0.190	-0.077	-0.201	-0.295	-0.054	-0.190	-0.222
Max	0.612	0.641	1.047	0.765	0.746	1.196	0.711	0.797	1.843	0.339	1.109	0.843
Skw	2.969	3.213	1.675	2.872	2.788	3.773	3.863	2.771	2.171	2.491	3.378	2.007
Kurt	14.956	19.878	4.470	13.595	11.212	26.921	28.325	15.421	9.653	8.680	18.859	5.779
ADF	-8.83**	-9.02**	-11.17**	-8.29**	-9.12**	-9.38**	-8.18**	-10.19**	-9.02**	-7.82**	-7.77**	-8.53**
PP	-516.84**	-546.97**	-428.3**	-551.84**	-506.87**	-496.64**	-563.72**	-553.95**	-374.97**	-546.66**	-379.61**	-575.57**

Note: Min = Minimum, SD = Standard deviation, Max = Maximum, Skw = Skewness, Kurt = Kurtosis, ADF = Augmented Dickey-Fuller test, PP = Phillips-Perron test. **, ***, * denote significance at 1%, 5% and 10% significance level.

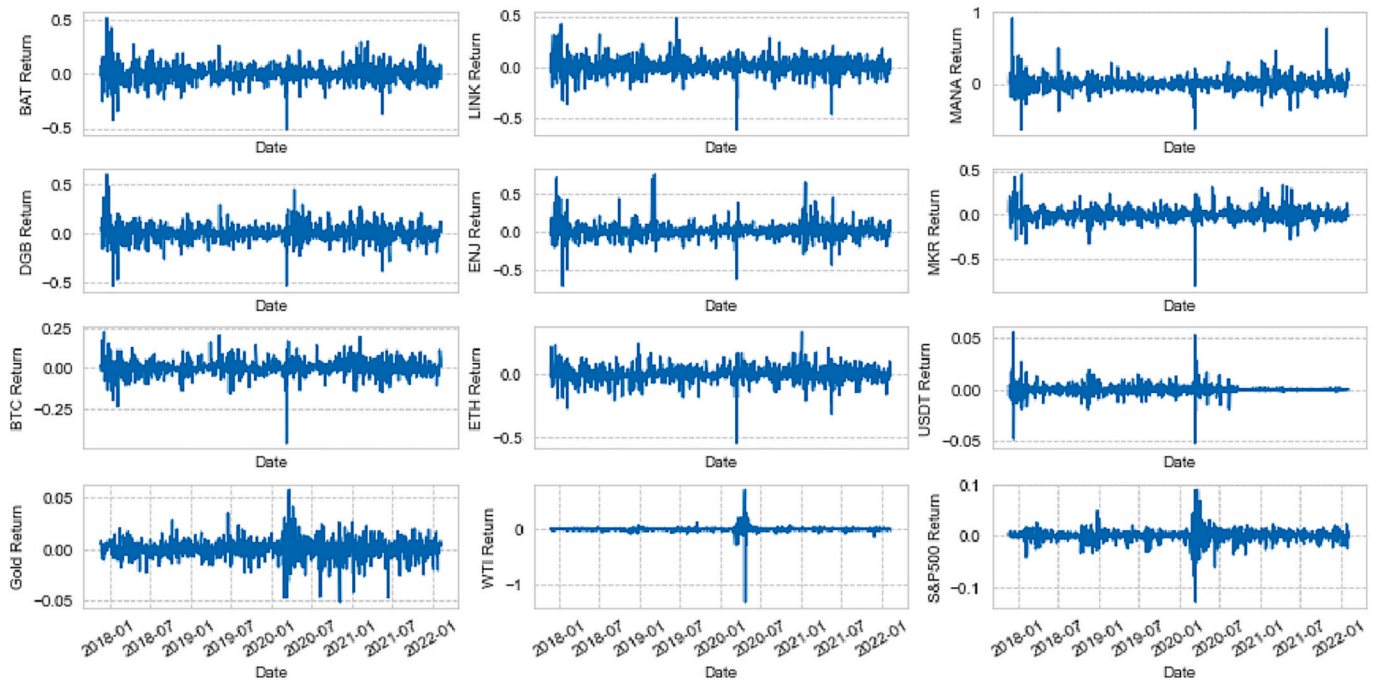


Fig. 1. Historical return of assets.

$h_{xy}^-(q)$, as well as the asymmetric cases of the Rényi's exponent, $\tau_{xy}^+(q)$ and $\tau_{xy}^-(q)$, can be calculated.

4. Empirical results

4.1. Summary statistics results

Tables 2, 3, and 4 present descriptive statistics and unit-root test results of assets return series of Panel A, Panel B, and Panel C, respectively. The mean return of NFTs (MANA & LINK) and DeFi (ENJ) surpass cryptocurrencies (BTC, ETH, and USD) and other traditional financial

assets (Gold, WTI, and S&P500) in the entire sample period. Before COVID-19, LINK had the highest return, whereas, during COVID-19, MANA had the highest return. The maximum return shows that NFT (MANA) has the highest return in the full sample (0.923), before COVID-19 (0.779), and during COVID-19 (0.779).

A recent study by Yousaf and Yarovaya (2022) also found a higher return of NFTs and DeFi than other assets. The standard deviation shows NFTs, DeFi, and cryptocurrencies returns are much higher than the returns of traditional assets. NFTs, DeFi, and cryptocurrencies are highly volatile compared to other assets. Therefore, DeFi and NFTs offer more returns at a higher risk compared to the other assets we have examined.

