A simple model of panic buying

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Abstract

We study a simple dynamic model to rationalize episodes of excess demand that resemble "panic buying" in actual markets. In such periods, consumers compete for a scarce good. Scarcity is triggered by an anticipated negative supply shock that takes place in a future period with positive probability. To avoid the risk of non-consumption, consumers can stockpile the good in earlier periods. We demonstrate that these stockpiling decisions can reinforce each other, creating a cascade of excess demand in several periods, similar to "panic buying" episodes observed in actual markets, e.g., during the Corona crisis. In our model, stockpiling is always detrimental to welfare, and we develop a suitable policy intervention.

Keywords: Consumer inventory, storable good, supply shock, excess demand, panic buying (JEL: D11, D15, H42)

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1 Introduction

We present a simple dynamic model to explain episodes of excess demand in the prequel of an anticipated negative supply shock. Such episodes are sometimes referred to as "panic buying" but our model can explain such behavior even when all consumers behave perfectly rational. Examples include the early stages of the COVID-19 pandemic, where consumers prepared for extended periods of stay-at-home orders and possible supply chain disruptions. For example, in Germany, empty shelves and a shortage of certain goods (such as toilet paper) were observed. Other examples of earlier panic-buying episodes are the years between the two world wars (Hughes, 1988), the global rice crisis of 2008 (Hansman et al., 2020), and the earthquakes in Chile (2010) and Japan (2011) (Cavallo et al., 2014; Hori and Iwamoto, 2014).

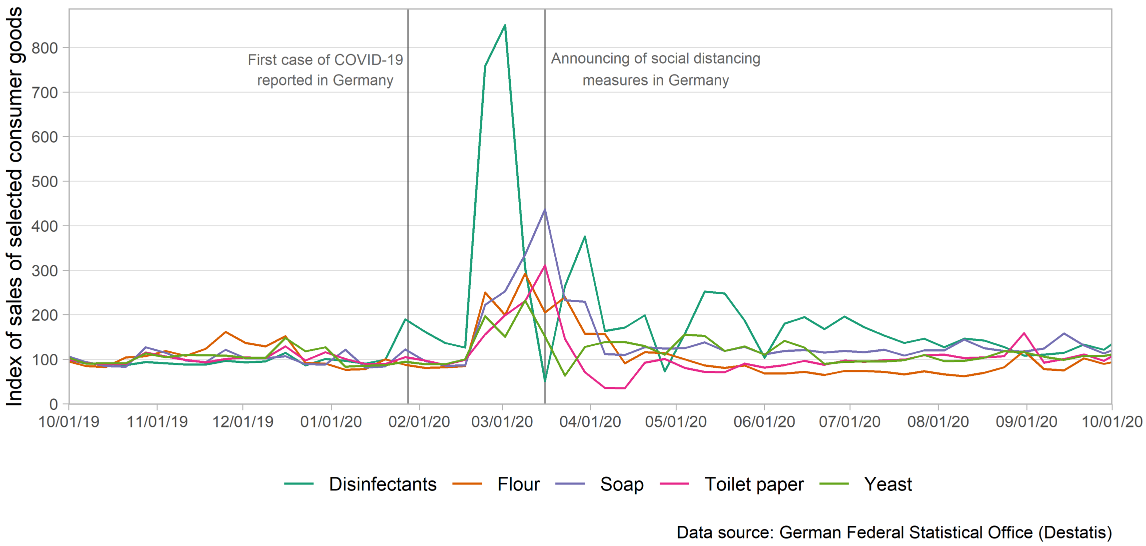
There are a number of stylized facts that are common to all these episodes: first, the most affected product categories are storable consumer goods, such as staple food, household supplies, and gasoline (e.g., Cavallo et al., 2014; O'Connell et al., 2021). Second, the demand for these products often peaked well before the actual supply shock was anticipated to take place, whereby in most cases, there was never really an actual reduction in supplies (Hori and Iwamoto, 2014; O'Connell et al., 2021).[1] Third, excess demand was driven by consumer stockpiling purchases and fueled by the fear of running out of the product. Last but not least, despite the huge upsurges in demands, prices stayed much on the same levels during these episodes because suppliers could not or did not want to raise them out of reputational concerns (Cavallo et al., 2014; Gagnon and López-Salido, 2020; Hansman et al., 2020).

Figure 1 illustrates one such panic buying episode based on supermarket scanner data from Germany during the first wave of the COVID-19 pandemic. As can be seen from the figure, the demand for household supplies rose sharply in the aftermath of the first reported Corona case in Germany and stayed high for 4–8 weeks, depending on the product category. However, for some product categories, the demand peak was even over before the first travel restrictions and stay-at-home orders became effective.

