



High Energy Physics – Theory

Compactification of 6d $\mathcal{N} = (1, 0)$ quivers, 4d SCFTs and their holographic dual Massive IIA backgrounds

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Abstract

In this paper we study an infinite family of Massive Type IIA backgrounds that holographically describe the twisted compactification of $\mathcal{N} = (1, 0)$ six-dimensional SCFTs to four dimensions. The analysis of the branes involved motivates an *heuristic* proposal for a four dimensional linear quiver QFT, that deconstructs the theory in six dimensions. For the case in which the system reaches a strongly coupled fixed point, we calculate some observables that we compare with holographic results. Two quantities measuring the number of degrees of freedom for the flow across dimensions are studied.

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Contents

1. Introduction	2
2. Supergravity backgrounds	3
2.1. Page fluxes and charges	5
2.2. The cases of H_2 and S^2 compactifications	9
3. Study of the dual field theory	10
3.1. Free energy and holographic central charge	10
3.2. Flow central charge	13

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3.3. A phenomenological proposal for the QFT 15

4. Conclusions 18

CRediT authorship contribution statement 19

Declaration of competing interest 19

Data availability 19

Acknowledgements 19

Appendix A. 19

 A.1. 7D $SU(2)$ Topologically Massive gauged Supergravity 19

 A.2. From SUSY variations to BPS equations 21

 A.3. Uplift to 10D Massive Type IIA 26

Appendix B. Numerical solution of the BPS system 30

 B.1. Infrared fixed point 30

 B.2. Linear perturbations 30

Appendix C. Analysis of the 4d QFT 31

 C.1. Central charges 32

References 33

1. Introduction

Maldacena’s AdS/CFT conjecture [1] motivates the study of both gravity and field theory topics. In particular, the study of supersymmetric and conformal field theories in diverse dimensions. In the past few years we witnessed the definition of new, characteristically non-Lagrangian CFTs, by the existence of a trustable background of Type II or M-theory, containing an AdS-factor.

In fact, this procedure has been applied to the possible space-time dimensions for which super conformal field theories exist ($d + 1 = 1, \dots, 6$). With eight Poincare supercharges, there exists a classification and an algorithmic way of associating a particular SCFT $_{d+1}$ with a Type II background containing an AdS $_{d+2}$ factor. At present there seems to be exceptions to this statement for the cases of supergravity solutions containing AdS $_3$ and AdS $_2$ spaces. See [2–14], for references working details of the cases $(d + 1) = 1, 2, 3, 4, 5, 6$. A comprehensive summary of various aspects of SCFTs in diverse dimensions can be found in [15].

A reasonable extension is the study of RG-flows away from these SCFTs $_{d+1}$. These flows can be between two conformal points or between a CFT and a gapped theory. Less conventional are the flows across dimensions, between a SCFT $_{D+1}$ and a SCFT $_{d+1}$ (there is also with the possibility of ending in gapped systems). There are numerous case-studies of this in the bibliography, see for example [16], for early examples working with twisted compactifications from the holographic point of view. The topic progressed considerably after the paper [17]. This was followed by many works studying compactifications (twisted or with fluxes) from a purely QFT point of view. In the particular case of compactifications of 6d to 4d systems (preserving minimal SUSY in both dimensions), we find the works [18]-[19]. For a very nice summary of these developments from a field theoretical perspective, see [20].

In this paper, we present an interesting example of flow across dimensions involving a twisted compactification. In particular, we start from an infinite family of six-dimensional $\mathcal{N} = (1, 0)$ SCFTs and compactify it on a two manifold of constant curvature. The end-point of the flow is an infinite family of strongly coupled four dimensional $\mathcal{N} = 1$ SCFTs (and possibly gapped QFTs, that we leave for future studies). The holographic study of the family of 4d SCFTs occupies an important part of this work, calculating observables that characterise it.

In more detail, the contents of the paper are distributed as follows.

In Section 2, we construct a new infinite family of Massive Type IIA backgrounds that represent the flow between a family of six-dimensional $\mathcal{N} = (1, 0)$ SCFTs and four dimensional $\mathcal{N} = 1$ SCFTs. These flows are new backgrounds, not present in the bibliography. The case of gapped four dimensional systems leads to singular backgrounds, hence we leave it to future study. The charges of the brane system are discussed, with emphasis on the effects of the twisted-compactification.

In Section 3, we present calculations of the holographic central charge in these supergravity backgrounds (the free energy of the dual CFT). These are calculations at the AdS₅ fixed point and along the flow. We also present a monotonic quantity interpolating between the conformal points at low and high energies. After this, based on the branes charges discussed in Section 2, we give a *phenomenological proposal* for a suitable quiver capturing the low energy dynamics. These 4d quiver QFTs are proposed to reach a conformal point at low energies, their strongly coupled dynamics being described by the infinite family of Massive Type IIA backgrounds with an AdS₅ factor (discussed in Section 2). We emphasise on the heuristic character of this proposal. Indeed, whilst the beta functions and R-symmetry anomalies of the proposed QFT are cancelled and the scaling of the free energy with the quiver parameters (rank of gauge groups and number of nodes) matches the holographic result, the precise coefficient of the free energy does not exactly match the one computed in the holographic dual. Hence the proposed quiver is only a first step towards the correct field theory dual to our infinite family of geometries. We discuss possible improvements in the conclusions and Appendices.

In Section 4, we summarise, present conclusions and propose some ideas for further research. Three very intensive appendices complement the presentation. The reader wishing to work on these topics should benefit from reading them in detail.

2. Supergravity backgrounds

We start this section by describing an infinite family of supergravity solutions, the analysis of which, is the main subject of the rest of this paper. This is a family of Massive Type IIA backgrounds, preserving four supersymmetries ($\mathcal{N} = 1$ in four dimensional notation). The construction of these backgrounds is described in great detail in Appendix A. From a quantum field theoretical perspective, these backgrounds are dual to twisted compactifications of six dimensional $\mathcal{N} = (1, 0)$ SCFTs at the origin of their tensor branch. We discuss this in more detail in Section 3.

Let us present the family of backgrounds in Massive Type IIA. These are written in terms of coordinates, parameters and functions,

$$\text{Coordinates: } (t, x_1, x_2, x_3, r, \theta_1, \phi_1, z, \theta_2, \phi_2). \quad \text{Parameters: } (\Psi_0, k). \tag{2.1}$$

$$\text{Functions: } \alpha(z), f(r), h(r), X(r) = e^{\frac{2}{5}\Phi(r)}, \omega(r, z) = \left(\frac{\alpha'(z)^2 - 2\alpha(z)\alpha''(z)X(r)^5}{\alpha'(z)^2 - 2\alpha(z)\alpha''(z)} \right).$$

The equations constraining these functions are written below. In terms of these coordinates and functions, the string-frame spacetime metric reads

$$ds_{st}^2 = 2\pi\sqrt{2}\sqrt{-\frac{\alpha(z)}{\alpha''(z)}} X(r)^{-\frac{1}{2}} e^{-\frac{4\Phi(r)}{5}} \left[e^{2f(r)} dx_{3,1}^2 + dr^2 \right]$$

$$\begin{aligned}
 & +e^{2h(r)} \left(d\theta_1^2 + \frac{1}{k} \sin^2(\sqrt{k}\theta_1) d\phi_1^2 \right) \Big] \\
 & + X(r)^{5/2} \left[\pi \sqrt{2} \sqrt{-\frac{\alpha''(z)}{\alpha(z)}} dz^2 \right. \\
 & \left. + \frac{\sqrt{2}\pi}{\omega(r,z)} \frac{\sqrt{-\alpha^3(z)\alpha''(z)}}{2\alpha(z)\alpha''(z) - \alpha'^2} \left(d\theta_2^2 + \sin^2(\theta_2) \left(d\phi_2 - \frac{1}{k} \cos(\sqrt{k}\theta_1) d\phi_1 \right)^2 \right) \right].
 \end{aligned} \tag{2.2}$$

The Neveu-Schwarz (B_2, Ψ) and Ramond (F_0, F_2, F_4) background fields are,

$$\begin{aligned}
 B_2 &= \left(\frac{\pi}{\omega(r,z)} \frac{\alpha(z)\alpha'(z)}{\alpha'(z)^2 - 2\alpha(z)\alpha''(z)} \sin(\theta_2) d\theta_2 - \pi \cos(\theta_2) dz \right) \\
 & \wedge \left(d\phi_2 - \frac{1}{k} \cos(\sqrt{k}\theta_1) d\phi_1 \right), \\
 e^{4\Psi(r,z)} &= \frac{X^5(r)}{\omega^2(r,z)} \left(\frac{-\alpha(z)}{\alpha''(z)} \right)^3 \left(\frac{e^{2\Psi_0}}{\alpha'(z)^2 - 2\alpha(z)\alpha''(z)} \right)^2, \\
 F_0 &= 2^{\frac{1}{4}} \frac{e^{-\Psi_0}}{\sqrt{\pi}} \alpha'''(z), \\
 F_2 &= 2^{\frac{1}{4}} \sqrt{\pi} e^{-\Psi_0} \alpha''(z) \left[\cos(\theta_2) \text{Vol}(\Sigma_k) - \text{Vol}(S_c^2) \right] \\
 & + F_0 \frac{\pi}{\omega(r,z)} \frac{\alpha(z)\alpha'(z)}{\alpha'(z)^2 - 2\alpha(z)\alpha''(z)} \text{Vol}(S_c^2), \\
 F_4 &= \left(\frac{2^{\frac{1}{4}} \pi^{\frac{3}{2}} e^{-\Psi_0}}{\omega(r,z)} \right) \left(\frac{\alpha(z)\alpha'(z)\alpha''(z)}{\alpha'(z)^2 - 2\alpha(z)\alpha''(z)} \right) \cos(\theta_2) \text{Vol}(\Sigma_k) \wedge \text{Vol}(S^2) \\
 & + 2^{\frac{1}{4}} \pi^{\frac{3}{2}} e^{-\Psi_0} \alpha''(z) \sin^2(\theta_2) dz \wedge d\phi_2 \wedge \text{Vol}(\Sigma_k).
 \end{aligned} \tag{2.3}$$

We have defined the volume elements,

$$\begin{aligned}
 \text{Vol}(S^2) &= \sin(\theta_2) d\theta_2 \wedge d\phi_2, \quad \text{Vol}(S_c^2) = \sin(\theta_2) d\theta_2 \wedge \left(d\phi_2 - \frac{1}{k} \cos(\sqrt{k}\theta_1) d\phi_1 \right), \\
 \text{Vol}(\Sigma_k) &= \frac{\sin(\sqrt{k}\theta_1)}{\sqrt{k}} d\theta_1 \wedge d\phi_1.
 \end{aligned} \tag{2.4}$$

The functions $f(r), h(r), \Phi(r)$ must satisfy first order (BPS) ordinary differential equations. Denoting the derivative respect to the coordinate r with a dot, they read,

$$\begin{aligned}
 \dot{f} &= \pm \frac{m}{2} e^{-2\Phi}, \quad \dot{h} = \pm \frac{1}{2} \left(\frac{1}{k} e^{-2h} + m e^{-2\Phi} \right), \\
 \dot{\Phi} &= \pm \left(-1 + \frac{1}{4k} e^{-2h} + m e^{-2\Phi} \right).
 \end{aligned} \tag{2.5}$$

We choose the positive sign from now on. The derivation of eqs. (2.5) and the origin of the parameter m are explained in Appendix A. The remaining BPS equation for $\alpha(z)$ is already written in eq. (2.3). In fact, the mass-parameter of massive Type IIA F_0 , that should be constant

by pieces for an interpretation in terms of localised D8 branes dictates that $\alpha'''(z)$ must be piece-wise constant. It is in the many possible choices for a piece-wise constant F_0 that the family of backgrounds is originated. More on this in Section 2.1 below.

The configurations in eqs. (2.2)-(2.5), are new solutions to the equations of motion of Massive IIA. In string frame these read,

$$\begin{aligned} \frac{1}{4}R + \nabla^2\Psi - (\nabla\Psi)^2 - \frac{1}{8}H_3^2 &= 0, \\ dF_p + H_3 \wedge *F_{p-2} &= 0, \quad d(e^{-2\Psi} * H_3) - (F_0 * F_2 + F_2 \wedge *F_4 + F_4 \wedge F_4) = 0, \\ R_{MN} + 2\nabla_M\nabla_N\Psi - \frac{1}{2}(H_3^2)_{MN} - \frac{1}{4}e^{2\Psi} \sum_p (F_p^2)_{MN} &= 0. \end{aligned} \tag{2.6}$$

In eq. (2.6) $p = 2, 4, 6, 8, 10$, and

$$(F_p^2)_{MN} = \frac{1}{(p-1)!} F_M^{N_1 \dots N_{p-1}} F_{NN_1 \dots N_{p-1}}, \quad (H_3^2)_{MN} = \frac{1}{2} H_M^{N_1 N_2} H_{NN_1 N_2}.$$

Regarding the volume form $\text{Vol}(\Sigma_k)$ in eq. (2.4), for the allowed values for the parameter k , namely $k = (1, -1)$, Σ_k is describing a two-sphere or a hyperbolic plane. The case $k = 0$, corresponding to a torus, is slightly more subtle and will be briefly addressed below.

This concludes the presentation of the backgrounds. To interpret these in terms of branes, we calculate the Page charges (quantised and gauge-variant) associated with this family of SUSY solutions.

2.1. Page fluxes and charges

The Page fluxes \widehat{F}_p , defined as a polyform $\widehat{F} = e^{-B_2} \wedge F$ are quantised. This implies a certain form for some of the functions in the background, as we discuss below.