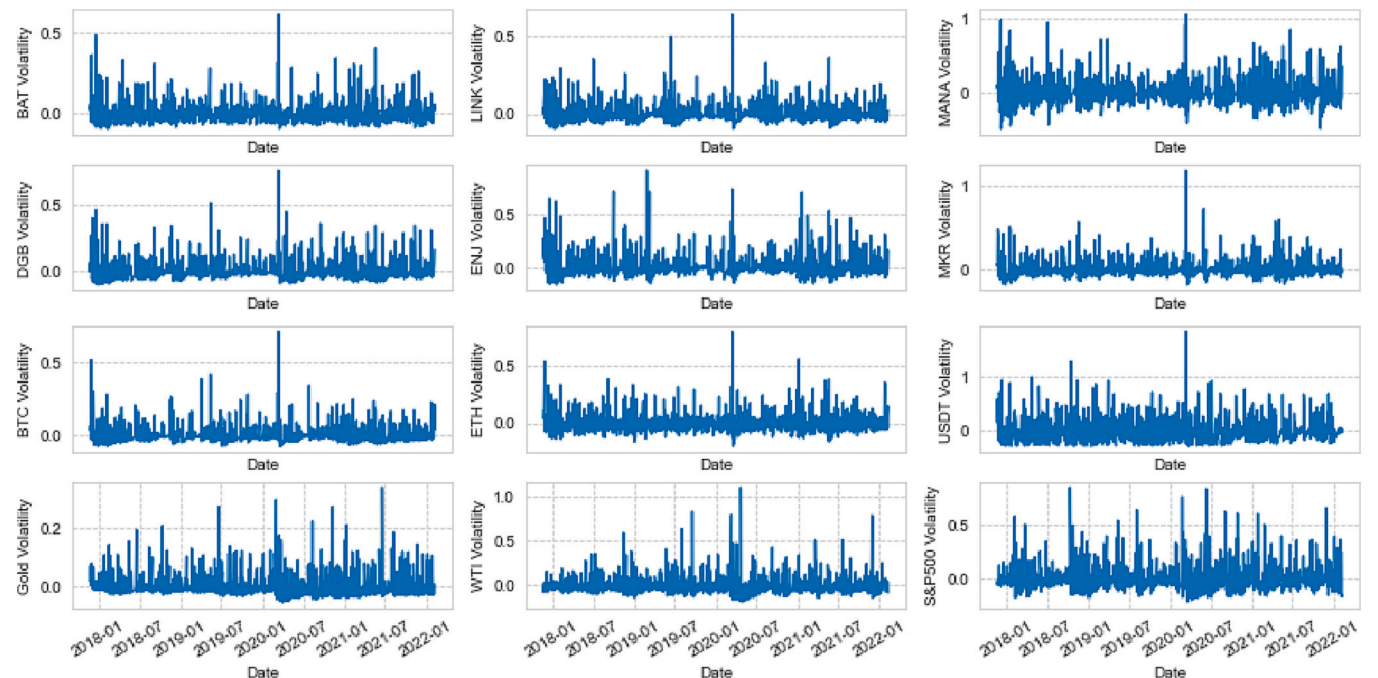


Fig. 2. Historical series of returns volatility.

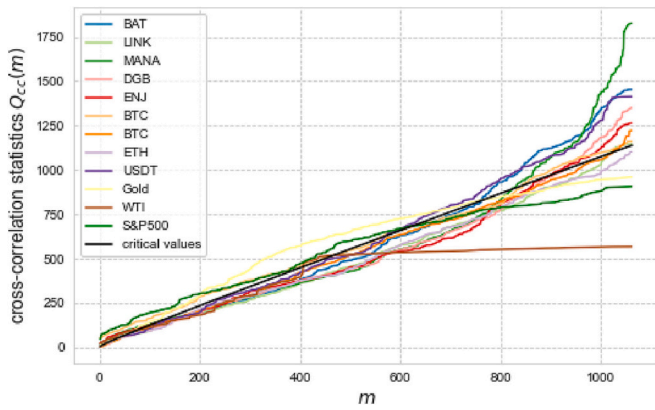


Fig. 3. Cross-Correlation of all assets.

As a result, the risk-return outlines of NFTs and Defi assets significantly differ from traditional financial assets. Skewness values suggest LINK, MKR, BTC, ETH, Gold, WTI, and S&P500 are negative in the whole sample while other assets have positive skewness. The kurtosis values show that most asset returns have fat tail distribution across all panels. The Augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) unit-root test are used to examine the stationarity of variables, and the results demonstrate that all variables return series are stationary.

Nevertheless, we also calculate the assets' daily returns volatility using the univariate TGARCH model using a full sample (2017/11/21 to 2022/02/08). Next, we split the estimated volatility into two panels where Panel B: Before COVID-19 (2017/11/21 to 2019/12/31), and Panel C: During COVID-19 (2020/01/01 to 2022/02/08). The descriptive statistics and unit-root test results of all assets' daily returns volatility are presented in Tables 5, 6, and 7 for Panel A, Panel B and Panel C, respectively. Result shows on average, USDT have negative volatility in full sample and during COVID-19, whereas USDT were positive before COVID-19. The ADF and PP unit-root test results confirms the stationarity of volatility series of all assets. Furthermore, the historical return of selected assets is presented in Fig. 1, and the volatility of all assets is presented in Fig. 2, where all assets show higher fluctuation during COVID-19 in both return and volatility.

4.2. Asymmetric detrended cross-correlation analysis (MF-ADCCA) results

The cross-correlations between price and volatility movement are reported first, to ensure that DFA-based methods are valid for the analyses. To check for cross-correlations between bivariate series, the statistical test suggested by Podobnik et al. (2009) is used. They defined the cross-correlation statistic for equal-length N series $\{x_i\}$ and $\{y_i\}$ as:

$$Q_{cc}(m) = N^2 \sum_{i=1}^m \frac{X_i^2}{N-i}; \tag{19}$$

Here X_i denotes the cross-correlation function, which is calculated as follows:

$$X_i = \frac{\sum_{k=i+1}^N x_k y_{k-1}}{\sqrt{\sum_{k=1}^N x_k^2 \sum_{k=1}^N y_k^2}} \tag{20}$$

The null hypothesis of the test is that the first m cross-correlation coefficients are nonzero, which can be tested using the statistic $Q_{cc}(m)$, which follows approximate $\chi^2(m)$ distribution with m degrees of freedom. If $Q_{cc}(m)$ is larger than the critical value of $\chi^2(m)$, the null hypothesis should be rejected, and the series shows a substantial cross-correlation.

The cross-correlation test statistics for volatility and price fluctuations of the NFTs, DeFi, cryptocurrencies, and traditional assets are calculated in Eqs. (19) and (20) with different degrees of freedom, m , starting from 1 to 1062. The $\chi^2(m)$ distribution critical values at the 5% level of significance are illustrated in Fig. 3. Results suggest that the statistic $Q_{cc}(m)$ differs significantly from the associated critical values for all NFTs, DeFi, cryptocurrencies, and traditional assets, suggesting that price and volatility movements have nonlinear cross-correlations. The test statistic for WTI and S&P500 deviates from the critical value less than other assets, implying that traditional assets have weaker nonlinear cross-correlations than NFTs, DeFi, and cryptocurrencies.

MF-ADCCA is estimated after validating the presence of nonlinear cross-correlations in the bivariate time series. The q th order fluctuation functions measured from the returns and volatility movements of the full sample panel with different q starting from -10 to 10 are shown in Fig. 4, and before COVID-19 and after COVID-19 results are presented in Fig. S.1 and Fig. S.5. The fluctuation functions are also illustrated with the overall trend in distinct bear and bull market conditions (downtrend & uptrend). Results suggest that the fluctuation functions typically approach a power-law against the scale in all circumstances, meaning that the bivariate series cross-correlations have a long-range power-law feature. Consequently, the MF-ADCCA is suitable for studying cross-correlations and the asymmetry across uptrend and downtrend cross-correlations. Fig. 4, S.1, and S.5 demonstrate how the function of power-law cross-correlations differs across market circumstances with varied trends. To estimate the level of asymmetry of the cross-correlations for the given q , we utilize the metric defined as:

$$\Delta h_{xy}(q) = \Delta h_{xy}^+(q) - \Delta h_{xy}^-(q) \tag{21}$$

The higher the value, the more asymmetric the features in terms of each trend. If $\Delta h_{xy}(q) < 0$ ($\Delta h_{xy}(q) > 0$), the cross-correlation in uptrend situations has a smaller (larger) exponent than in uptrend conditions. While the bivariate time series have the same scaling exponent, theoretically, $\Delta h_{xy}(q)$ is equal to zero, and the two-time series exhibit symmetric cross-correlations. Moreover, we also compute the cases of $q = -10$ (small fluctuations), $q = 2$ (corresponding to the Hurst exponent), and $q = 10$ (large fluctuations). The results of asymmetric cross-correlation and efficiency ranking are presented in Tables 8, 9, and 10 for Panel A, Panel B and Panel C, respectively. Tables 8, 9, and 10 show that, irrespective of small and large fluctuations, $\Delta h_{xy}(q)$ is positive for all of the NFTs, DeFi, cryptocurrencies and traditional assets in panel A (except BAT, BTC, USDT, WTI and S&P500). The Panel B result shows before COVID-19 BAT, LINK, MANA, MKR, ETH, USDT, WTI and S&P500 are negative. Moreover, in panel C during COVID-19 BAT, MANA, MKR, WTI and S&P500 have negative $\Delta h_{xy}(q)$. Fig. 5, Fig. S.2 and Fig. S.6 validate these findings where $\Delta h_{xy}^+(q)$ is greater than $\Delta h_{xy}^-(q)$. Price-volatility in the uptrend markets' cross-correlations are somewhat more persistent at all levels of volatility than downtrend markets' cross-correlations.

The existence of multifractality is estimated by examining whether or not the generalized Hurst exponents are order q dependent. Given that $h_{xy}(q)$ decreases as q increases in Fig. 5, Fig. S.2 and Fig. S.6, $h_{xy}(q)$ is not perpetual for q , and thus multifractal properties exist in the bivariate series cross-correlations of NFTs, DeFi, cryptocurrencies, and traditional assets. Moreover, the mass function of all assets is illustrated in Fig. 6, Fig. S.3, and Fig. S.7. To describe the variation from efficiency and monofractality statistically, the market efficiency measure (MDM) of Wang, Liu, and Gu (2009) is employed as follows:

$$D_{xy} = \frac{1}{2} (|h_{xy}(-10) - 0.5| + |h_{xy}(10) - 0.5|) \tag{22}$$

The association among the series is efficient if D_{xy} is zero or close to zero. D_{xy} values greater than one indicate greater inefficiency, while lower values imply less inefficiency. This indicator is helpful in determining the degree of (in)efficiency (Y. Wang et al., 2009). Results of all panels' D_{xy} are ranked according to their degree of efficiency. In Panel A,

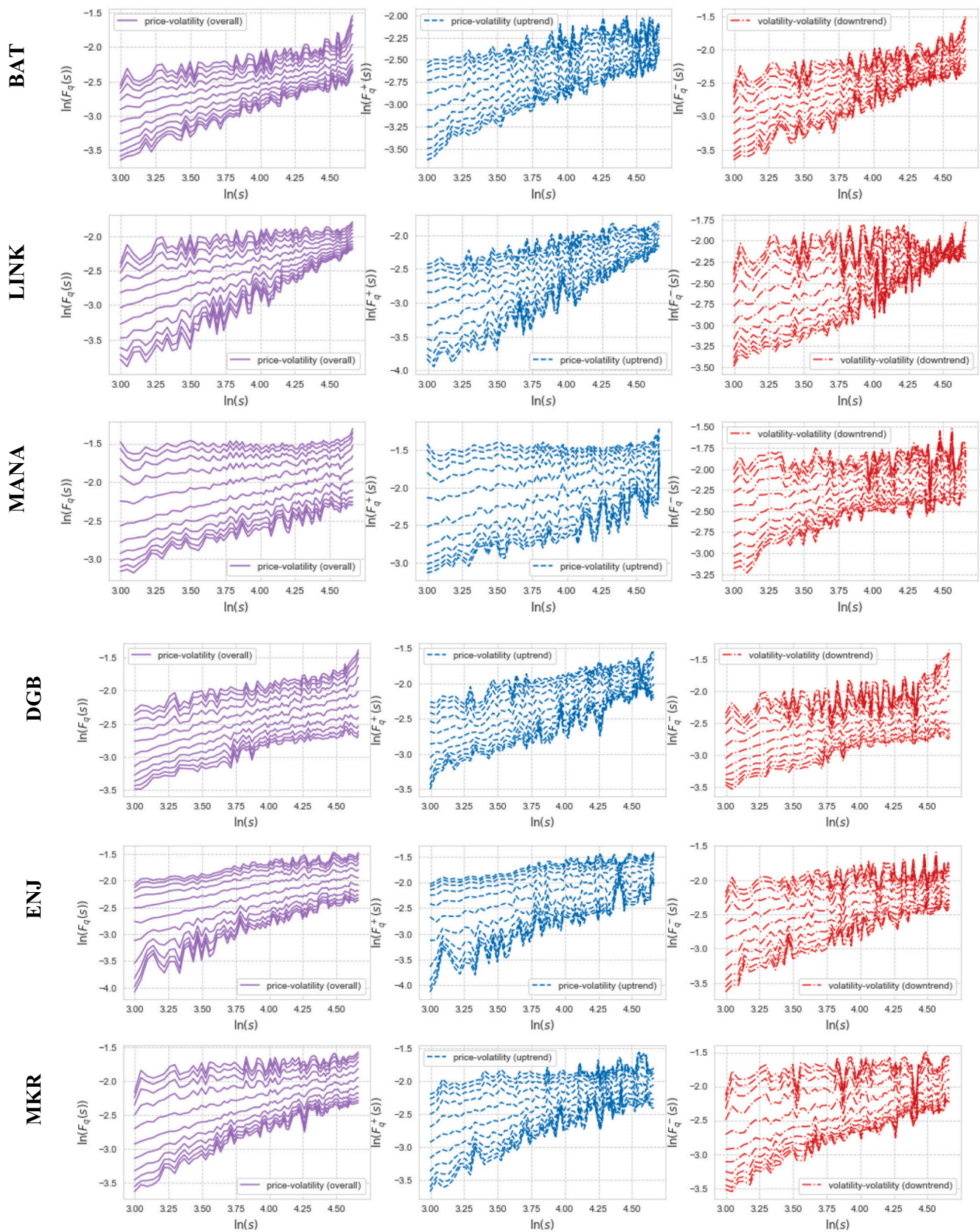


Fig. 4. A. Log-log plots of NFTs (Panel A).
 B. Log-log plots of DeFi (Panel A).
 C. Log-log plots of Cryptocurrencies (Panel A).
 D. Log-log plots of Other Assets (Panel A).