The Page fluxes are gauge-variant. They do change under a gauge transformation of B_2 . We use this in our favour, performing a particular transformation that makes explicit the quantised charges present and the role of the function $\alpha(z)$. To write the Page fluxes in a concise fashion, it proves useful to define a one form Θ_1 and its exterior derivative,

$$\Theta_1 = -\cos(\theta_2) \left(d\phi_2 - \frac{1}{k} \cos(\sqrt{k}\theta_1) d\phi_1 \right), \quad d\Theta_1 = \text{Vol}(S_c^2) - \cos(\theta_2) \text{Vol}(\Sigma_k). \tag{2.7}$$

It is also convenient to change the B_2 -field by a large gauge transformation (this has no effect on $H_3 = dB_2$). Below, we explain the purpose of such transformation,

$$\begin{aligned} B_{2,new} &= B_{2,old} - \pi d \left[(z - \Delta)\Theta_1 \right], \quad \Delta \text{ is a constant,} \tag{2.8} \\ B_{2,new} &= \left(\frac{\pi}{\omega(r,z)} \frac{\alpha(z)\alpha'(z)}{\alpha'(z)^2 - 2\alpha(z)\alpha''(z)} \text{Vol}(S_c^2) - \pi(z - \Delta)d\Theta_1 \right), \\ H_3 &= \pi \left[-\frac{1}{\omega^2} \frac{\alpha(z)\alpha'(z)}{\alpha'(z)^2 - 2\alpha(z)\alpha''(z)} \frac{\partial\omega}{\partial X} X'(r) dr + \frac{d}{dz} \left(\frac{1}{\omega} \frac{\alpha(z)\alpha'(z)}{\alpha'(z)^2 - 2\alpha(z)\alpha''(z)} \right) dz \right] \\ &\quad \wedge \text{Vol}(S_c^2) \\ &\quad - \pi \left(\frac{1}{\omega} \frac{\alpha(z)\alpha'(z)}{\alpha'(z)^2 - 2\alpha(z)\alpha''(z)} \right) \sin(\theta_2) d\theta_2 \wedge \text{Vol}(\Sigma_k) - \pi dz \wedge \text{Vol}(S_c^2) \\ &\quad + \pi \cos(\theta_2) dz \wedge \text{Vol}(\Sigma_k). \end{aligned}$$

With this new- B_2 we compute the Page flux \widehat{F}_2 and obtain,

$$\widehat{F}_2 = F_2 - B_2 F_0 = -2^{\frac{1}{4}} \sqrt{\pi} e^{-\psi_0} (\alpha'' - (z - \Delta) \alpha''') d\Theta_1. \tag{2.9}$$

Similarly we calculate the \widehat{F}_4 Page flux,

$$\widehat{F}_4 = F_4 - B_2 \wedge F_2 + \frac{1}{2} B_2 \wedge B_2 F_0, \tag{2.10}$$

$$\begin{aligned} \widehat{F}_4 = & 2^{\frac{1}{4}} \pi^{\frac{3}{2}} e^{-\psi_0} \alpha''(z) \sin^2 \theta_2 dz \wedge d\phi_2 \wedge \text{Vol}(\Sigma_k) \\ & + 2^{1/4} \pi^{3/2} e^{-\psi_0} (z - \Delta) (2\alpha''(z) - \alpha'''(z)(z - \Delta)) \cos \theta_2 \text{Vol}(S_c^2) \wedge \text{Vol}(\Sigma_k). \end{aligned}$$

Finally, for F_0 we have the same as in eq. (2.3), $\widehat{F}_0 = 2^{\frac{1}{4}} \frac{e^{-\psi_0}}{\sqrt{\pi}} \alpha'''(z)$. We now calculate the charges associated with these Page fluxes, and impose their quantisation.

2.1.1. Page charges

To calculate the charges, we need to compute the integrals (as in the rest of the paper we set $g_s = \alpha' = 1$),

$$Q_{NS5} = \frac{1}{4\pi^2} \int_{M_3} H_3, \quad Q_{D_p} = \frac{1}{(2\pi)^{7-p}} \int_{M_{8-p}} \widehat{F}_{8-p}, \quad p = 4, 6, 8. \tag{2.11}$$

These integrals need to be defined over suitable cycles, some of which contain the sub-manifold Σ_k . Then, it is useful to first calculate the volume of the two-manifold Σ_k defined in eq. (2.4). As we stated above, in the cases $k = (1, 0, -1)$ the space described is a two-sphere, a torus or a hyperbolic plane. Its volume is calculated using Gauss-Bonnet’s theorem and the fact that Σ_k has curvature $R = 2k$. For a genus g Riemann surface, we have

$$\int_{\Sigma_k} d^2x \sqrt{g} R = 8\pi(1 - g) \quad \longrightarrow \quad \int_{\Sigma_k} d^2x \sqrt{g} = \frac{4\pi}{k}(1 - g).$$

For the sphere, $k = 1$ and $g = 0$, while for the hyperbolic plane, $k = -1$ with $g > 1$. The case of the torus needs some care, as we have $k = 0, g = 1$ (and the volume is 4π). We will not discuss the case of T^2 in what follows (except for the purpose of making an intuitive argument below). This allows us to write (for S^2, H_2)

$$\int_{\Sigma_k} d^2x \sqrt{g} = 4\pi |g - 1|. \tag{2.12}$$

After these preliminaries, we calculate the Page charges. Inspecting the H_3 -field in eq. (2.8) we find two possible three-cycles on which the integral can be performed. These three-cycles are

$$\mathcal{M}_1 = (\theta_1, \phi_1, z)|_{\theta_2=0}, \quad \text{and} \quad \mathcal{M}_2 = (\theta_2, \phi_2, z)|_{r \rightarrow +\infty}. \tag{2.13}$$

We find that there are two sets of NS-five branes. Their total numbers being,

$$\begin{aligned} N_{NS5}^{(1)} &= \frac{1}{4\pi^2} \int_{\mathcal{M}_1} H_3 = \frac{1}{4\pi} \text{Vol}(\Sigma_k) P = |g - 1| P, \\ N_{NS5}^{(2)} &= \frac{1}{4\pi^2} \int_{\mathcal{M}_2} H_3 = P. \end{aligned} \tag{2.14}$$

To calculate $N_{NS5}^{(2)}$ a boundary condition $\alpha(z = 0) = \alpha(z = P) = 0$ has been imposed, more on this below. We have also taken the orientation of the manifolds such that all the charges are positive. Importantly, we have set the range of the z -coordinate to be $z \in [0, P]$, with P an integer.

For the charges of D6-branes, we have a pair of two-cycles on which to integrate the Page flux of eq. (2.9),

$$\mathcal{M}_3 = (\theta_1, \phi_1)|_{\theta_2=0}, \quad \text{and} \quad \mathcal{M}_4 = (\theta_2, \phi_2).$$

Performing the integrals, we find,

$$\begin{aligned} N_{D6}^{(1)} &= \frac{1}{2\pi} \int_{\mathcal{M}_3} \widehat{F}_2 = 2^{\frac{5}{4}} \sqrt{\pi} e^{-\Psi_0} |g - 1| (\alpha'' - (z - \Delta)\alpha'''), \\ N_{D6}^{(2)} &= \frac{1}{2\pi} \int_{\mathcal{M}_4} \widehat{F}_2 = 2^{\frac{5}{4}} \sqrt{\pi} e^{-\Psi_0} (\alpha'' - (z - \Delta)\alpha'''). \end{aligned} \tag{2.15}$$

For the D8 branes we use eq. (2.3) and find,

$$N_{D8} = 2\pi \int dz F'_0 = 2^{\frac{5}{4}} \sqrt{\pi} e^{-\Psi_0} \int dz \alpha^{(4)}. \tag{2.16}$$

Finally, for the D4 branes, the four-cycle is $\mathcal{M}_5 = (\theta_2, \phi_2, \theta_1, \phi_1)$, at constant values of r, z . Calculating

$$Q_{D4} = \frac{1}{8\pi^3} \int_{\mathcal{M}_5} \widehat{F}_4 = 0.$$

In other words, there is no charge of D4 branes in the system.¹

Inspecting the charges in eqs. (2.14), (2.15) and (2.16), suggests to set $e^{\Psi_0} = 2^{\frac{5}{4}} \sqrt{\pi}$. Imposing quantisation, it is clear from eqs. (2.15) that the function $\alpha''(z)$ must be a linear function with integer coefficients N_l . In fact, we can divide the range of the z -coordinate in intervals of unit size. In each interval, $\alpha''(z)$ should be a linear function. The charge of D8 branes in eq. (2.16) suggests that $\alpha''(z)$ should be piecewise linear and continuous (with integer coefficients), the third derivative piecewise constant (integer and generically discontinuous), whilst the fourth-derivative a sum of delta functions with integer coefficients. In other words, if we choose for $\alpha''(z)$,

$$\alpha''(z) = \begin{cases} N_1 z & , 0 \leq z < 1 \\ N_1 + (N_2 - N_1)(z - 1) & , 1 \leq z < 2 \\ \vdots & \\ N_l + (N_{l+1} - N_l)(z - l) & , l \leq z < l + 1 \\ \vdots & \\ N_{P-1}(P - z) & , (P - 1) \leq z < P. \end{cases} \tag{2.17}$$

¹ One might wonder about computing Q_{D4} integrating \widehat{F}_4 over the manifold $\mathcal{M}_6 = [z, \theta_1, \phi_1, \phi_2]_{\theta_2 = \frac{\pi}{2}}$. This is not a well defined four-cycle, as it has a boundary.

This implies,

$$\alpha'''(z) = \begin{cases} N_1 & , 0 \leq z < 1 \\ N_2 - N_1 & , 1 \leq z < 2 \\ \vdots & \\ N_{l+1} - N_l & , l \leq z < l + 1 \\ \vdots & \\ -N_{P-1} & , (P - 1) \leq z < P, \end{cases} \tag{2.18}$$

and

$$\alpha^{(4)}(z) = \sum_{l=1}^{P-1} (2N_l - N_{l+1} - N_{l-1}) \delta(z - l). \tag{2.19}$$

At this point we choose a convenient coefficient Δ in the large gauge transformation of B_2 —see eq. (2.8). In fact, choosing $\Delta = l$ for $z \in [l, l + 1]$ we have in each interval

$$(\alpha'' - (z - l)\alpha''') = N_l.$$

In summary, we have two kinds of NS-five branes, their total number is given in eqs. (2.14). In the interval $z \in [l, l + 1]$ we have two types of D6 brane charges and one kind of D8 charge given by (in each interval),

$$\begin{aligned} N_{D6}^{(1)}[l, l + 1] &= |g - 1|N_l, \quad N_{D6}^{(2)}[l, l + 1] = N_l, \\ N_{D8}[l, l + 1] &= 2N_l - N_{l+1} - N_{l-1}. \end{aligned} \tag{2.20}$$

Though it does not feature in the calculation of Page charges, the function $\alpha(z)$ is obtained after two integrations of eq. (2.17). The integration constants must be chosen such that $\alpha(z)$ is a piecewise continuous cubic function with continuous derivative $\alpha'(z)$. To avoid singular behaviours (not associated with the presence of localised D8 branes), it must satisfy $\alpha(0) = \alpha(P) = 0$.

This is a good point to discuss the physical effect of the large gauge transformation on the B_2 -field—see eq. (2.8). Being a gauge transformation, it does not affect the Physics of our system, but it makes the counting of charges more transparent. Had we not performed it and calculated Page charges with the $B_{2,old}$, we would have obtained a combination between charges of D6 branes induced on the D8 branes and those of ‘actual’ D6 branes. The large gauge transformation separates these, making the counting clearer.

To better understand these systems, it is a good (and intuitive) guide to go back to the case of the torus. In this case, we are compactifying a six dimensional $\mathcal{N} = (1, 0)$ SCFT on T^2 without any flux that breaks SUSY. In other words, we would end with an $\mathcal{N} = 2$ four dimensional SCFT. For more on this perspective see, for example [21]. What follows in the next paragraph is an *intuitive* argument.

The compactification on T^2 (which sets $k = 0$) should be handled with some care. For example, the ‘twisting’ in the one-form Θ_1 is absent, leading to $\Theta_1 = -\cos\theta_2 d\phi_2$. Also, in the BPS eqs. (2.5), the terms $\frac{e^{-2h}}{k}$ are absent. This gives $2\Phi(r) = 0$ with $X(r) = 1$ (for the parameter $m = 1$) and $2f(r) = 2h(r) = r$. This leads to a background metric — see eq. (2.2) — of the form $\text{AdS}_7 \times S^2 \times R_z$. For this case the genus is $g = 1$ and the charges in eqs. (2.14) and (2.20)

indicate the presence of only one type of NS-five branes and D6 branes, together with D8 branes. The reader will recognise that this is the compactification on T^2 of the backgrounds in [13]-[14].

Intuitively, this is what our background in eqs. (2.2)-(2.5) is describing for large values of the r -coordinate. Of course, since we compactify on a curved two manifold (either S^2 or H_2), the twisting needs to be performed even at high energies (large values of the r -coordinate). The system preserves four supercharges all along the flow. The twisted compactification of the six dimensional system of NS-D6-D8 induces new sets of NS and D6 branes. We identify these new branes as those whose charge comes with the $(g - 1)$ -factor in front—see eqs. (2.14) and (2.15).

Let us be more precise about the compactifications on S^2 and H_2 .

2.2. The cases of H_2 and S^2 compactifications

In both cases ($k = 1$ or $k = -1$), we can solve two of the equations in (2.5), finding

$$e^{-2\Phi} = \frac{2k\dot{h} - e^{-2h}}{km}, \quad f = \frac{m}{2} \int e^{-2\Phi} dr, \tag{2.21}$$

whilst $e^{2h(r)} = \mathcal{G}(r)$ must satisfy a nonlinear second order differential equation,

$$\ddot{\mathcal{G}} + \frac{(\dot{\mathcal{G}})^2}{\mathcal{G}} - \frac{5\dot{\mathcal{G}}}{2k\mathcal{G}} - 2\dot{\mathcal{G}} + \frac{3}{2\mathcal{G}} + \frac{2}{k} = 0. \tag{2.22}$$

The numerical resolution of this system is studied in Appendix B. Let us gain some understanding by discussing asymptotic solutions. For large values of the coordinate r we find an asymptotic solution,

$$e^{2h(r)} \sim e^r, \quad e^{2f(r)} \sim e^{2f_0+r}, \quad e^{2\Phi} \sim \frac{km}{k - e^{-r}} \sim m. \tag{2.23}$$

In the case of the compactification on the hyperbolic plane ($k = -1$), we find an exact, fixed-point solution. This is the same solution found in [22], after conventions are matched.

$$e^{2f(r)} = e^{\frac{2}{3}r}, \quad e^{2h(r)} = \frac{3}{4}, \quad e^{2\Phi(r)} = \frac{3m}{4}. \tag{2.24}$$

As is usual, the r -coordinate plays the role of energy-coordinate. The solution in eq. (2.23) asymptotes to $\widehat{\text{AdS}}_7 \times S^2_c \times R_z$ when replaced in eqs. (2.2)-(2.3), describing a six dimensional SCFT formulated on $R^{1,3} \times H_2$. The $\widehat{\text{AdS}}_7$ is written as a foliation over this six-space. On the other end, when the fixed point solution of eq. (2.24) is replaced in the family of backgrounds of Massive Type IIA of eqs. (2.2)-(2.3), the space time takes the form $\text{AdS}_5 \times \mathcal{M}_5$. These describe the dual to a family of four dimensional $\mathcal{N} = 1$ SCFTs. The numerical solution connecting the large- r asymptotics in eq. (2.23) with the fixed-point exact solution (2.24) is described in Appendix B. We present the plot for the functions $f(r)$, $h(r)$, $g(r)$ in Fig. 1.

The numerical study of the case of the two-sphere ($k = 1$) leads to badly singular behaviour as we decrease the r -coordinate. We believe that a background more elaborated than that in eqs. (2.2)-(2.3) could resolve this singular behaviour, but leave this for a future investigation. In what follows we focus our attention on the backgrounds describing the twisted compactification on H_2 (preserving 4 SUSYs) of a six dimensional $\mathcal{N} = (1, 0)$ 6d SCFT.

We close this section emphasising that we have presented a new infinite family of backgrounds in eqs. (2.2)-(2.5). Each solution in this family is labelled by each possible $\alpha(z)$. At the IR fix point, this is a Massive IIA background with an AdS_5 factor preserving four supercharges.

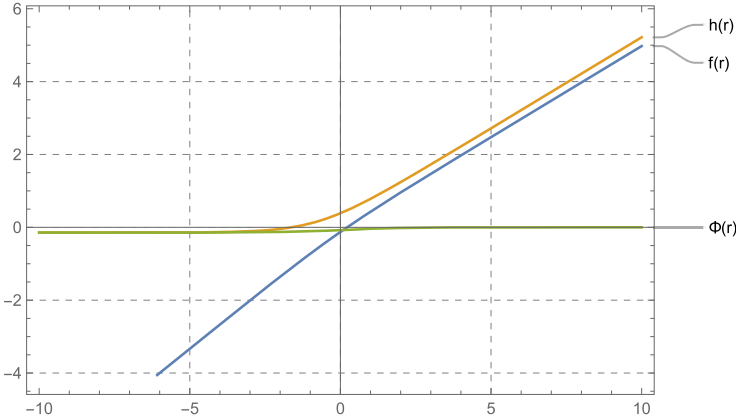


Fig. 1. The plot for the case of the H_2 compactification ($k = -1$) of the functions $f(r)$, $h(r)$, $\Phi(r)$. The UV asymptotics at $r \rightarrow +\infty$ and the IR at $r \rightarrow -\infty$. We clearly see the attainment of both IR and UV fixed points.

3. Study of the dual field theory

We start this section with the holographic calculation of the free energy of the field theory. In particular we start with the free energy of the 4d SCFT that results in the low energy limit of our compactification. After that, we study the same quantity along the flow described by our full solution. We then define a monotonic quantity that captures the flow between the low energy four dimensional SCFT and the high energy six dimensional conformal point (as suggested by the family of supergravity backgrounds).

After these holographic calculations we use the charges calculated in Section 2 to give an *heuristic* proposal for a 4d quiver that at low energies becomes conformal and captures some aspects of the fixed point AdS₅ backgrounds. This proposal is provisional and certainly incomplete. In fact, whilst various observables and scaling behaviours do match between supergravity and QFT calculations, the precise coefficient of the free energy does not, suggesting that some extra fields should supplement our QFT proposal.

3.1. Free energy and holographic central charge

One meaningful observable of SCFTs is the free energy. For the case of SCFTs in diverse dimensions this was calculated holographically. See for example the papers [4], [6], [7], [9], [11], [12], [13]. These calculations were checked against field theoretical computations (typically a global anomaly coefficient or a localisation calculation in matrix models). These checks of the AdS/CFT correspondence are valid in the regime of parameters for which the supergravity background is a reliable representation of the QFT dynamics. This is typically for long linear quivers $P \rightarrow \infty$ and for large rank of the gauge nodes $N_i \rightarrow \infty$.

The calculations done with the supergravity backgrounds are either computing a regularised on-shell action for a putative reduced supergravity or alternatively the calculation of the Newton constant associated with a reduced theory of gravity in lower dimensions.

In particular, it was shown in [25], [26] that for any generic holographic background dual to a QFT in $d + 1$ spacetime dimensions, with metric and dilaton given by,

$$ds^2 = a(r, y^i) \left(dx_{d,1}^2 + b(r) dr \right) + g_{ij} dy^i dy^j, \quad \Psi(r, y^i), \tag{3.1}$$

we can define quantities V_{int}, \widehat{H} according to

$$V_{int} = \int dy^i \sqrt{e^{-4\Psi} a(r, y^i)^d \det[g_{int}]}, \quad \widehat{H} = V_{int}^2. \tag{3.2}$$

From these we define the holographic central charge (or free energy),

$$c_{hol} = d^d \frac{b(r)^{\frac{d}{2}} H^{\frac{2d+1}{2}}}{G_N^{(10)} (H')^d}, \tag{3.3}$$

where $G_N^{(10)} = 8\pi^6$ is in our conventions, the ten-dimensional Newton constant.

For the case of our backgrounds in eq. (2.2), we compare with eq. (3.1) and read,

$$a(r, z) = 2\beta\pi\sqrt{2}\sqrt{-\frac{\alpha(z)}{\alpha''(z)}} X(r)^{-\frac{1}{2}} e^{-\frac{4\Phi(r)}{5} + 2f(r)}, \quad b(r) = e^{-2f(r)}, \quad d = 3. \tag{3.4}$$

The metric of the internal space has determinant,

$$\det(g_{int}) = \frac{16\sqrt{2}\pi^5}{\omega^2} e^{4h(r) + \Phi(r)} \frac{\sqrt{-\alpha(z)^7 \alpha''(z)}}{(\alpha'(z)^2 - 2\alpha(z)\alpha''(z))^2} \frac{\sin^2(\sqrt{k}\theta_1)}{k} \sin^2(\theta_2). \tag{3.5}$$

This leads, after using eqs. (3.2)-(3.3), to

$$H = \widehat{N}^2 e^{6f(r) + 4h(r) - 4\Phi(r)}, \quad \widehat{N} = 16\sqrt{2}\pi^4 e^{-2\Psi_0} \int [-\alpha(z)\alpha''(z)] dz \text{Vol}(\Sigma_k) \text{Vol}(S^2). \\ c_{hol} = \frac{27\widehat{N}}{8G_N^{(10)}} \frac{e^{2h(r) - 2\Phi(r)}}{(3\dot{f} + 2\dot{h} - 2\dot{\Phi})^3}. \tag{3.6}$$

After using the BPS equations in (2.5), the holographic central charge reads

$$c = \frac{27\widehat{N}}{G_N^{(10)}} \frac{e^{2h(r) - 2\Phi(r)}}{\left(4 + \frac{1}{k}e^{-2h(r)} + m e^{-2\Phi(r)}\right)^3}. \tag{3.7}$$

Since this computes the central charge of the four dimensional SCFTs — we have set $d = 3$ in eq. (3.4) — we evaluate this quantity for $k = -1$, the hyperbolic plane twisted compactification of the 6d SCFTs (setting $m = 1$), and find

$$c_{hol} = \frac{27\widehat{N}}{64 G_N^{(10)}}. \tag{3.8}$$

Let us analyse the results in eqs. (3.7)-(3.8).

First, notice that the factor \widehat{N} in eq. (3.6), has a part proportional to $\left(-\text{Vol}(S^2) \int dz \alpha'' \alpha\right)$. We recognise this as information coming from the UV, six dimensional SCFT. Indeed, this factor appears when calculating the free energy of a six dimensional (1, 0) SCFT, see equation (2.14) in the paper [14] or equation (4.10) in the first paper referred in [13]. This is coming from the UV-part of the flow. Note that \widehat{N} in eq. (3.6), also contains a factor of $\text{Vol}\Sigma_k = 4\pi|(g - 1)|$. This suggests that the number of fields gets multiplied by $(g - 1)$. Both these factors inform the phenomenological proposal for the QFT in Section 3.3.

Let us evaluate the holographic central charge in eq. (3.8) for two examples. Whilst these examples are not generic, they capture many aspects of the dynamics of the QFTs. We also discuss these examples (purely from a QFT perspective) in Appendix C.

3.1.1. Example 1

We study the case corresponding to a function $\alpha(z)$ given by,

$$\alpha(z) = \begin{cases} \frac{N}{6}(1 - P^2)z + \frac{N}{6}z^3 & , 0 \leq z \leq (P - 1) \\ -\frac{1}{6}(2P^2 - 3P + 1)(P - z) + \frac{N}{6}(P - 1)(P - z)^3 & , (P - 1) \leq z \leq P. \end{cases} \quad (3.9)$$

This function is associated with a 6d SCFT consisting of a linear quiver with gauge groups of rank $N_j = jN$, ending with a flavour group $SU(PN)$. The function $\alpha(z)$ vanishes at $z = 0$ and $z = P$, is continuous at $z = (P - 1)$ and the derivative $\alpha'(z)$, is continuous at the same point. The second derivative is

$$\alpha''(z) = \begin{cases} Nz & , 0 \leq z \leq (P - 1) \\ N(P - 1)(P - z) & , (P - 1) \leq z \leq P. \end{cases} \quad (3.10)$$

We now use the expressions in eqs. (3.6)-(3.8), the fact that $e^{2\psi_0} = 4\sqrt{2}\pi$, $G_N = 8\pi^6$ and find,

$$\widehat{N} = \frac{64}{45}\pi^5 N^2 P^5 (g - 1), \quad c_{hol,ex1} = \frac{3}{40\pi} N^2 P^5 (g - 1) \left(1 + O\left(\frac{1}{P^2}\right) \right). \quad (3.11)$$

We find a scaling with the length of the quiver and the number of nodes, that is reminiscent of what occurs for this kind of quiver for the case of six dimensional SCFTs. Let us see a second example.

3.1.2. Example 2

Consider the function $\alpha(z)$,

$$\alpha(z) = \begin{cases} \frac{N}{2}(1 - P)z + \frac{N}{6}z^3 & , 0 \leq z \leq 1 \\ \frac{N}{6} - \frac{PN}{2}z + \frac{N}{2}z^2 & , 1 \leq z \leq (P - 1) \\ -\frac{N}{2}(P - 1)(P - z) + \frac{N}{6}(P - z)^3 & , (P - 1) \leq z \leq P. \end{cases} \quad (3.12)$$

From the perspective of the 6d SCFT, this function describes a linear quiver with $(P - 1)$ gauge nodes of rank $N_j = N$ and flavour nodes $F_J = N(\delta_{J,1} + \delta_{J,P-1})$. The one in eq. (3.12) is a function that vanishes at $z = 0$ and $z = P$, is continuous at $z = 1$ and at $z = (P - 1)$ and the derivative $\alpha'(z)$ is continuous at the same two points. The second derivative is

$$\alpha''(z) = \begin{cases} Nz & , 0 \leq z \leq 1 \\ N & , 1 \leq z \leq (P - 1) \\ N(P - z) & , (P - 1) \leq z \leq P. \end{cases} \quad (3.13)$$

Using eqs. (3.6)-(3.8), we calculate,

$$c_{hol,ex2} = \frac{9}{32\pi} N^2 P^3 (g - 1) \left(1 + O\left(\frac{1}{P^2}\right) \right). \quad (3.14)$$

This 4d SCFT has a scaling characteristic of a six dimensional linear quiver of the same type described above.

This result, together with the one in the first example and their respective comparison with the field theoretical calculation in eqs. (C.9)-(C.11), inform our phenomenological proposal of Section 3.3. Let us now discuss the r -dependence of the holographic central charge in eq. (3.6).

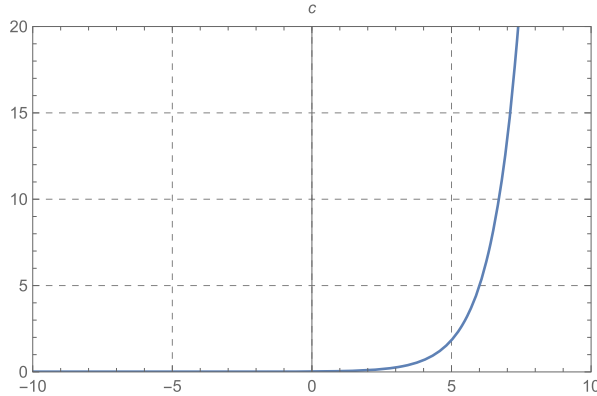


Fig. 2. Central Charge calculated for the numerical solutions of Fig. 1. Here we see that the central charge is divergent in the UV.

3.1.3. Energy dependence of the holographic central charge

Let us now interpret the r -dependence of our free energy/holographic central charge in eq. (3.7). Inspecting this quantity, we find at the IR fixed point the result in eq. (3.8). We associate with this the free energy of a family of four dimensional SCFTs preserving four Poincare supercharges. The family of SCFTs in 4d is labelled by the different functions $\alpha(z)$ that we choose as input, as in the examples above. On the other end, at high energies, we find using eq. (2.23), that the quantity in eq. (3.7) diverges. The plot of Fig. 2 with the numerical solution found in Appendix B shows this.

The divergence of this monotonically increasing (towards the UV) free energy is understandable. Massive fields that originated in the wrapping of both D6's and NS-five branes on the surface $\Sigma_k = H_2$ generically have a mass inversely proportional to the size of Σ_k and are frozen at low energies (of course, there are also massless fields present in the 4d SCFT). When flowing towards the UV these massive fields become active. The number of these Kaluza-Klein modes grows with energy and leads to the divergence of c_{hol} . Somewhat, the quantity defined in eqs. (3.4)-(3.8) for a four dimensional QFT (we have set $d = 3$) is not able to recognise that the system is approximately approaching a six dimensional SCFT.