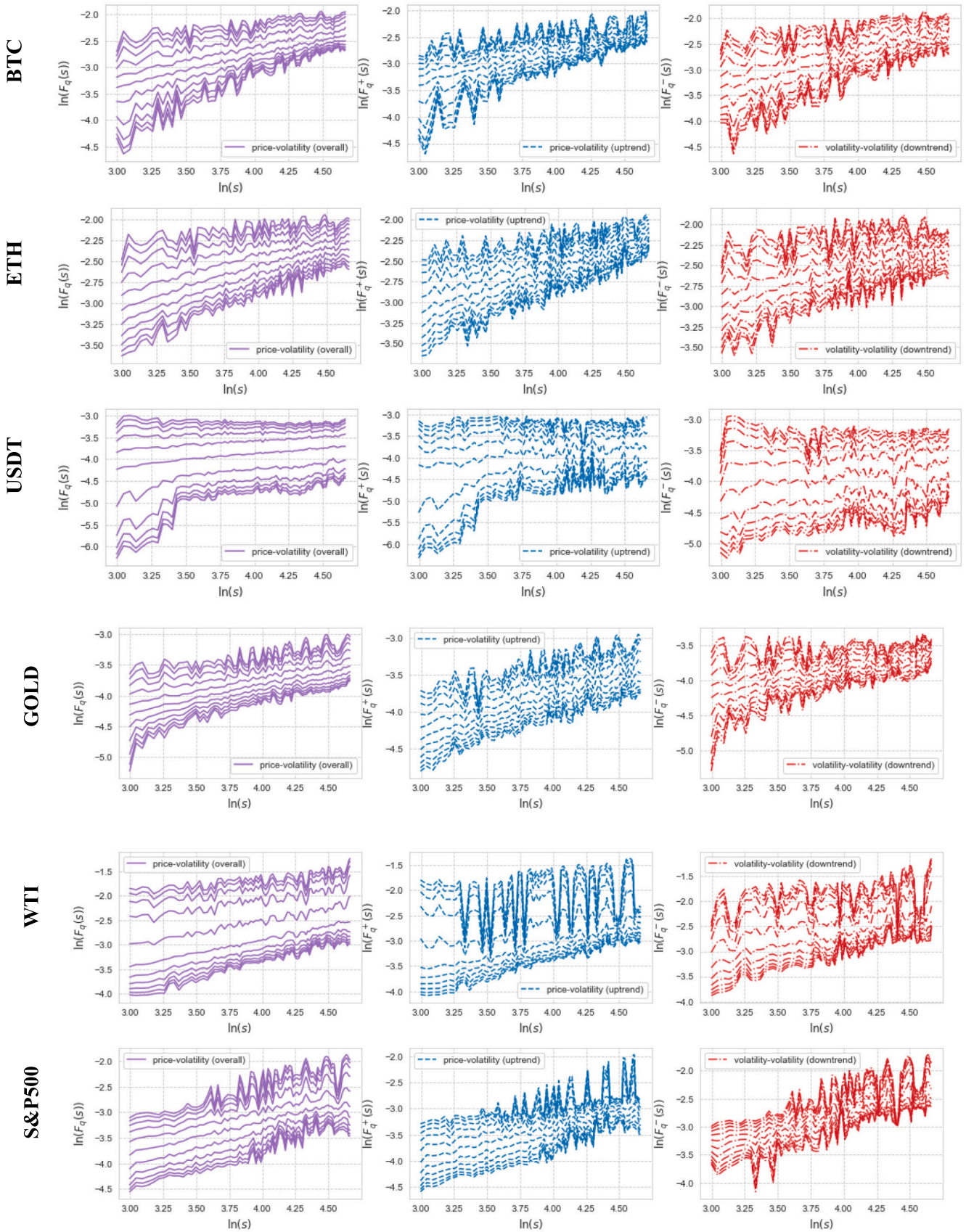


Fig. 4. (continued).

Table 8
Efficiency ranking of Panel A: Full sample.

	BAT	LINK	MANA	DGB	ENJ	MKR	BTC	ETH	USDT	Gold	WTI	SP500
Panel A: Full Sample												
$\Delta h_{xy}(-10)$	-0.220	0.230	0.144	0.276	0.313	0.258	-0.035	0.152	0.537	0.020	0.025	0.060
$\Delta h_{xy}(2)$	-0.183	0.142	0.061	0.059	0.178	0.094	-0.079	0.111	-0.018	0.208	-0.003	-0.093
$\Delta h_{xy}(10)$	-0.116	0.338	-0.030	0.003	0.196	0.091	0.055	0.182	-0.007	0.322	-0.147	-0.155
D_{xy}	0.156	0.411	0.229	0.063	0.288	0.288	0.389	0.225	0.422	0.144	0.213	0.241
Rank	3	11	6	1	9	8	10	5	12	2	4	7

Table 9
Efficiency ranking of Panel B: Before COVID-19.

	BAT	LINK	MANA	DGB	ENJ	MKR	BTC	ETH	USDT	Gold	WTI	SP500
$\Delta h_{xy}(-10)$	-0.114	0.187	-0.165	0.122	0.919	0.471	0.881	0.283	0.021	-0.185	-0.034	-0.314
$\Delta h_{xy}(2)$	-0.100	-0.098	-0.021	0.061	0.176	-0.066	0.046	-0.044	-0.068	0.167	-0.125	0.046
$\Delta h_{xy}(10)$	-0.081	-0.029	0.346	0.099	0.377	-0.092	0.001	-0.283	-0.227	0.137	-0.029	-0.061
D_{xy}	0.300	0.265	0.293	0.105	0.533	0.344	0.459	0.229	0.226	0.224	0.207	0.097
Rank	9	7	8	2	12	10	11	6	5	4	3	1

Table 10
Efficiency ranking of Panel A: Full sample.

	BAT	LINK	MANA	DGB	ENJ	MKR	BTC	ETH	USDT	Gold	WTI	SP500
$\Delta h_{xy}(-10)$	-0.165	0.501	-0.368	-0.127	0.119	-0.001	-0.134	-0.035	1.486	0.385	0.305	0.124
$\Delta h_{xy}(2)$	-0.111	0.164	0.075	0.324	0.273	0.015	0.058	0.134	0.785	0.369	0.128	-0.103
$\Delta h_{xy}(10)$	-0.024	0.412	-0.029	0.583	0.107	-0.039	0.362	0.182	1.222	0.424	-0.004	-0.186
D_{xy}	0.236	0.437	0.253	0.206	0.120	0.404	0.478	0.271	0.937	0.308	0.348	0.204
Rank	4	10	5	3	1	9	11	6	12	7	8	2

DGN ranked first with $D_{xy}=0.063$ and most efficient assets and USDT ranked last with $D_{xy}=0.422$. Before COVID-19, the ranking in Panel B suggests S&P500 ranked first with $D_{xy}=0.097$ and ENJ ranked last with $D_{xy}=0.533$. In contrast, during COVID-19 ENJ ranked first with $D_{xy}=0.12$ and USDT ranked last with $D_{xy}=0.937$. A recent study by [Mensi et al. \(2021\)](#) also found a decline in the efficiency of the stock market during COVID-19. The result shows there is a big change in ENJ before and during COVID-19. Previous studies by [Maouchi et al. \(2021\)](#) and [Yousaf, Nekhili and Gubareva \(2022\)](#) also found similar behavior of DeFi.