Similar arguments were expressed by the authors of [27]. In that paper the authors define a quantity $c_{monotonic}$ using the Entanglement Entropy. The quantity $c_{monotonic}$ in [27] and our eq. (3.7), are related by a proportionality factor when studied in our family of backgrounds.

This calls for a quantity that is actually able to detect both fixed points (this was also one of the objectives in [27]). A quantity called c_{flow} in [26], [28] can play this role. This quantity is monotonic, but decreases towards the UV fixed point. We study this below.

3.2. Flow central charge

Let us now define a quantity that is capable of detecting both the UV and IR fixed points of our flow across dimensions. With this in mind, we write a generic holographic metric dual to a QFT defined on anisotropic spacetime—see [26], [28] for details,

$$ds^2 = -a_0 dt_0^2 + \sum_{i=1}^{d-p} a_i dx_i^2 + a_{d-p+1} ds_p^2 + \prod_{i=1}^{d-p} (a_i a_{d-p+1}^p)^{\frac{1}{d}} \tilde{b} dr^2 + h_{ab} dy^a dy^b, \quad (3.15)$$

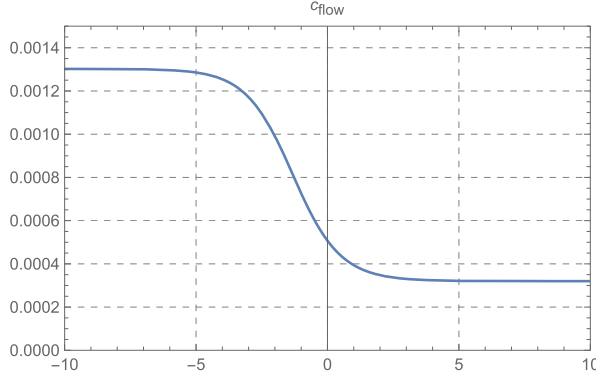


Fig. 3. Flow central charge calculated for the numerical solutions. We clearly see the attainment of both IR and UV fixed points.

where t and x_i are coordinates of the macroscopic space, p is the dimension of the compactification manifold and ds_p^2 its line element. The metric and the coordinates of the internal manifold are denoted by h_{ab} and y^a respectively. The dilaton is $\Psi(r, y^a)$. We define the quantities,

$$G_{ij}dX^i dX^j = \sum_{i=1}^{d-p} a_i dx_i^2 + a_{d-p+1} ds_p^2 + h_{ab} dy^a dy^b, \tag{3.16}$$

$$\tilde{H} = \left(\int \prod_{a=1}^{8-d} dX^a \sqrt{e^{-4\Psi} \det(G_{ij})} \right)^2, \quad c_{flow} = d^d \frac{\tilde{b}^{\frac{d}{2}} \hat{H}^{\frac{2d+1}{2}}}{G_N^{(10)} (\hat{H}')^d}.$$

Specialising these quantities in our metric of eq. (2.2), we have

$$d = 5, \quad p = 2, \quad a_0 = a_1 = a_2 = a_3 = 2\pi\sqrt{2} \sqrt{-\frac{\alpha(z)}{\alpha''(z)}} X(r)^{-\frac{1}{2}} e^{-\frac{4\Phi(r)}{5} + 2f(r)},$$

$$a_4 = a_0 e^{2h(r) - 2f(r)}, \quad \tilde{b} = e^{-\frac{6}{5}f(r) - \frac{4}{5}h(r)},$$

$$\tilde{H} = \hat{\mathcal{N}}^2 e^{6f(r) + 4h(r) - 4\Phi(r)}, \quad \hat{\mathcal{N}} = 16\sqrt{2}\pi^4 e^{-2\Psi_0} \int (-\alpha(z)\alpha''(z)) dz \text{Vol}(\Sigma_k) \text{Vol}(S^2).$$

Using eq. (3.16) we compute,

$$c_{flow} = \frac{27\hat{\mathcal{N}}}{32G_N^{(10)}} \frac{e^{-2\Phi(r)}}{(3\dot{f} + 2\dot{h} - 2\dot{\Phi})^5}, \quad \text{use BPS in eq. (2.5) to find,}$$

$$c_{flow} = \frac{27\hat{\mathcal{N}}}{G_N^{(10)}} \frac{e^{-2\Phi(r)}}{\left(4 + \frac{1}{k}e^{-2h(r)} + m e^{-2\Phi(r)}\right)^5}. \tag{3.17}$$

Evaluating in the IR fixed point of eq. (2.24) and in the asymptotic UV-expansion of eq. (2.23), leads to

$$c_{flow,IR} = \frac{9\hat{\mathcal{N}}}{256 G_N^{(10)}}, \quad c_{flow,UV} = \frac{27\hat{\mathcal{N}}}{5^5 G_N^{(10)}}. \tag{3.18}$$

We can evaluate this quantity numerically on the flow solutions discussed in Appendix B. The plot in Fig. 3 shows this. This plot is very similar to the analogous quantity calculated from the

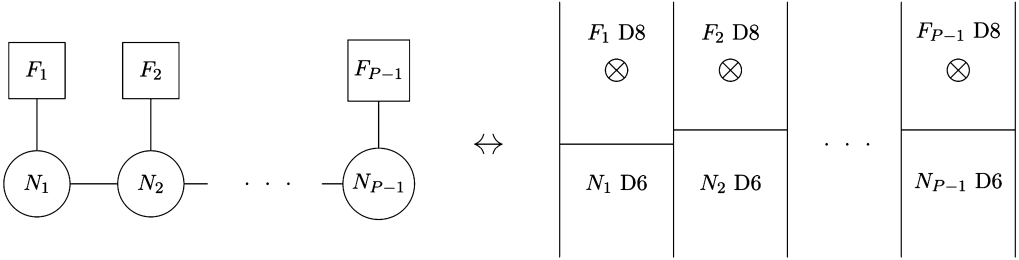


Fig. 4. We show a *rough* approximation to the UV field theory. This would be just the torus reduction of a family of $\mathcal{N} = (1, 0)$ six dimensional SCFTs. As we explain below, this picture is deformed by other brane stacks appearing.

Entanglement Entropy in reference [27]. The discussion about the factors in $\widehat{\mathcal{N}}$ is similar to that in the previous subsection. The qualitative lessons for the QFT along the flow are similar. In summary, we have defined a quantity for an anisotropic QFT realising a flow across dimensions. It should be nice to tackle this same problem from a purely field theoretical perspective.

3.3. A phenomenological proposal for the QFT

We wish to put forwards a proposal for the brane system and the field theory dual to the background in eqs. (2.2)-(2.3). Our proposal is motivated by the scaling of the holographic central charge, calculated in eqs. (3.8), (3.11), (3.14). We find that our proposal is at best provisional. In fact, whilst the scaling with free energy of the quiver parameters matches with the holographic computations, the precise coefficients do not match. We suggest possible modifications of our proposal to be studied in the future.

We will be particularly interested on the fixed point solution of eq. (2.24), that gives a family of backgrounds with an AdS_5 factor preserving four Poincare supercharges.

As we advanced in the previous section, for large values of the r -coordinate, the system is dual to a QFT that is to a good approximation six-dimensional. In fact, using the solution of eq. (2.23) in the background of eqs. (2.2)-(2.3), we find that the space asymptotes to $AdS_7 \times S^2 \times R_z$. The six dimensional SCFT is defined on a spacetime of the form $R^{1,3} \times H_2$. This forces the twisting of the SCFT (to partially preserve SUSY). The different fibrations in the Massive IIA background (encoded by the one form Θ_1) are the effects of the twist in the holographic perspective. Following the formalism developed in the references [13]-[14], we can *roughly* think that the field theory is encoded in a long linear quiver and Hanany-Witten set up of D6-D8-NS5 branes depicted in Fig. 4.

The numbers N_1, N_2, \dots, N_P and F_1, \dots, F_P must satisfy the relation $F_i = 2N_i - N_{i-1} - N_{i+1}$ in order for the six dimensional field theory to be free of gauge anomalies. This condition is ensured by the function $\alpha''(z)$ chosen in eq. (2.17). Indeed, note that from the fourth-derivative $\alpha^{(4)}(z)$ we derive the relation between flavours and colours required, see eq. (2.19). The function $\alpha(z)$, continuous and cubic by pieces is chosen such that the z -coordinate begins and ends smoothly. This is imposed by the conditions $\alpha(0) = \alpha(P) = 0$. There is one UV conformal point for each choice of $\alpha(z)$. Of course, this is just a *very rough* picture.

Indeed, these UV conformal points are deformed (either by VEVs or by relevant operators). The dimension of these operators can be read from the near- AdS_7 expansion of the metric. These deformations (analogously, the presence of the fibrations) topologically twist the 6d CFT and trigger a RG flow, that ends in a CFT_4 . As we discussed around eqs. (2.14), (2.20), a new set

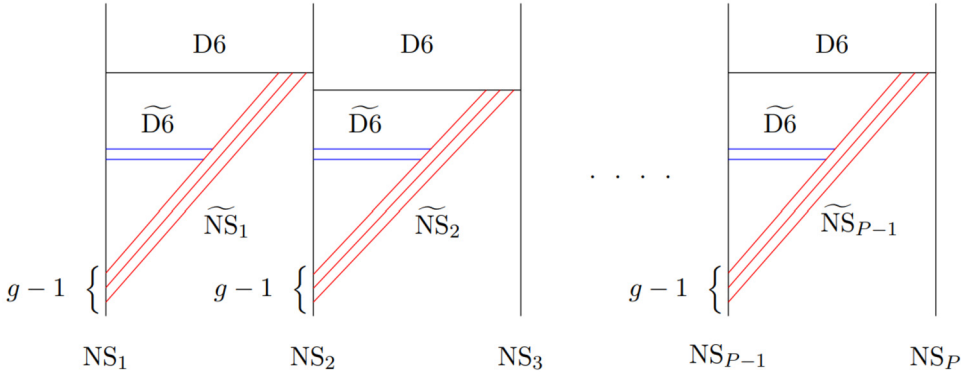


Fig. 5. We draw the new set of branes—denoted with a tilde—which are induced by the twisting. These new branes give place to new gauge groups and new matter fields represented by strings extending between different stacks of branes.

of NS and D6-branes appears due to the twisted compactification. These new branes lead to the presence of new gauge groups, not present in the original linear quiver without twisting.

Following this line of arguments, our system is represented by the Hanany-Witten set up of Fig. 5.

As expressed in eq. (2.14), a new set of NS-five branes appears due to the twisting. These new $(g - 1) \times P$ NS₅ branes share the $R^{1,3}$ directions with the other branes and extend along the coordinates (θ_2, ϕ_2) . They are orthogonal to the original NS branes that extend along $(R^{1,3}, \theta_1, \phi_1)$. This leads to the generation of new gauge groups. In fact, the D6 branes originally extended along $(R^{1,3}, \theta_1, \phi_1, z)$ can now extend between the two stacks of NS-five branes. This generates a new stack of D6 branes extended along $(R^{1,3}, \theta_2, \phi_2, z)$. We associate with these the new gauge groups. As calculated in eq. (2.20), there are $(g - 1) \times N_l$ of them, for each interval. There are also D8 sources, playing the role of flavour groups. The system must cancel all gauge anomalies. We emphasise the $(g - 1) \times P \times P$ -replication of the original quiver. This reflects in the free energy calculated in Section 3.1. It should be interesting to carefully quantise the open strings between these different stacks of branes, to have a more concrete handle on the low energy QFT.

A natural question that arises is the following: can we find a Lagrangian description for the 4d QFT? If so, we could propose a (low energy) duality between a 4d Lagrangian QFT and a 6d SCFT on a geometry of the form $R^{1,3} \times \Sigma_\kappa$. This is perhaps too much to ask. We should be content with a 4d quiver (even when strongly coupled) that captures aspects of the compactified 6d theory. In fact, the above analysis leads us to propose a quiver, that consist on $P^2(g - 1)$ copies of the six-dimensional ‘mother’ theory. Of course, the system is now four dimensional and preserves $\mathcal{N} = 1$ SUSY. In the usual notation (lines with arrows indicating chiral multiplets, circles denoting vector multiplets, boxes indicating flavour symmetries), we propose the quiver in Fig. 6.

In Appendix C, we assign values to the anomalous dimensions and R-charges of each of these four dimensional fields. Using those values we calculate beta functions and R-symmetry anomalies for each gauge group (finding vanishing values). Also a suitable superpotential is proposed, this indicates an interaction between different rows of the quiver in Fig. 6. With the R-charges assignation above, we also calculate a_{CFT} and c_{CFT} , the a, c -central charges of the quiver in Fig. 6. We compare some of these results with the analogous holographic ones in Section 3.1. Whilst the scaling with the quiver parameters (number and rank of gauge nodes) is matched, the precise numerical coefficient does not coincide.

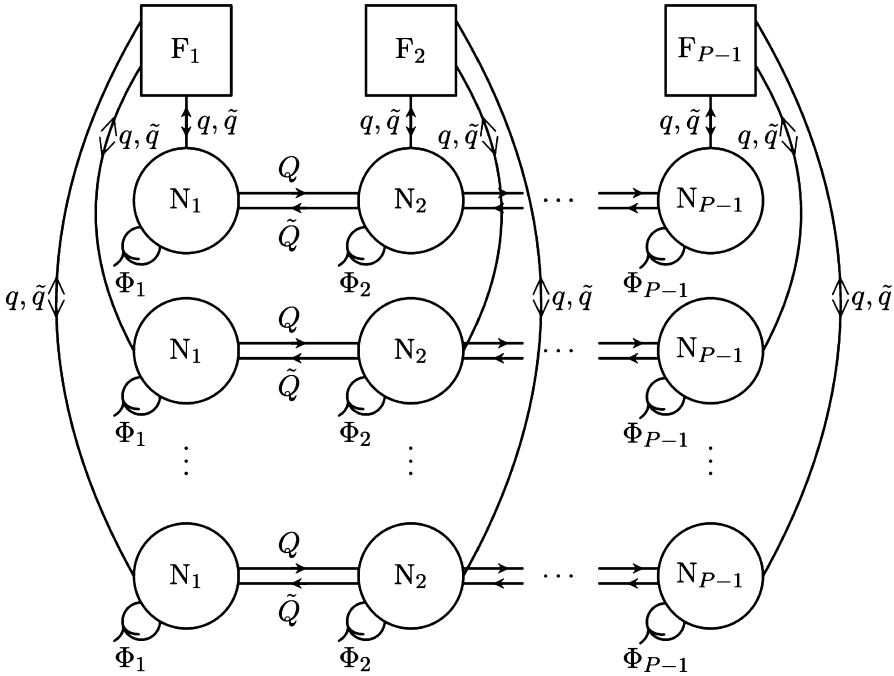


Fig. 6. The proposed quiver field theory. The adjoint fields Φ_i get a mass. The bifundamentals fields are denoted by (Q, \tilde{Q}) , the fundamentals by (q, \tilde{q}) and the vector multiplets represented by the circles. All the fields are in four dimensional $\mathcal{N} = 1$ notation.

Let us close this section with three important comments.

- It may be the case that other fields that do not contribute to the beta functions and the R-symmetry anomaly, but still contribute to the a, c central charges, are present in our quiver of Fig. 6. To decide if they should be there, we should compute the integral of the anomaly polynomial I_8 on the compactification manifold Σ_{-} . We get a hint at those fields, by comparing the material of Section 3.1 with that in Appendix C.
- The scaling with the number of nodes P of the a-central charge (the free energy) is two powers higher than usual (see Appendix C for a field theoretical derivation of this result, and Section 3.1 for a holographic viewpoint on this). In fact, it actually scales as a six dimensional CFT. The quiver we write is *deconstructing* two dimensions, in the sense of [23]. What is interesting about this case is that this is occurring due to the twisting, whilst in [23] it is an effect of going to a particular point in the moduli space, expanding, etc.
- Finally, as explained in Appendix C, a mass term for Φ_i in eq. (C.3) appears. This generates a new set of $(P - 1) - U(1)$'s. These are all anomalous, except for one. The R-symmetry we have taken is the one after this mixing has taken place. Importantly, this makes the beautiful result of Bobev and Cricigno [24], not applicable here. In fact, the authors of [24] assume the absence of mixings. In this sense, the coefficient in their eq. (3.6) for the a central charge, is not enforced on us.²

² Special thanks to Nikolay Bobev for a discussion on this.

Let us close this work by presenting some conclusions.

4. Conclusions

Let us start with a brief overview of the contents of this paper.

In Section 2, we construct an infinite family of Massive Type IIA backgrounds. These describe holographically the flow between a family of six-dimensional $\mathcal{N} = (1, 0)$ SCFTs and four dimensional $\mathcal{N} = 1$ SCFTs. The charges of the brane system are discussed. New sets of branes appear induced by the twisted-compactification and are carefully discussed.

In Section 3, we present and calculate two quantities that measure the number of degrees of freedom along the flow. One of these quantities coincides with the holographic central charge for the 4d SCFTs, and diverges in the UV. The second quantity detects both fixed points, it is monotonous, its value being bigger in the IR than in the UV. We also present a phenomenological proposal for a family of dual four dimensional SCFTs of the linear quiver type. The form and the parameters characterising the quiver are inherited from the six dimensional description. The 4d QFTs are proposed to reach a conformal point at low energies compared to the inverse size of the compactification manifold. These 4d SCFTs are holographically described by an infinite family of Massive Type IIA backgrounds with an AdS_5 factor. We use the quiver field theory description to calculate field theory observables—see Appendix C for details. Interestingly our field theoretical calculation of the a and c central charges in comparison with the holographic one, suggests that other fields should intervene in the dynamics, aside from those in the proposed quiver.

Detailed appendices complement the presentation. We trust that the reader wishing to work on these topics should enjoy and profit from them.

This paper suggests various interesting topics to work on, as follows up of the material presented here. We make a small list below.

- It should be interesting to further study the two-sphere compactifications. These do not lead to conformal 4d fixed points. It is important to de-singularise those solutions.
- It is important to study the string quantisation in the Hanany-Witten set up that arises as a result of the twisted compactification. The massless modes would give firm clues about the four dimensional quiver field theory, for which we proposed a particular type of quiver.
- The presence of extra fields contributing to the central charges (as emphasised in Appendix C) is one of the most urgent topics suggested by this work. May be a treatment along the lines of [19] proves effective. In fact, involving the anomaly polynomials I_8 and I_6 for the SCFT₆ and SCFT₄ may show the need to add flip fields.
- More generally, bringing together the field theoretical techniques of [18]-[19] with the holographic results expressed in this paper or in [28], seems like a problem to study.
- It should be interesting to attempt this kind of compactifications with systems in different dimensions. The systematic study of examples might indicate dimension-dependent characteristics.
- It should be interesting to exploit the recently discovered plethora of embeddings of 5d supergravity into Massive IIA [29]. Finding interesting compactifications of our family of backgrounds, black holes, and other solutions may illuminate various aspects of the $\mathcal{N} = 1$, four dimensional family of SCFTs.

We hope to return to these problems soon.

CRediT authorship contribution statement

All authors of this manuscript are responsible for hundred per cent of all the contents and process involved in this paper.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix A

In this appendix we outline in detail the construction of the infinite family of solutions of Massive Type IIA used in this paper. We begin by considering 7D $SU(2)$ Topologically Massive gauged Supergravity. We propose an ansatz and derive the BPS equations from the SUSY variations. Finally, we outline the uplift of the 7D background to 10D Massive Type IIA, following the procedure used in [30].

Starting in Appendix A.1.1 we change slightly the notation respect to the main body of the paper. We denote with a prime ($'$) the derivative with respect to the r -coordinate.

A.1. 7D $SU(2)$ Topologically Massive gauged Supergravity

The Lagrangian, in string frame, of the bosonic fields of the 7D $SU(2)$ Topologically Massive Supergravity is given by,

$$\mathcal{L} = \sqrt{-g} e^{-2\Phi} \left[R - \frac{1}{8} F_{\mu\nu}^i F^{\mu\nu i} + 4\partial^\mu \Phi \partial_\mu \Phi - \left(\frac{m^2}{2} e^{-4\Phi} - 4m e^{-2\Phi} - 4 \right) \right] - \frac{1}{2} e^{2\Phi} * G_4 \wedge G_4 + \frac{1}{4} F^i \wedge F^i \wedge B_3 - \frac{m}{2} G_4 \wedge B_3, \quad (\text{A.1})$$

where $G_4 = dB_3$, $F = dA + iA \wedge A$, and A is a $su(2)$ valued gauge 1-form, i.e. $A = A^i T^i$. The SUSY variations of the fermionic fields of the theory are

$$\delta\lambda = \left[\Gamma^\mu \partial_\mu \Phi - \frac{i}{4} \Gamma^{\mu\nu} F_{\mu\nu} + \frac{1}{48} e^{2\Phi} \Gamma^{\mu\nu\lambda\rho} G_{\mu\nu\lambda\rho} - m e^{-2\Phi} + 1 \right] \epsilon, \tag{A.2}$$

$$\delta\Psi_\mu = \left[D_\mu + i A_\mu - \frac{i}{2} \Gamma^\nu F_{\mu\nu} + \frac{1}{96} e^{2\Phi} \Gamma_\mu^{\nu\lambda\rho\sigma} G_{\nu\lambda\rho\sigma} - \frac{m}{4} e^{-2\Phi} \Gamma_\mu \right] \epsilon, \tag{A.3}$$

where the covariant derivative is given by

$$D_\mu \epsilon = \left(\partial_\mu + \frac{1}{4} \omega_\mu^{ab} \Gamma_{ab} \right) \epsilon. \tag{A.4}$$

Here μ, ν are spacetime indices while a, b are tangent space ones.

We want to derive the BPS equations using a method similar to the one showed in [31], where the author studied the same Supergravity theory but for vanishing topological mass ($m = 0$). For this, we need to redefine our 1-form gauge field as follows: first we express the $su(2)$ generators, T^i , in terms of the Pauli matrices, σ^i , as $T^i = \frac{\sigma^i}{2}$. Then we derive the expression for the components of the field strength along the algebra generators. Using $A \wedge A = \frac{1}{2}[A, A]$, we have

$$F = dA + \frac{i}{2}[A, A] = dA^i \frac{\sigma^i}{2} + \frac{i}{8} [\sigma^j, \sigma^k]^i A^j \wedge A^k. \tag{A.5}$$

Using the algebra of the Pauli matrices, $[\sigma^i, \sigma^j] = 2i \varepsilon^{ijk} \sigma^k$, and expanding $F = F^i \frac{\sigma^i}{2}$, we get

$$F^i = dA^i - \frac{1}{2} \varepsilon^{ijk} A^j \wedge A^k. \tag{A.6}$$

In order to get both the same field strength and SUSY variations as in [31] in the limit $m = 0$, we redefine $A^i \rightarrow -A^i$, which leads to

$$F^i = - \left(dA^i + \frac{1}{2} \varepsilon^{ijk} A^j \wedge A^k \right) = -F'^i. \tag{A.7}$$

Note that this change doesn't affect the Lagrangian since it is quadratic in F , while the SUSY variations now change, since they are linear in A and F . Using these redefinitions and expanding in the algebra generators, one finds

$$\delta\lambda = \left[\Gamma^\mu \partial_\mu \Phi + \frac{i}{8} \Gamma^{\mu\nu} F_{\mu\nu}^i \sigma^i + \frac{1}{48} e^{2\Phi} \Gamma^{\mu\nu\lambda\rho} G_{\mu\nu\lambda\rho} - m e^{-2\Phi} + 1 \right] \epsilon, \tag{A.8}$$

$$\delta\Psi_\mu = \left[D_\mu - \frac{i}{2} A_\mu^i \sigma^i + \frac{i}{4} \Gamma^\nu F_{\mu\nu}^i \sigma^i + \frac{1}{96} e^{2\Phi} \Gamma_\mu^{\nu\lambda\rho\sigma} G_{\nu\lambda\rho\sigma} - \frac{m}{4} e^{-2\Phi} \Gamma_\mu \right] \epsilon, \tag{A.9}$$

note that we dropped the ' of F' .

A.1.1. Background ansatz

For the background metric we consider the following fibred version of $\mathbb{R}^{3,1} \times \Sigma_k \times \mathbb{R}$

$$ds^2 = e^{2f(r)} dx_{3,1}^2 + e^{2h(r)} \left(d\theta^2 + \frac{1}{k} \sin^2(\sqrt{k}\theta) d\phi^2 \right) + dr^2, \tag{A.10}$$

where k is proportional to the curvature of Σ_k . Taking $k = -1, 0, 1$, gives $\Sigma_k = H^2, \mathbb{R}^2$ or S^2 , respectively. Because we are interested in studying twisted compactifications we will only consider the cases $k = \pm 1$.

The rest of the background fields are $B_3 = 0$, $\Phi = \Phi(r)$ and

$$A^1 = -k a(r) d\theta, \quad A^2 = \frac{1}{\sqrt{k}} a(r) \sin(\sqrt{k}\theta) d\phi, \quad A^3 = -\frac{1}{k} \cos(\sqrt{k}\theta) d\phi, \quad (\text{A.11})$$

which leads to the field strengths³

$$F^1 = -k a'(r) dr \wedge d\theta, \quad F^2 = \frac{1}{\sqrt{k}} a'(r) \sin(\sqrt{k}\theta) dr \wedge d\phi, \\ F^3 = \left(1 - k a^2(r)\right) \frac{\sin(\sqrt{k}\theta)}{\sqrt{k}} d\theta \wedge d\phi. \quad (\text{A.12})$$

In what follows we will need the vielbeins

$$e^{\widehat{m}} = e^{f(r)} dx^m, \quad e^{\widehat{\theta}} = e^{h(r)} d\theta, \quad e^{\widehat{\phi}} = \frac{1}{\sqrt{k}} e^{h(r)} \sin(\theta) d\phi, \quad e^{\widehat{r}} = dr, \quad (\text{A.13})$$

and the spin connection

$$\omega^{\widehat{m}\widehat{r}} = f'(r) e^{f(r)} dx^m, \quad \omega^{\widehat{\theta}\widehat{\phi}} = -\cos(\sqrt{k}\theta) d\phi, \quad \omega^{\widehat{\theta}\widehat{r}} = h'(r) e^{h(r)} d\theta, \\ \omega^{\widehat{\phi}\widehat{r}} = \frac{1}{\sqrt{k}} h'(r) e^{h(r)} \sin(\sqrt{k}\theta) d\phi. \quad (\text{A.14})$$

In the vielbein basis, we have

$$A^1 = -k a(r) e^{-h(r)} e^{\widehat{\theta}}, \quad A^2 = a(r) e^{-h(r)} e^{\widehat{\phi}}, \quad A^3 = \frac{1}{\sqrt{k}} e^{-h(r)} \cotg(\sqrt{k}\theta) e^{\widehat{\phi}}, \quad (\text{A.15})$$

$$F^1 = k a'(r) e^{-h(r)} e^{\widehat{\theta}} \wedge e^{\widehat{r}}, \quad F^2 = -a'(r) e^{-h(r)} e^{\widehat{\phi}} \wedge e^{\widehat{r}}, \\ F^3 = \left(1 - k a^2(r)\right) e^{-2h(r)} e^{\widehat{\theta}} \wedge e^{\widehat{\phi}}, \quad (\text{A.16})$$

$$\omega^{\widehat{m}\widehat{r}} = f'(r) e^{\widehat{m}}, \quad \omega^{\widehat{\theta}\widehat{\phi}} = -\sqrt{k} e^{-h(r)} \cotg(\sqrt{k}\theta) e^{\widehat{\phi}}, \quad \omega^{\widehat{\theta}\widehat{r}} = h'(r) e^{\widehat{\theta}}, \quad \omega^{\widehat{\phi}\widehat{r}} = h'(r) e^{\widehat{\phi}}. \quad (\text{A.17})$$

A.2. From SUSY variations to BPS equations

Substituting the above ansatz into the expressions for the SUSY variations, one derives BPS equations as outlined below. We begin with the Dilatino variation and move onto the Gravitino variations.