The asymmetric market trends of all panels are illustrated in [Fig. 7](#), [Fig. S.4](#), and [Fig. S.8](#) using the singularity spectra $f_{xy}(\alpha)$, $f_{xy}^+(\alpha)$, and $f_{xy}^-(\alpha)$ as an additional examination of the multifractal characteristics. As expected, the spectra are quite broad, and the width varies depending on NFTs, DeFi, cryptocurrencies, and traditional assets market trends. [Tables 11, 12, and 13](#) show the values of the asymmetric spectrum parameter A_α and Δ_α for the three financial market trends: downtrend, uptrend, and overall for Panel A, Panel B and Panel C. In the full sample, USDT has the highest degree of multifractality, with a value of $\Delta_\alpha=1.070$ in the overall trend, and Gold has the lowest degree of multifractality with a value of $\Delta_\alpha=0.193$. Moreover, before COVID-19 (Panel B), ENJ has the highest degree of multifractality, with a value of $\Delta_\alpha=1.757$ in the overall trend, and S&P500 has the lowest degree of multifractality with a value of $\Delta_\alpha=0.307$. Furthermore, during COVID-19 (Panel C), USDT has the highest degree of multifractality, with a value of $\Delta_\alpha=1.070$ in the overall trend, and DGB has the lowest degree of multifractality with a value of $\Delta_\alpha=0.193$.

We also go over the multifractal properties of another metric of the asymmetric spectrum parameter, $A_\alpha = \frac{\Delta_{\alpha L} - \Delta_{\alpha R}}{\Delta_{\alpha L} + \Delta_{\alpha R}}$, here $\Delta_{\alpha L} = \alpha_0 - \alpha_{min}$, $\Delta_{\alpha R} = \alpha_{max} - \alpha_0$, and α_0 is the value of α at the singular spectrum's maximum. The asymmetry here is due to the distortion of the singularity spectrum $f_{xy}(\alpha)$, not too different from market trends. The metric A_α provides information for identifying the bivariate series compositions. If $A_\alpha > 0$ ($A_\alpha < 0$), the singular spectrum has a left-sided (right-sided) asymmetry, indicating that $q > 0$ ($q < 0$) determines the scaling

properties, and thus larger (smaller) fluctuations dictate the multifractal properties ([Kakinaka & Umeno, 2021](#)).

As a result, in the left-sided case, multifractality is associated with larger fluctuations, whereas in the right-sided case, the opposite is true. When $A_\alpha = 0$, the breadth of the spectrum on the left and right sides is equal, and small and large variations work on multifractality in a similar manner. Because the left-sided spectrum is more prevalent in real-world financial data, it is realistic to assume odd characteristics in greater fluctuations and noise-like properties in smaller variations ([Kakinaka & Umeno, 2021](#)). The full sample overall trend shows that with LINK, ENJ, BTC, USDT, and S&P500's the asymmetric spectrum parameters A_α , and A_α^+ have negative values, whereas BTC and S&P500 have negative values in A_α^- . On the other hand, other assets have positive values. [Kakinaka and Umeno \(2021\)](#) and [Kakinaka and Umeno \(2022\)](#) also found similar properties of cryptocurrency.

Moreover, before COVID-19, DGB and ETH had positive values A_α , A_α^+ , and A_α^- of the asymmetric spectrum parameters and other assets had negative values. In contrast, during COVID-19 (Panel C), DGB and Gold had positive values A_α , A_α^+ , and A_α^- of the asymmetric spectrum parameters and other assets had negative values. Because their right-skewed spectra indicate that smaller events play a bigger part in the underlying multifractality, the results show that NFTs and DeFi are more sensitive to smaller events and these markets' price-volatility behavior is quite complex. Some cryptocurrencies and traditional assets, on the other hand, had left-skewed spectra, which represents weak multifractal features and denotes larger events contribute more to the multifractal behavior. Studies by [Mensi et al. \(2019\)](#), [El Alaoui et al., \(2019\)](#), and [Kakinaka and Umeno \(2022\)](#) also found weak multifractal features of cryptocurrencies.

5. Discussion and policy implications

Multifractality and efficiency of financial assets are very crucial for investors and policymakers. Attempts to explain the elements that contribute to increased efficiency and multifractality of new blockchain-based asset class NFTs and DeFi are still in their early stages, and we