A.2.1. Dilatino variation

Substituting the ansatz into the Dilatino variation leads to

$$\delta\lambda = \left[\Gamma_{\widehat{r}} \Phi'(r) + \frac{i}{4} \left(k a'(r) e^{-h(r)} \Gamma_{\widehat{\theta}\widehat{r}} \sigma^1 - a'(r) e^{-h(r)} \Gamma_{\widehat{\phi}\widehat{r}} \sigma^2 \right. \right. \\ \left. \left. + \left(1 - k a^2(r)\right) e^{-2h(r)} \Gamma_{\widehat{\theta}\widehat{\phi}} \sigma^3 \right) - m e^{-2\Phi(r)} + 1 \right] \epsilon. \quad (\text{A.18})$$

³ In the appendixes, we have denoted the derivative with respect to r with a prime. In the main body of the paper we have reserved the primes for derivatives with respect to z , whilst denoting the r -derivatives with a dot.

We now impose the projection

$$\Gamma_{\widehat{\theta}\widehat{\phi}}\epsilon = \frac{1}{k}\sigma^1\sigma^2\epsilon. \tag{A.19}$$

Recalling that the Pauli matrices satisfy $\sigma^1\sigma^2\sigma^3 = i$, we have

$$\begin{aligned} \delta\lambda = & \left[\Gamma_{\widehat{r}}\Phi'(r) + \frac{i}{4}a'(r)e^{-h(r)} \left(k\Gamma_{\widehat{\theta}\widehat{r}}\sigma^1 - \Gamma_{\widehat{\phi}\widehat{r}}\sigma^2 \right) \right. \\ & \left. - \frac{1}{4k} \left(1 - k a^2(r) \right) e^{-2h(r)} - m e^{-2\Phi(r)} + 1 \right] \epsilon, \end{aligned} \tag{A.20}$$

then, by multiplying by $\Gamma_{\widehat{r}}$, we get

$$\begin{aligned} \Gamma_{\widehat{r}}\delta\lambda = & \left[\Phi'(r) - \frac{i}{4}a'(r)e^{-h(r)} \left(k\Gamma_{\widehat{\theta}}\sigma^1 - \Gamma_{\widehat{\phi}}\sigma^2 \right) \right. \\ & \left. + \left(1 - m e^{-2\Phi(r)} - \frac{1}{4k} \left(1 - k a^2(r) \right) e^{-2h(r)} \right) \Gamma_{\widehat{r}} \right] \epsilon. \end{aligned} \tag{A.21}$$

Using the projection (A.19) and the fact that $k = \pm 1$ one can show that

$$\left(k\Gamma_{\widehat{\theta}}\sigma^1 - \Gamma_{\widehat{\phi}}\sigma^2 \right) \epsilon = 2k\Gamma_{\widehat{\theta}}\sigma^1\epsilon. \tag{A.22}$$

Replacing this in the variation, yields

$$\begin{aligned} \Gamma_{\widehat{r}}\delta\lambda = & \left[\Phi'(r) - \frac{i}{2}k a'(r)e^{-h(r)}\Gamma_{\widehat{\theta}}\sigma^1 \right. \\ & \left. + \left(1 - m e^{-2\Phi(r)} - \frac{1}{4k} \left(1 - k a^2(r) \right) e^{-2h(r)} \right) \Gamma_{\widehat{r}} \right] \epsilon. \end{aligned} \tag{A.23}$$

A.2.2. Gravitino variation

We now switch our focus to the Gravitino variations, focusing on the \widehat{m} , $\widehat{\theta}$, $\widehat{\phi}$ and \widehat{r} components in turn. From the $m = 0, 1, 2, 3$ components of the variation of the gravitino in tangent space indices, we have

$$\delta\psi_{\widehat{m}} = \left(\frac{1}{2}f'(r)\Gamma_{\widehat{m}\widehat{r}} - \frac{m}{4}e^{-2\Phi(r)}\Gamma_{\widehat{m}} \right) \epsilon, \tag{A.24}$$

and multiplying by $2\Gamma_{\widehat{r}\widehat{m}}$, gives

$$2\Gamma_{\widehat{r}\widehat{m}}\delta\psi_{\widehat{m}} = \left(f'(r) - \frac{m}{2}e^{-2\Phi(r)}\Gamma_{\widehat{r}} \right) \epsilon. \tag{A.25}$$

From the $\widehat{\theta}$ component, we have

$$\begin{aligned} \delta\psi_{\widehat{\theta}} = & \left[\frac{1}{2}h'(r)\Gamma_{\widehat{\theta}\widehat{r}} + \frac{i}{2}k a(r) e^{-h(r)}\sigma^1 \right. \\ & \left. + \frac{i}{4} \left(k a'(r) e^{-h(r)}\Gamma_{\widehat{r}}\sigma^1 + \left(1 - k a^2(r) \right) e^{-2h(r)}\Gamma_{\widehat{\phi}}\sigma^3 \right) - \frac{m}{4}e^{-2\Phi(r)}\Gamma_{\widehat{\theta}} \right] \epsilon. \end{aligned} \tag{A.26}$$

Multiplying by $2\Gamma_{\widehat{r}\widehat{\theta}}$, we get

$$2\Gamma_{\widehat{r}\widehat{\theta}}\delta\psi_{\widehat{\theta}} = \left[h'(r) + ika(r)e^{-h(r)}\Gamma_{\widehat{r}\widehat{\theta}}\sigma^1 + \frac{i}{4}\left(-ka'(r)e^{-h(r)}\Gamma_{\widehat{\theta}}\sigma^1 + (1-ka^2(r))e^{-2h(r)}\Gamma_{\widehat{r}}\Gamma_{\widehat{\theta}\widehat{\phi}}\sigma^3\right) - \frac{m}{4}e^{-2\Phi(r)}\Gamma_{\widehat{r}} \right] \epsilon. \tag{A.27}$$

Using the projection (A.19) leads to

$$2\Gamma_{\widehat{r}\widehat{\theta}}\delta\psi_{\widehat{\theta}} = \left[h'(r) + ika(r)e^{-h(r)}\Gamma_{\widehat{r}\widehat{\theta}}\sigma^1 - \frac{i}{2}ka'(r)e^{-h(r)}\Gamma_{\widehat{\theta}}\sigma^1 - \frac{1}{2k}\left(1-ka^2(r)\right)e^{-2h(r)}\Gamma_{\widehat{r}} - \frac{m}{2}e^{-2\Phi(r)}\Gamma_{\widehat{r}} \right] \epsilon. \tag{A.28}$$

From the $\widehat{\phi}$ component we have

$$\delta\psi_{\widehat{\phi}} = \left[\frac{1}{2}\left(h'(r)\Gamma_{\widehat{\phi}\widehat{r}} - \sqrt{k}e^{-h(r)}\cotg(\sqrt{k}\theta)\Gamma_{\widehat{\theta}\widehat{\phi}}\right) - \frac{i}{2}\left(a(r)e^{-h(r)}\sigma^2 + \frac{1}{\sqrt{k}}e^{-h(r)}\cotg(\sqrt{k}\theta)\sigma^3\right) + \frac{i}{4}\left(-a'(r)e^{-h(r)}\Gamma_{\widehat{r}}\sigma^2 - \left(1-ka^2(r)\right)e^{-2h(r)}\Gamma_{\widehat{\theta}}\sigma^3\right) - \frac{m}{4}e^{-2\Phi(r)}\Gamma_{\widehat{\phi}} \right] \epsilon. \tag{A.29}$$

Using (A.19), the second and the fourth term cancel. This is the effect of the topological twist. We are left with

$$\delta\psi_{\widehat{\phi}} = \left[\frac{1}{2}h'(r)\Gamma_{\widehat{\phi}\widehat{r}} - \frac{i}{2}a(r)e^{-h(r)}\sigma^2 + \frac{i}{4}\left(-a'(r)e^{-h(r)}\Gamma_{\widehat{r}}\sigma^2 - \left(1-ka^2(r)\right)e^{-2h(r)}\Gamma_{\widehat{\theta}}\sigma^3\right) - \frac{m}{4}e^{-2\Phi(r)}\Gamma_{\widehat{\phi}} \right] \epsilon, \tag{A.30}$$

then multiplying by $2\Gamma_{\widehat{r}\widehat{\phi}}$, gives

$$2\Gamma_{\widehat{r}\widehat{\phi}}\delta\psi_{\widehat{\phi}} = \left[h'(r) - ia(r)e^{-h(r)}\Gamma_{\widehat{r}\widehat{\phi}}\sigma^2 + \frac{i}{2}\left(a'(r)e^{-h(r)}\Gamma_{\widehat{\phi}}\sigma^2 + \left(1-ka^2(r)\right)e^{-2h(r)}\Gamma_{\widehat{r}}\Gamma_{\widehat{\theta}\widehat{\phi}}\sigma^3\right) - \frac{m}{2}e^{-2\Phi(r)}\Gamma_{\widehat{r}} \right] \epsilon. \tag{A.31}$$

Using the projection again, leads to

$$2\Gamma_{\widehat{r}\widehat{\phi}}\delta\psi_{\widehat{\phi}} = \left[h'(r) - ia(r)e^{-h(r)}\Gamma_{\widehat{r}\widehat{\phi}}\sigma^2 + \frac{i}{2}a'(r)e^{-h(r)}\Gamma_{\widehat{\phi}}\sigma^2 - \frac{1}{2k}\left(1-ka^2(r)\right)e^{-2h(r)} - \frac{m}{2}e^{-2\Phi(r)}\Gamma_{\widehat{r}} \right] \epsilon, \tag{A.32}$$

and due to (A.22), we have $k\Gamma_{\widehat{\theta}}\sigma^1\epsilon = -\Gamma_{\widehat{\phi}}\sigma^2\epsilon$, giving

$$2\Gamma_{\widehat{r}\widehat{\phi}}\delta\psi_{\widehat{\phi}} = \left[h'(r) + ika(r)e^{-h(r)}\Gamma_{\widehat{r}\widehat{\theta}}\sigma^1 - \frac{i}{2}ka'(r)e^{-h(r)}\Gamma_{\widehat{\theta}}\sigma^1 - \frac{1}{2}\left(1-ka^2(r)\right)e^{-2h(r)} - \frac{m}{2}e^{-2\Phi(r)}\Gamma_{\widehat{r}} \right] \epsilon, \tag{A.33}$$

so we see that $\Gamma_{\hat{r}\hat{\theta}}\delta\psi_{\hat{\theta}} = \Gamma_{\hat{r}\hat{\phi}}\delta\psi_{\hat{\phi}}$.

Finally, from the \hat{r} component of the gravitino variation, we have

$$\delta\psi_{\hat{r}} = \left[\partial_r + \frac{i}{4} \left(-k a'(r) e^{-h(r)} \Gamma_{\hat{\theta}} \sigma^1 + a'(r) e^{-h(r)} \Gamma_{\hat{\phi}} \sigma^2 \right) - \frac{m}{4} e^{-2\Phi(r)} \Gamma_{\hat{r}} \right] \epsilon, \tag{A.34}$$

which can be rewritten as

$$\delta\psi_{\hat{r}} = \left[\partial_r + \frac{i}{4} a'(r) e^{-h(r)} \left(-k \Gamma_{\hat{\theta}} \sigma^1 + \Gamma_{\hat{\phi}} \sigma^2 \right) - \frac{m}{4} e^{-2\Phi(r)} \Gamma_{\hat{r}} \right] \epsilon, \tag{A.35}$$

and using the projection, we find

$$\delta\psi_{\hat{r}} = \left[\partial_r - \frac{i}{2} k a'(r) e^{-h(r)} \Gamma_{\hat{\theta}} \sigma^1 - \frac{m}{4} e^{-2\Phi(r)} \Gamma_{\hat{r}} \right] \epsilon. \tag{A.36}$$

After imposing $\delta\lambda = 0$ and $\delta\psi_{\mu} = 0$, the results (A.23), (A.25) and (A.28) lead to the following BPS equations

$$\Phi'(r)\epsilon - \frac{i}{2} k a'(r) e^{-h(r)} \Gamma_{\hat{\theta}} \sigma^1 \epsilon + \left(1 - m e^{-2\Phi(r)} - \frac{1}{4k} \left(1 - k a^2(r) \right) e^{-2h(r)} \right) \Gamma_{\hat{r}} \epsilon = 0, \tag{A.37}$$

$$f'(r)\epsilon - \frac{m}{2} e^{-2\Phi(r)} \Gamma_{\hat{r}} \epsilon = 0, \tag{A.38}$$

$$h'(r)\epsilon + i k a(r) e^{-h(r)} \Gamma_{\hat{r}\hat{\theta}} \sigma^1 \epsilon - \frac{i}{2} k a'(r) e^{-h(r)} \Gamma_{\hat{\theta}} \sigma^1 \epsilon - \frac{1}{2k} \left(1 - k a^2(r) \right) e^{-2h(r)} \Gamma_{\hat{r}} \epsilon - \frac{m}{2} e^{-2\Phi(r)} \Gamma_{\hat{r}} \epsilon = 0, \tag{A.39}$$

$$\partial_r \epsilon = \frac{i}{2} k a'(r) e^{-h(r)} \Gamma_{\hat{\theta}} \sigma^1 \epsilon + \frac{m}{4} e^{-2\Phi(r)} \Gamma_{\hat{r}} \epsilon. \tag{A.40}$$

A.2.3. Rearranging the SUSY variations

Following the procedure given in [31], we now rearrange the SUSY variations to obtain the BPS equations. For this, we use the properties of the Γ matrices to simplify the equations obtained in the previous subsection.

Let us start by rewriting the dilatino variation (A.23) as

$$\Gamma_{\hat{r}} \epsilon = \beta \epsilon + \tilde{\beta} i \Gamma_{\hat{\theta}} \sigma^1 \epsilon, \tag{A.41}$$

with

$$\beta = \frac{-\Phi'(r)}{1 - m e^{-2\Phi(r)} - \frac{1}{4k} (1 - k a^2(r)) e^{-2h(r)}}, \tag{A.42}$$

$$\tilde{\beta} = \frac{\frac{1}{2} k a'(r) e^{-h(r)}}{1 - m e^{-2\Phi(r)} - \frac{1}{4k} (1 - k a^2(r)) e^{-2h(r)}}. \tag{A.43}$$

By applying $\Gamma_{\hat{r}}$ to (A.41), we get

$$\epsilon = \beta \Gamma_{\hat{r}} \epsilon - i \tilde{\beta} \Gamma_{\hat{\theta}} \sigma^1 \Gamma_{hr} \epsilon. \tag{A.44}$$

Using (A.41), gives

$$\epsilon = \beta \left(\beta \epsilon + \tilde{\beta} i \Gamma_{\hat{\theta}} \sigma^1 \epsilon \right) - i \tilde{\beta} \Gamma_{\hat{\theta}} \sigma^1 \left(\beta \epsilon + \tilde{\beta} i \Gamma_{\hat{\theta}} \sigma^1 \epsilon \right), \tag{A.45}$$

and by simplifying, we obtain

$$\epsilon = (\beta^2 + \tilde{\beta}^2)\epsilon, \tag{A.46}$$

from which we see that

$$\beta^2 + \tilde{\beta}^2 = 1. \tag{A.47}$$

Multiplying (A.38) by $\Gamma_{\hat{r}}$ we get

$$f'(r)\Gamma_{\hat{r}}\epsilon = \frac{m}{2} e^{-2\Phi(r)}\epsilon. \tag{A.48}$$

Combining this with (A.38) leads to

$$\left(f'^2(r) - \frac{m^2}{4} e^{-4\Phi(r)} \right) \epsilon = 0, \tag{A.49}$$

which means that

$$f'(r) = \pm \frac{m}{2} e^{-2\Phi(r)}. \tag{A.50}$$

Now we go back to (A.48) and substitute in (A.41) as follows

$$f'(r) \left(\beta + \tilde{\beta} i \Gamma_{\hat{\theta}} \sigma^1 \right) \epsilon - \frac{m}{2} e^{-2\Phi(r)} \epsilon = 0. \tag{A.51}$$

Collecting terms with and without $i \Gamma_{\hat{\theta}} \sigma^1$, we get

$$\left(f'(r) \beta - \frac{m}{2} e^{-2\Phi(r)} \right) \epsilon + f'(r) \tilde{\beta} i \Gamma_{\hat{\theta}} \sigma^1 \epsilon = 0, \tag{A.52}$$

from where we get two equations

$$f'(r) \beta = \frac{m}{2} e^{-2\Phi(r)}, \tag{A.53}$$

$$f'(r) \tilde{\beta} = 0. \tag{A.54}$$

From (A.50) and (A.53) we get $\beta = \pm 1$, and from (A.54) we get $\tilde{\beta} = 0$, which in (A.43) gives

$$a'(r) = 0, \tag{A.55}$$

and hence, (A.41) becomes

$$\Gamma_{\hat{r}}\epsilon = \beta\epsilon. \tag{A.56}$$

Now we turn to (A.39). By replacing (A.56) and $a'(r) = 0$, we get

$$h'(r)\epsilon - i\beta k a(r) e^{-h(r)} \Gamma_{\hat{\theta}} \sigma^1 \epsilon - \beta \frac{1}{2k} \left(1 - k a^2(r) \right) e^{-2h(r)} \epsilon - \beta \frac{m}{2} e^{-2\Phi(r)} \epsilon = 0. \tag{A.57}$$

Again, by collecting terms with and without $i \Gamma_{\hat{\theta}} \sigma^1$, we get

$$\left(h'(r) - \beta \frac{1}{2k} \left(1 - k a^2(r) \right) e^{-2h(r)} - \beta \frac{m}{2} e^{-2\Phi(r)} \right) \epsilon - i\beta k a(r) e^{-h(r)} \Gamma_{\hat{\theta}} \sigma^1 \epsilon = 0. \tag{A.58}$$

From which we get

$$h'(r) - \beta \frac{1}{2k} (1 - k a^2(r)) e^{-2h(r)} - \beta \frac{m}{2} e^{-2\Phi(r)} = 0, \tag{A.59}$$

$$a(r) e^{-h(r)} = 0. \tag{A.60}$$

From the second result, we necessarily get $a(r) = 0$. After implementing this condition into (A.42) and (A.59), we finally derive the following BPS equations for $f(r)$ (which we got from (A.50)), $h(r)$ and $\Phi(r)$

$$f'(r) = \pm \frac{m}{2} e^{-2\Phi(r)}, \tag{A.61}$$

$$h'(r) = \pm \frac{1}{2} \left(\frac{1}{k} e^{-2h(r)} + m e^{-2\Phi(r)} \right), \tag{A.62}$$

$$\Phi'(r) = \pm \left(-1 + \frac{1}{4k} e^{-2h(r)} + m e^{-2\Phi(r)} \right). \tag{A.63}$$

The minus sign can be absorbed by $r \rightarrow -r$.

A.3. Uplift to 10D Massive Type IIA

A.3.1. Einstein frame and normalizations

The procedure which we will follow to uplift our 7D topologically massive solution is given in [30], however, the action we used for the 7D $\mathcal{N} = 2$ $SU(2)$ Gauged SUGRA is written in string frame, while in [30] is written in Einstein frame, hence we move our action to Einstein frame and then we compare normalizations and relevant constants. To move to Einstein frame we use the transformation

$$g_{\mu\nu}^{(E)} = e^{-\alpha\Phi(r)} g_{\mu\nu}^{(S)}, \tag{A.64}$$

in the action (A.1). Then, by comparing with [30] we see that $\alpha = \frac{4}{5}$, $m = 1$, $g = \sqrt{2}$, $h = \frac{1}{2}$ and that our Dilaton $\Phi(r)$ is related to the one in [30], which we call $\varphi(r)$, by

$$\varphi(r) = \frac{2\sqrt{10}}{5} \Phi(r). \tag{A.65}$$

Also, from the kinetic term for the gauge field, we see that in [30] the gauge field is normalised in a way in which the coupling constant appears in the definition of F rather than as a coefficient of the kinetic term F^2 , while in (A.1) we consider the opposite. To match conventions we re-scale our gauge field and field strength as

$$A \rightarrow -\frac{1}{g} A, \quad F \rightarrow -\frac{1}{g} F. \tag{A.66}$$

With this, our 7D ansatz now reads

$$ds^2 = e^{-\frac{4}{5}\Phi(r)} \left(e^{2f(r)} ds_{3,1}^2 + e^{2h(r)} \left(d\theta^2 + \frac{1}{k} \sin^2(\sqrt{k}\theta) d\phi^2 \right) + dr^2 \right), \tag{A.67}$$

together with the gauge field

$$A_1 = 0, \quad A_2 = 0, \quad A_3 = \frac{1}{g} \frac{1}{k} \cos(\sqrt{k}\theta) d\phi, \tag{A.68}$$

which leads to

$$F_1 = 0, \quad F_2 = 0, \quad F_3 = -\frac{1}{g} \text{Vol}(\Sigma_k^2). \tag{A.69}$$

A.3.2. Uplift of 7D topologically Massive to Massive IIA

The massive Type IIA ansatz is given by

$$ds^2 = \frac{1}{\ell} X(r)^{-\frac{1}{2}} e^{2A(\rho)} ds_7^2 + X(r)^{\frac{5}{2}} ds_{M_3}^2, \tag{A.70}$$

with $\ell = 4$, ds_7^2 a solution of the 7D $\mathcal{N} = 2$ Gauged Supergravity presented in the previous section, and

$$ds_{M_3}^2 = d\rho^2 + \frac{1 - x(\rho)^2}{16\omega} e^{2A(\rho)} Ds_{S^2}^2, \tag{A.71}$$

where $X(r)$ is given by the 7D dilaton

$$X(r) = e^{\frac{2}{5}\Phi(r)}, \tag{A.72}$$

and

$$\omega = X(r)^5 (1 - x(\rho)^2) + x(\rho)^2. \tag{A.73}$$

The covariantized metric $Ds_{S^2}^2$ on the sphere is constructed as follows. First we consider the normal unitary vector to the sphere

$$y^i = (\cos(\phi_2) \sin(\theta_2), \sin(\phi_2) \sin(\theta_2), \cos(\theta_2)), \tag{A.74}$$

then, the covariant line element is given by

$$Ds_{S^2}^2 = Dy^i Dy^i, \quad Dy^i = dy^i + g \epsilon^{ijk} y^k A^j, \tag{A.75}$$

where A^i is the component along the i th Pauli matrix of the 7D gauge field. This spacetime is supported by the 10D Dilaton Ψ

$$e^{2\Psi} = \frac{X(r)^{\frac{5}{2}}}{\omega} e^{2\psi(\rho)}, \tag{A.76}$$

and the background forms

$$B_2 = \left[\frac{1}{16} e^{2A(\rho)} \frac{x(\rho) \sqrt{1 - x(\rho)^2}}{\omega} \text{Vol}_2 - \frac{1}{2} e^{A(\rho)} d\rho \wedge \left(a - \frac{1}{g} y^i A^i \right) \right], \tag{A.77}$$

$$F_2 = -q \left(\text{Vol}_2 + g y^i F^i \right) + \frac{1}{16\omega} F_0 e^{2A(\rho)} x(\rho) \sqrt{1 - x(\rho)^2} \text{Vol}_2, \tag{A.78}$$

$$F_4 = \frac{\ell}{4} \left[-g \frac{q}{16\omega} e^{2A(\rho)} x(\rho) \sqrt{1 - x(\rho)^2} y^i F^i \wedge \text{Vol}_2 - g \frac{q}{4} e^{A(\rho)} d\rho \wedge \epsilon^{ijk} F^i y^j Dy^k \right] + G_4 \text{ terms}, \tag{A.79}$$

where Vol_2 is the volume element of the covariantised sphere, $a = \frac{1}{2} \cos(\theta_2) d\phi_2$ and F_0 is the Ramond mass (which is constant). Note that we have not written explicitly the terms proportional to the 7D 4-form, since we will not use them.

This configuration is a solution to the equations of motion

$$\frac{1}{4}R + \nabla^2\Psi - (\nabla\Psi)^2 - \frac{1}{8}H_3^2 = 0, \tag{A.80}$$

$$dF_p + H_3 \wedge *F_{p-2} = 0, \tag{A.81}$$

$$d(e^{-2\Psi} * H_3) - (F_0 * F_2 + F_2 \wedge *F_4 + F_4 \wedge F_4) = 0, \tag{A.82}$$

$$R_{MN} + 2\nabla_M\nabla_N\Psi - \frac{1}{2}(H_3^2)_{MN} - \frac{1}{4}e^{2\Psi} \sum_p (F_p^2)_{MN}, \tag{A.83}$$

with $p = 2, 4, 6, 8, 10$, and

$$(F_p^2)_{MN} = \frac{1}{(p-1)!} F_M^{N_1 \dots N_{p-1}} F_{MN_1 \dots N_{p-1}}, \tag{A.84}$$

and similarly for $(H_3^2)_{MN}$, provided $\psi(\rho)$, $x(\rho)$ and $A(\rho)$ satisfy

$$\frac{d}{d\rho}\psi(\rho) = \frac{1}{4} \frac{e^{-A(\rho)}}{\sqrt{1-x(\rho)^2}} \left(12x(\rho) + (2x(\rho)^2 - 5)F_0 e^{A(\rho)+\psi(\rho)} \right), \tag{A.85}$$

$$\frac{d}{d\rho}x(\rho) = -\frac{1}{2} e^{-A(\rho)} \sqrt{1-x(\rho)^2} \left(4+x(\rho) F_0 e^{A(\rho)+\psi(\rho)} \right), \tag{A.86}$$

$$\frac{d}{d\rho}A(\rho) = \frac{1}{4} \frac{e^{-A(\rho)}}{\sqrt{1-x(\rho)^2}} \left(4x(\rho) - F_0 e^{A(\rho)+\psi(\rho)} \right). \tag{A.87}$$

A.3.3. Explicit uplift of the interpolating background

We now write explicitly the field configuration for the uplift of (A.67)-(A.69). The spacetime metric reads (here we rename $(\theta, \phi) \rightarrow (\theta_1, \phi_1)$ with respect to the 7D solution)

$$ds^2 = \frac{1}{\ell} X(r)^{-\frac{1}{2}} e^{2A(\rho)} e^{-\frac{4\Phi(r)}{5}} \left[e^{2f(r)} dx_{3,1}^2 + dr^2 + e^{2h(r)} \left(d\theta_1^2 + \frac{1}{k} \sin^2(\sqrt{k}\theta_1) d\phi_1^2 \right) \right] \\ + X(r)^{5/2} \left[d\rho^2 + \frac{1-x(\rho)^2}{16\omega} e^{2A(\rho)} \left(d\theta_2^2 + \sin^2(\theta_2) \left(d\phi_2 - \frac{1}{k} \cos(\sqrt{k}\theta_1) d\phi_1 \right)^2 \right) \right], \tag{A.88}$$

with $\ell = 4$, while the background forms read

$$B_2 = \left(\frac{1}{16\omega} e^{2A(\rho)} x(\rho) \sqrt{1-x(\rho)^2} \sin(\theta_2) d\theta_2 - \frac{1}{4} e^{A(\rho)} \cos(\theta_2) d\rho \right) \\ \wedge \left(d\phi_2 - \frac{1}{k} \cos(\sqrt{k}\theta_1) d\phi_1 \right), \\ F_2 = \frac{1}{4} e^{A(\rho)-\psi(\rho)} \sqrt{1-x(\rho)^2} \left[\cos(\theta_2) \text{Vol}(\Sigma_k) - \text{Vol}(S_c^2) \right] \\ + \frac{1}{16\omega} F_0 e^{2A(\rho)} x(\rho) \sqrt{1-x(\rho)^2} \text{Vol}(S_c^2), \tag{A.89} \\ F_4 = \frac{e^{3A(\rho)-\psi(\rho)}}{64\omega} \cos(\theta_2) x(\rho) \left(1-x(\rho)^2 \right) \text{Vol}(\Sigma_k) \wedge \text{Vol}(S^2) \\ + \frac{e^{2A(\rho)-\psi(\rho)}}{16} \sin^2(\theta_2) \sqrt{1-x^2(\rho)} d\rho \wedge d\phi_2 \wedge \text{Vol}(\Sigma_k),$$

where

$$\text{Vol}(S^2) = \sin(\theta_2)d\theta_2 \wedge d\phi_2, \tag{A.90}$$

is the volume of the 2-sphere of the internal manifold, while

$$\text{Vol}(S_c^2) = \sin(\theta_2)d\theta_2 \wedge \left(d\phi_2 - \frac{1}{k} \cos(k\theta_1)d\phi_1 \right), \tag{A.91}$$

is the volume of the covariantised 2-sphere. Note that

$$\text{Vol}(\Sigma_k) \wedge \text{Vol}(S_c^2) = \text{Vol}(\Sigma_k) \wedge \text{Vol}(S^2). \tag{A.92}$$