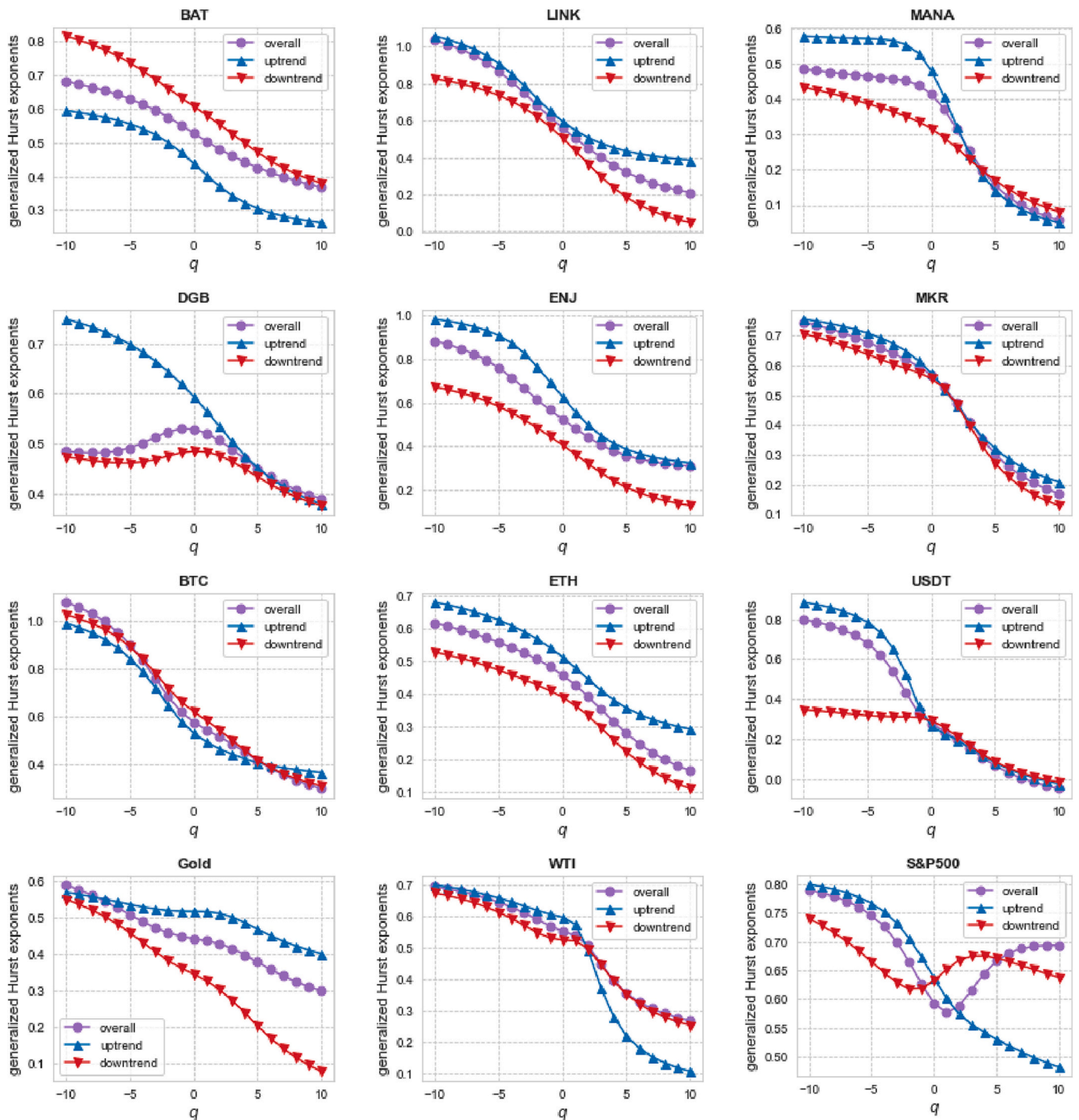


Fig. 5. Generalized Hurst exponents (Panel A).

have only a limited grasp of what causes an asset to be efficient or inefficient. The findings of our study uncover some salient information concerning the multifractality and efficiency of NFTs, DeFi, and a heterogeneous class of assets. The market efficiency result using MDM shows that in the full sample, DGB is the most efficient asset class. However, before COVID-19, S&P500 is the most efficient asset class, whereas ENJ is the most efficient asset class during the COVID-19 period. A possible cause for the efficiency of S&P500 before COVID-19 driven by the random walk of the assets, a previous study by Diniz-Maganini et al. (2021) also found that the stock market is more efficient than other assets. Interestingly, over the whole sample and during

COVID-19, DeFi (DGB and ENJ) exhibits the highest level of efficiency, which suggests investors are moving toward DeFi from traditional assets. These findings indicate that after the COVID-19 outbreak, investors move their investments to DeFi, and this efficiency surge indicates investors' herding behavior (Al-Yahyaee, Mensi, & Yoon, 2018; Naeem et al., 2021). A recent study by Maouchi et al. (2021) also found a digital bubble in the DeFi market due to COVID-19.

We also find that Tether outperforms Gold, WTI, and the S&P500 in terms of multifractality. The higher level of multifractality indicates there is a higher level of herding behavior persistent in the cryptocurrency market. This result implies that cryptocurrency investors move

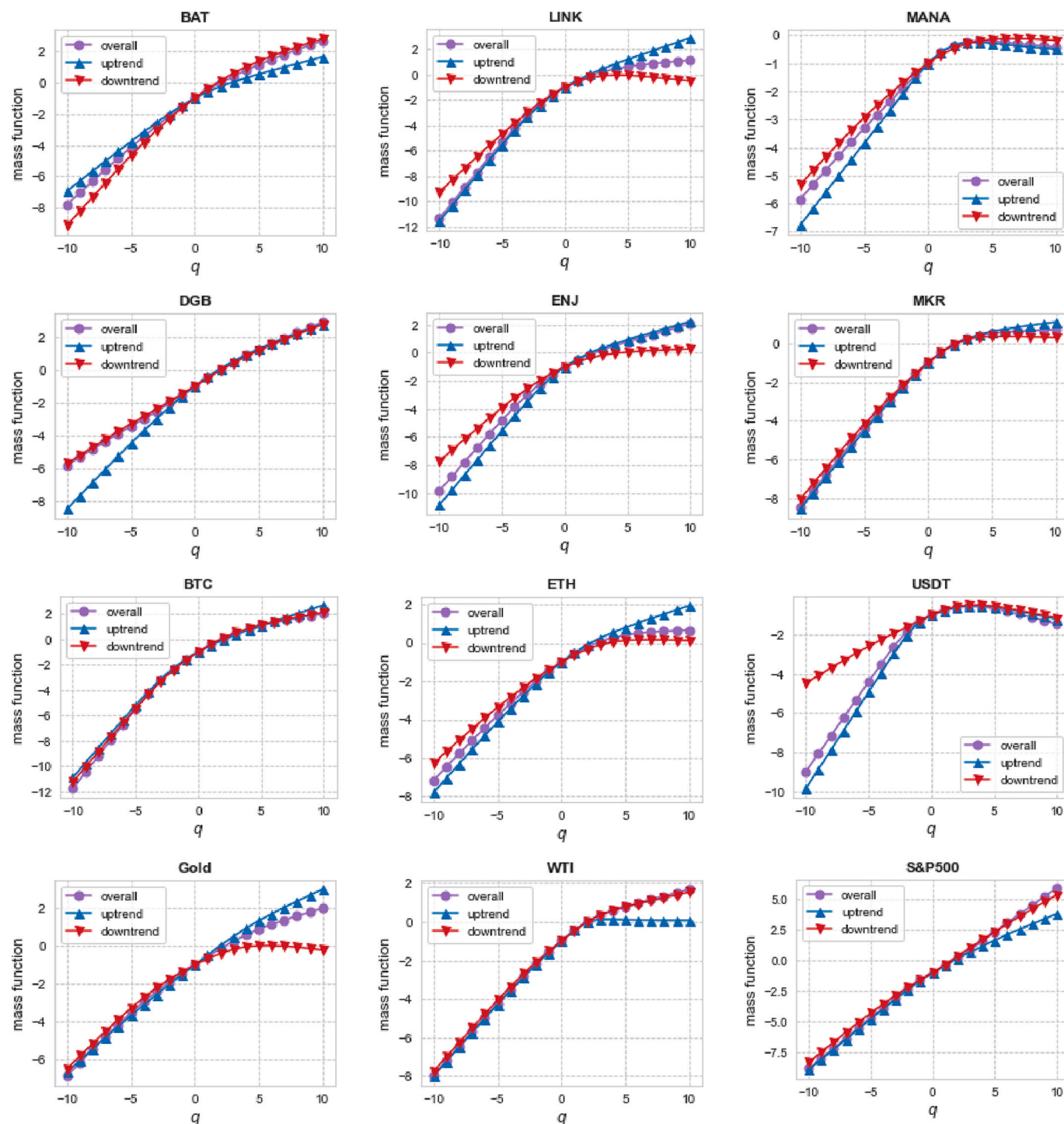


Fig. 6. Mass Function Plots (Panel A).

their investments due to strong herding behavior (Al-Yahyaee et al., 2018; Kakinaka & Umeno, 2022; Naeem et al., 2021). More importantly, traditional assets have substantially weaker nonlinearity than NFTs, and DeFi in terms of their volatility dynamics. The reason behind the higher nonlinearity of NFTs and DeFi is strong cross-correlations between return and volatility. Nevertheless, NFTs and DeFi show a higher sensitivity to small fluctuations in bull markets, which implies these assets are highly event-dependent and there is an existence of herding behavior (Kakinaka & Umeno, 2021, 2022).