A.3.4. Page fluxes

In order to get a quantised number of charges, we need to consider the Page fluxes, given by

$$\widehat{F}_p = F_p \wedge e^{-B_2}, \tag{A.93}$$

for the RR fields. Explicitly we have

$$\begin{aligned} \widehat{F}_2 = F_2 - B_2 F_0 &= \frac{1}{4} e^{A(\rho) - \psi(\rho)} \sqrt{1 - x(\rho)^2} \left[\cos(\theta_2) \text{Vol}(\Sigma_k) - \text{Vol}(S_c^2) \right] \\ &\quad + F_0 \frac{e^{A(\rho)}}{4} \cos(\theta_2) d\rho \wedge \left(d\phi_2 - \frac{1}{k} \cos(\sqrt{k}\theta_1) d\phi_1 \right), \end{aligned} \tag{A.94}$$

$$\begin{aligned} \widehat{F}_4 = F_4 - F_2 \wedge B_2 + \frac{1}{2} B_2 \wedge B_2 F_0 \\ = \frac{1}{16} e^{2A(\rho) - \psi(\rho)} \sqrt{1 - x(\rho)^2} d\rho \wedge d\phi_2 \wedge \text{Vol}(\Sigma_k). \end{aligned} \tag{A.95}$$

A.3.5. Rewriting in terms of $\alpha(z)$

Finally, it can be shown (as in [32]) that the 10D BPS equations can be solved in terms of just one function, $\alpha(z)$, provided (A, x, ψ) are of the following form

$$A(\rho) = \frac{1}{2} \ln \left(8\pi \sqrt{2} \sqrt{-\frac{\alpha(z)}{\alpha''(z)}} \right), \tag{A.96}$$

$$\psi(\rho) = \frac{1}{4} \ln \left(e^{4\psi_0} \frac{\left(-\frac{\alpha(z)}{\alpha''(z)} \right)^3}{\left(\alpha'(z)^2 - 2\alpha(z)\alpha''(z) \right)^2} \right), \tag{A.97}$$

$$x(\rho) = \sqrt{1 + \frac{2\alpha(z)\alpha''(z)}{\alpha'(z)^2 - 2\alpha(z)\alpha''(z)}}, \tag{A.98}$$

where the coordinate z is related to ρ via the following change of coordinates

$$d\rho = \sqrt{\pi \sqrt{2}} \left(-\frac{\alpha''(z)}{\alpha(z)} \right)^{\frac{1}{4}} dz. \tag{A.99}$$

This is a solution of the 10D BPS equations provided $\alpha(z)$ satisfies

$$\alpha'''(z) = 2^{-\frac{1}{4}} \sqrt{\pi} e^{\psi_0} F_0. \tag{A.100}$$

In terms of this new variable, the metric and background fields are the ones in (2.2)-(2.3).

Appendix B. Numerical solution of the BPS system

Here we give a detailed derivation of the numerical solutions that describe the flow from AdS₇ to AdS₅ × H². The starting point is to consider a linear perturbation around the IR fixed point and use that as initial conditions for the numerical solution.

B.1. Infrared fixed point

We are looking for solutions to the BPS equations for $f(r)$, $h(r)$ and $\Phi(r)$, which we quote here for convenience,

$$f' = \frac{m}{2} e^{-2\Phi}, \tag{B.1}$$

$$h' = \frac{1}{2} \left(\frac{1}{k} e^{-2h} + m e^{-2\Phi} \right), \tag{B.2}$$

$$\Phi' = \left(-1 + \frac{1}{4k} e^{-2h} + m e^{-2\Phi} \right), \tag{B.3}$$

that have constant $h(r)$ and $\Phi(r)$. This will correspond to the IR fixed point, and we note that it exists only for $k = -1$. The solutions

$$f(r) = \frac{2}{3}r, \quad h(r) = \frac{1}{2} \ln \left(\frac{3}{4} \right), \quad \Phi(r) = \frac{1}{2} \ln \left(\frac{3m}{4} \right), \tag{B.4}$$

are exact solutions to the BPS equations, and the spacetime obtained from this corresponds to AdS₅ × H².

B.2. Linear perturbations

Here we consider linear deviations from the IR fixed point

$$f(r) = \frac{2}{3}r + \epsilon a(r), \tag{B.5}$$

$$h(r) = \frac{1}{2} \ln \left(\frac{3}{4} \right) + \epsilon b(r), \tag{B.6}$$

$$\Phi(r) = \frac{1}{2} \ln \left(\frac{3m}{4} \right) + \epsilon c(r). \tag{B.7}$$

Replacing in the BPS equations and keeping only first order terms in ϵ , we obtain linear equations for the linear perturbations

$$a' = -\frac{4}{3}c, \tag{B.8}$$

$$b' = \frac{4}{3}(b - c), \tag{B.9}$$

$$c' = \frac{1}{3}(2b - 8c). \tag{B.10}$$

This system has as solution

$$a(r) = \frac{C_1}{3}(\sqrt{7} + 2)e^{-\frac{2}{3}(\sqrt{7}+1)r} - \frac{C_2}{3}(\sqrt{7} - 2)e^{\frac{2}{3}(\sqrt{7}-1)r}, \tag{B.11}$$

$$b(r) = C_1 e^{-\frac{2}{3}(\sqrt{7}+1)r} + C_2 e^{\frac{2}{3}(\sqrt{7}-1)r}, \tag{B.12}$$

$$c(r) = \frac{C_1}{2}(\sqrt{7} + 3)e^{-\frac{2}{3}(\sqrt{7}+1)r} - \frac{C_2}{2}(\sqrt{7} - 3)e^{\frac{2}{3}(\sqrt{7}-1)r}. \tag{B.13}$$

We are interested in solutions to vanish for $r \rightarrow -\infty$, which is the location of the fixed point, hence we set $C_1 = 0$ and also without loss of generality we can set $C_2 = 1$.

Finally, to obtain the numerical solutions we use the perturbations around the fixed point (B.5)-(B.7) as initial conditions at $r = -8$. Fig. 1 shows these numerical solutions.

Appendix C. Analysis of the 4d QFT

Referring to the four dimensional $\mathcal{N} = 1$ quiver in Fig. 6, we assign R-charges and anomalous dimensions for each of the fields. For the adjoint scalars Φ_i , the vector multiplets W_i , the bifundamentals between gauge nodes (Q, \tilde{Q}) and the fundamentals (q, \tilde{q}) we propose

$$\text{The R-charges: } R[\Phi_i] = 1, \quad R[W_i] = 1, \quad R[Q] = R[\tilde{Q}] = R[q] = R[\tilde{q}] = \frac{1}{2}. \tag{C.1}$$

$$\text{The anomalous dimensions: } \gamma_{\Phi_i} = 1, \quad \gamma_Q = \gamma_{\tilde{Q}} = \gamma_q = \gamma_{\tilde{q}} = -\frac{1}{2}. \tag{C.2}$$

Notice that the dimension of any combination of fields \mathcal{O} satisfies

$$\dim \mathcal{O} = \frac{3}{2} R_{\mathcal{O}}.$$

As should occur at conformal points. In particular the following superpotential terms are present,

$$\mathcal{W} \sim \mu_i \Phi_i \Phi_i + q_i \tilde{q}_j q_j \tilde{q}_i + \text{all other combinations.} \tag{C.3}$$

The R-charge of each of these possible terms is $R[\mathcal{W}] = 2$ and their dimension $\dim[\mathcal{W}] = 3$. The mass term for all the adjoint scalars decouples them from the IR dynamics. They do not participate in the quantities computed below. The quartic term, on the other hand, generates interactions between all the different $P^2(g - 1)$ horizontal lines of the quiver, by closing loops using the flavour groups. As one can start seeing, the behaviour of these quivers is somewhat reminiscent of those in [33]. Let us calculate the beta functions and the R-symmetry anomaly for each gauge group of the quiver in Fig. 6.

C.0.1. Beta functions

For any particular node we use the NSVZ beta function $\beta \sim 3N_c - N_{fi}(1 - \gamma_i)$, we find

$$\beta_i \sim 3N_i - (N_{i+1} + N_{i-1} + F_i)(1 - (-\frac{1}{2})) = \frac{3}{2} [2N_i - F_i - N_{i+1} - N_{i-1}] = 0. \tag{C.4}$$

As we start with a balanced quiver in the UV (the six dimensional quiver must cancel gauge anomalies), the beta function in four dimensions vanishes for each gauge group. It is nice to see how the six dimensional anomaly transmuted into the four dimensional beta function condition for conformality.

C.0.2. R-symmetry anomaly

We calculate using the expression $\Delta\Theta = T(R_i)R(f_i)$. We use that $T(adj_i) = 2N_i$ and $T(fund) = 1$. We find

$$\Delta\Theta_i = 2N_i \times 1 + (F_i + N_{i+1} + N_{i-1}) \times 2 \times \left(-\frac{1}{2}\right) = 2N_i - F_i - N_{i+1} - N_{i-1} = 0. \tag{C.5}$$

The comments written below eq. (C.4) also apply here. Both these calculations add to the proposal that the quiver in Fig. 6 describes the dynamics of our AdS₅ fixed point.

Other interesting quantities are the central charges a and c .

C.1. Central charges

They are defined as,

$$a = \frac{3}{32\pi} [3\text{Tr}R^3 - \text{Tr}R], \quad c = \frac{1}{32\pi} [9\text{Tr}R^3 - 5\text{Tr}R]. \tag{C.6}$$

Calculating explicitly for the quiver in Fig. 6, we find

$$\begin{aligned} \text{Tr}R &= P^2(g-1) \left[\sum_{j=1}^{P-1} (N_j^2 - 1) \times 1 + N_j F_j \times \left(-\frac{1}{2}\right) \times 2 + \sum_{j=1}^{P-2} N_j N_{j+1} \times \left(-\frac{1}{2}\right) \times 2 \right] \\ &= P^2(g-1) \left[\sum_{j=1}^{P-1} (N_j^2 - 1 - N_j F_j) - \sum_{j=1}^{P-2} N_j N_{j+1} \right]. \\ \text{Tr}R^3 &= P^2(g-1) \left[\sum_{j=1}^{P-1} (N_j^2 - 1) \times 1^3 + N_j F_j \times \left(-\frac{1}{2}\right)^3 \times 2 \right. \\ &\quad \left. + \sum_{j=1}^{P-2} N_j N_{j+1} \times \left(-\frac{1}{2}\right)^3 \times 2 \right] \\ &= P^2(g-1) \left[\sum_{j=1}^{P-1} (N_j^2 - 1 - \frac{1}{4} N_j F_j) - \frac{1}{4} \sum_{j=1}^{P-2} N_j N_{j+1} \right]. \end{aligned}$$

Using eqs. (C.6) we have,

$$a = \frac{3}{16\pi} P^2(g-1) \left[\sum_{j=1}^{P-1} (N_j^2 - 1) + \frac{1}{8} N_j F_j + \frac{1}{8} \sum_{j=1}^{P-2} N_j N_{j+1} \right], \tag{C.7}$$

$$c = \frac{1}{8\pi} P^2(g-1) \left[\sum_{j=1}^{P-1} (N_j^2 - 1) + \frac{11}{16} N_j F_j + \frac{11}{16} \sum_{j=1}^{P-2} N_j N_{j+1} \right]. \tag{C.8}$$

To gain some intuition, we discuss explicitly two illustrative examples.

C.1.1. Example 1

In this case we take

$$N_j = jN, \quad F_j = NP\delta_{j,P-1}.$$

We find, for large values of N and P (the holographic limit!)

$$a = \frac{9}{128\pi} P^5 N^2 (g - 1) \left(1 + O\left(\frac{1}{P}\right) \right), \quad c \sim a. \tag{C.9}$$

The comparison between eqs. (C.9) and (3.11) suggests that in the QFT side there must be other fields that whilst not contributing to the beta function and R-anomaly, it makes a contribution to the a and c central charges. The contribution of these fields, that we denote as \mathcal{X} should modify the result in eq. (C.9) (at leading order in P) according to,

$$a = \frac{9}{128\pi} N^2 P^5 (g - 1) (1 + \delta_{\mathcal{X}}), \quad 15\delta_{\mathcal{X}} = 1. \tag{C.10}$$

C.1.2. Example 2

In this case we take

$$N_j = N, \quad F_j = N(\delta_{j,1} + \delta_{j,P-1}).$$

We find, in the holographic limit,

$$a = \frac{27}{128\pi} P^3 N^2 (g - 1) \left(1 + O\left(\frac{1}{P}\right) \right), \quad c \sim a. \tag{C.11}$$

The comparison between eqs. (C.11) and (3.14) suggests that in the QFT side there must be other fields (not contributing to the beta function and R-anomaly), but adding a contribution to the a and c central charges. The contribution of these fields, that we denote as \mathcal{X} should be of the form (at leading order in P)

$$a = \frac{27}{128\pi} N^2 P^3 (g - 1) (1 + \delta_{\mathcal{X}}), \quad 3\delta_{\mathcal{X}} = 1. \tag{C.12}$$

This suggests that in our definition of the QFT quiver, new fields should enter adding these small contributions to the a central charge. Indeed, the presence of ‘flip fields’ that couple to irrelevant operators of the baryonic type is common in field theories of the class S_k . These fields are gauge singlets and decouple at low energies. They do not contribute to the beta functions or R-symmetry anomaly, but do change the result of a, c . It would be nice to see that their contribution can be $\delta_{\mathcal{X}}$ given in eqs. (C.10)-(C.12).

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