Our empirical findings also offer significant policy implications for academics, investors, portfolio managers, and regulators. From a

theoretical standpoint, NFTs and DeFi support the efficient market hypothesis and fractal market hypothesis, as small fluctuations dominate in bear markets and large fluctuations dominate in bull markets. From the investor's and portfolio manager's perspective, there is a diversification benefit by adding NFTs and DeFi as they are efficient compared to other assets during financial turmoil, such as COVID-19. From the regulatory and policymaker perspective, our findings show that while NFTs and the DeFi bubble raised during COVID-19 (Maouchi et al., 2021), NFTs and DeFi markets are still developing so policymakers should closely monitor the market. The abnormal growth of NFTs and DeFi could have a negative impact on other cryptocurrencies and financial

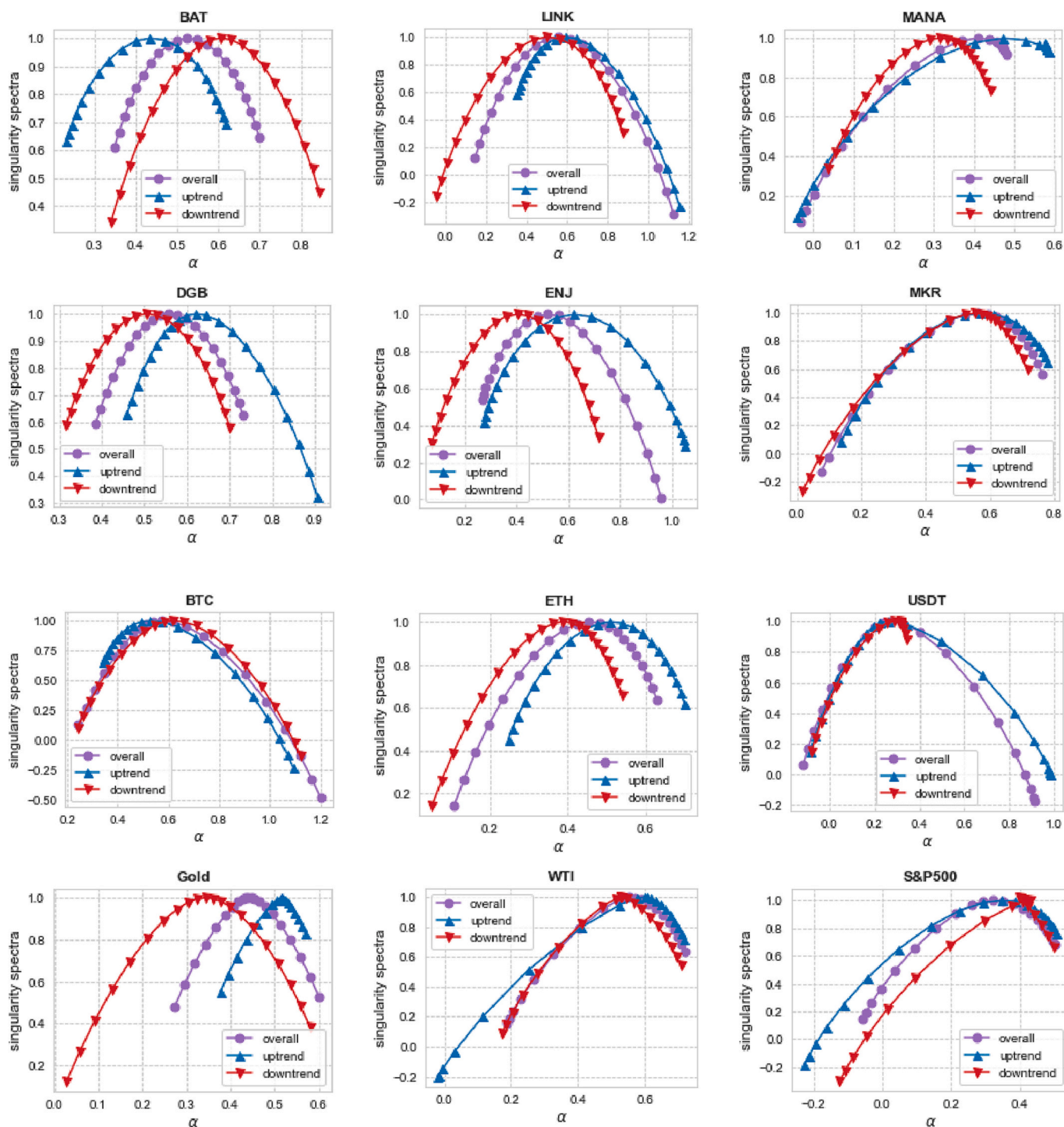


Fig. 7. Singularity Spectrum Plots (Panel A).

markets, which is cause for concern.

6. Conclusion

This study examines the efficiency and asymmetric multifractal features of NFTs, DeFi, cryptocurrencies, and traditional assets. Specifically, we examine the multifractal features of asymmetric cross-correlations when the market condition is in an upward or downward trend and the overall market trend. Results shows existence of power-law cross-correlations as well as multifractal features among NFTs, DeFi, cryptocurrencies, and traditional assets. We observed that

bivariate series exhibit distinct features in negative and positive market trends. However, Tether has the highest degree of multifractality in the entire sample period, and gold has the lowest degree of multifractality. Moreover, during COVID-19, Tether has remained an asset of the highest multifractality. When we rank assets according to the degree of efficiency, S&P 500 is the most efficient asset with its lowest degree of multifractality before the COVID-19. While the Enjin Coin (ENJ) showed the highest degree of multifractality before COVID-19, we notice that the ENJ is the most efficient asset class during the pandemic period. DeFi, NFTs, S&P 500, and gold are relatively more efficient assets than cryptocurrencies for the entire sample period. We also observe that the

Table 11
The asymmetric degree of singularity spectra of Panel A: Full sample.

		BAT	LINK	MANA	DGB	ENJ	MKR	BTC	ETH	USDT	Gold	WTI	SP500
Overall	α_0	0.525	0.554	0.410	0.528	0.520	0.564	0.574	0.455	0.274	0.441	0.553	0.590
	α_{max}	0.699	1.125	0.480	0.531	0.959	0.764	1.200	0.630	0.915	0.602	0.719	0.827
	α_{min}	0.348	0.145	-0.031	0.359	0.265	0.075	0.244	0.109	-0.119	0.274	0.188	0.573
	A_α	0.008	-0.166	0.726	0.972	-0.266	0.419	-0.309	0.328	-0.240	0.019	0.374	-0.864
Uptrend	Δ_α	0.384	0.802	0.628	0.433	0.780	0.641	0.753	0.453	1.070	0.193	0.736	0.356
	α_0^+	0.435	0.590	0.474	0.591	0.619	0.569	0.528	0.509	0.262	0.516	0.597	0.633
	α_{max}^+	0.616	1.154	0.587	0.770	1.050	0.778	1.092	0.701	0.985	0.570	0.716	0.827
	α_{min}^+	0.232	0.352	-0.040	0.337	0.270	0.137	0.339	0.248	-0.086	0.377	-0.020	0.471
Downtrend	A_α^+	0.056	-0.407	0.638	0.171	-0.104	0.347	-0.498	0.154	-0.351	0.441	0.677	-0.088
	Δ_α^+	0.505	0.920	0.403	0.131	0.645	0.702	0.880	0.490	0.424	0.553	0.534	0.147
	α_0^-	0.604	0.499	0.314	0.485	0.401	0.554	0.618	0.387	0.291	0.343	0.526	0.634
	α_{max}^-	0.844	0.874	0.440	0.485	0.715	0.718	1.120	0.541	0.343	0.580	0.706	0.758
	α_{min}^-	0.339	-0.046	0.037	0.353	0.071	0.016	0.240	0.051	-0.081	0.028	0.172	0.612
	A_α^-	0.049	0.184	0.373	1.000	0.024	0.531	-0.142	0.373	0.753	0.140	0.327	-0.695
	Δ_α^-	0.351	0.980	0.512	0.171	0.694	0.689	0.956	0.521	1.034	0.328	0.531	0.255

Table 12
The asymmetric degree of singularity spectra of Panel B: Before COVID-19.

		BAT	LINK	MANA	DGB	ENJ	MKR	BTC	ETH	USDT	Gold	WTI	SP500
Overall	α_0	0.506	0.587	0.406	0.490	0.600	0.500	0.663	0.567	0.284	0.445	0.508	0.632
	α_{max}	0.885	0.922	0.810	0.545	1.689	0.973	1.554	0.758	0.488	0.780	0.818	0.680
	α_{min}	0.191	0.303	0.112	0.271	0.377	0.162	0.441	0.231	0.041	0.270	0.347	0.420
	A_α	-0.093	-0.081	-0.160	0.600	-0.661	-0.167	-0.600	0.274	0.086	-0.315	-0.315	0.632
Uptrend	Δ_α	0.645	0.708	0.492	0.394	1.757	1.032	1.452	0.799	0.603	0.405	0.507	0.307
	α_0^+	0.478	0.569	0.353	0.537	0.774	0.551	0.741	0.611	0.277	0.519	0.438	0.615
	α_{max}^+	0.804	0.994	0.681	0.657	2.198	1.218	1.905	0.882	0.496	0.643	0.833	0.650
	α_{min}^+	0.159	0.286	0.189	0.263	0.441	0.185	0.453	0.083	-0.106	0.238	0.326	0.342
Downtrend	A_α^+	-0.011	-0.201	-0.337	0.393	-0.621	-0.292	-0.603	0.324	0.274	0.388	-0.558	0.776
	Δ_α^+	0.684	0.446	1.119	0.379	1.183	0.361	0.392	0.148	0.285	0.776	0.513	0.607
	α_0^-	0.533	0.622	0.468	0.485	0.535	0.501	0.645	0.545	0.297	0.352	0.590	0.564
	α_{max}^-	0.938	0.749	0.832	0.547	1.160	0.653	0.852	0.582	0.470	0.899	0.848	1.033
	α_{min}^-	0.255	0.303	-0.286	0.168	-0.023	0.292	0.460	0.434	0.185	0.123	0.335	0.426
	A_α^-	-0.186	0.427	0.348	0.671	-0.058	0.157	-0.058	0.507	-0.214	-0.409	-0.007	-0.545
	Δ_α^-	0.694	0.619	0.698	0.274	1.312	0.811	1.114	0.527	0.446	0.509	0.470	0.260

Table 13
The asymmetric degree of singularity spectra of Panel C: During COVID-19.

		BAT	LINK	MANA	DGB	ENJ	MKR	BTC	ETH	USDT	Gold	WTI	SP500
Overall	α_0	0.487	0.565	0.476	0.614	0.514	0.660	0.522	0.481	0.340	0.521	0.565	0.564
	α_{max}	0.761	1.258	0.514	0.878	0.607	1.030	1.366	0.731	2.011	0.859	1.029	0.789
	α_{min}	0.209	0.156	0.017	0.409	0.324	0.046	0.119	0.072	-0.295	0.133	0.159	0.546
	A_α	0.006	-0.256	0.846	-0.124	0.342	0.248	-0.354	0.243	-0.449	0.070	-0.066	-0.846
Uptrend	Δ_α	0.509	0.737	0.528	0.091	0.360	0.819	0.483	0.435	1.593	0.464	0.895	0.371
	α_0^+	0.464	0.634	0.500	0.702	0.665	0.685	0.522	0.547	0.584	0.674	0.604	0.589
	α_{max}^+	0.701	1.256	0.521	0.790	0.672	1.020	1.003	0.714	2.027	0.865	1.019	0.796
	α_{min}^+	0.192	0.519	-0.007	0.699	0.312	0.201	0.520	0.278	0.434	0.401	0.124	0.424
Downtrend	A_α^+	0.070	-0.689	0.919	-0.930	0.964	0.182	-0.991	0.234	-0.812	0.179	0.074	-0.113
	Δ_α^+	0.690	0.617	0.731	0.906	0.349	0.804	1.065	0.697	1.251	0.525	0.500	0.071
	α_0^-	0.542	0.503	0.424	0.500	0.329	0.617	0.555	0.419	0.152	0.322	0.483	0.614
	α_{max}^-	0.892	0.646	0.803	0.978	0.622	1.051	1.165	0.789	0.337	0.478	0.675	0.665
	α_{min}^-	0.202	0.029	0.072	0.071	0.273	0.247	0.100	0.091	-0.914	-0.047	0.175	0.594
	A_α^-	-0.014	0.535	-0.036	-0.054	-0.682	-0.079	-0.144	-0.060	0.704	0.406	0.235	-0.441
	Δ_α^-	0.553	1.102	0.497	0.470	0.283	0.984	1.247	0.659	2.306	0.726	0.870	0.244

price and volatility movement of NFTs, DeFi, and traditional assets follow nonlinear cross-correlations, but traditional assets, particularly gold and S&P500, show relatively weaker nonlinear cross-correlations than NFTs, DeFi, and cryptocurrencies. Finally, NFTs and DeFi show a higher sensitivity to smaller events in bull markets.

Based on our findings, more research in this area is clearly needed. Future studies may examine the NFTs and DeFi market sentiment and forecasting methodology for NFTs and DeFi prices.

Declaration of Competing Interest

on behalf of all authors, the corresponding author states no conflict

of interest.

Data availability

Data will be made available on request.

Appendix A. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.irfa.2023.102642>.

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