In this paper, we present a simple dynamic model that may help to rationalize these stylized facts and we develop a policy recommendation based on our findings. In our model, there is a market for a homogeneous storable good that is affected by a negative random supply shock at a certain point in time. The possibility of this shock is anticipated from the

[1] During the first wave of the COVID-19 pandemic, for instance, visits of supermarkets were exempted from stay-at-home orders and supermarket retailers never faced objective supply shortages.

Figure 1: Panic buying during the COVID-19 pandemic in Germany



Notes: The vertical axis shows the weekly indexes of sales for selected consumer products in Germany (expressed as percentages of the average sales in the prior 6 months) around the outbreak of the COVID-19 pandemic. Indexes are based on digital point-of-sale data from a number of sales outlets throughout Germany. The German government announced the first nationwide social distancing measures on 03/15/20 and entered into force on 03/22/20.

start of the game.In line with the stylized facts, we assume that the suppliers of this good have limited capacities in every period and keep their prices fixed at all times during the run-up to the event.[2] The supply shock, if it occurs, reduces the suppliers' capacities in the period when it occurs. The good is demanded by a population of homogeneous consumers who want to consume one unit in every period. To prepare for the shock, the consumers can visit the suppliers' shops at negligible costs in every period. However, storing a unit of the good for the next period always comes at a cost.

We study the Markov perfect equilibria of this setting where consumers rationally anticipate the potential occurrence of the supply shock in the known period. Hence, our analysis is consistent with empirical evidence suggesting that stockpiling purchases follow some individually rational plans even in times of crises (e.g., Hansman et al., 2020; O'Connell et al., 2021). In fact, the rationality assumption is the cornerstone for our central result: consumers' anticipation of the supply shock creates interdependencies in their demands with the consequence that, under certain conditions, a cascade of excess demands emerges that peaks well before the actual shock is predicted to take place. The intuition for

[2]See Section 7 for a discussion of our main modelling assumptions.

this cascade is as follows. If the expected supply shock is sufficiently large, consumers are trying to prepare for it by building up inventory in the period before. However, this creates excess demand already in that period, which, since it is anticipated, triggers another round of inventory purchases in the period before, etc.

We show that such a "panic buying" episode is most likely to emerge if the negative supply shock is expected to be large, and consumers' storage costs per unit of the good are small, relative to the utility loss incurred by a consumer who fails to consume the good in a period. Nevertheless, we also show that even when storage costs are small, the aggregate welfare losses due to stockpiling can be large and even exceed the direct welfare costs of the supply shock. The reason is as follows. Stockpiling does not resolve the problem of the negative supply shock because it does not create any of the extra units that would be needed to offset the resulting supply shortages. Stockpiling merely shifts the problem of non-consumption from one consumer to the next. As a result, all storage costs are socially wasteful in our model. Moreover, stockpiling creates an additional welfare cost when the supply shock is stochastic because for every stockpiling purchase by one consumer in the prequel of the supply shock, another consumer foregoes consumption in that period. Therefore, stockpiling can induce inefficient consumption losses. This is easiest to see in the case where a negative supply shock is expected to arrive , but in the end it never occurs. Then, if the storage cost is small or the cost of non-consumption is high, consumers prepare for the shock by stockpiling. This leads to non-consumption losses on the side of other consumers who visit a shop in these earlier periods but have to leave empty handed. The resulting welfare costs of these consumption outages can be substantial, as we illustrate with numerical examples.

Based on these insights, we also develop a policy intervention that seeks to prevent stockpiling purchases by consumers. To this end, we consider a benevolent social planner that provides the good in the period of the supply shock at some additional costs, e.g., by purchasing it overseas. We show that even if the quantity provided does not entirely neutralize the negative supply shock, it can still render any stockpiling purchases superfluous, thereby increasing welfare significantly. The policy is particularly effective in the case of an unlikely but possibly sizable supply shock that is anticipated by consumers long before it occurs. We show that the welfare impact of every additional unit provided can, in this case, even be a multiple of the original consumption value of the good. Hence, our policy findings provide an ex-post rationalization for the sometimes very costly programs to provide essential consumer products in times of crisis.[3]

Our model and findings are related to several branches in the literature: first, the theoretical literature on storable goods. One line of work in this branch studies the intertemporal dependence in a single consumer's demand that is typical when a good can be stored for future consumption (e.g. Erdem et al., 2003; Hendel and Nevo, 2006; Crawford, 2018). Another line investigates the implications of this within-subject interdependence for the profit-motivated manipulation of prices by sellers (Hong et al., 2002; Anton and Varma, 2005; Dudine et al., 2006; Liu and Van Ryzin, 2008; Hendel and Nevo, 2013). None of these papers focuses, however, on the demand interdependencies *between* different consumers that arise when consumers anticipate other consumers' storage demands.[4] Our paper is, to the best of our knowledge, the first to develop a dynamic model of consumer competition for a scarce good in the prequel of a negative supply shock. We show that stockpiling, driven by consumers' attempts to limit their private losses, can lead to sizable welfare losses even when the market satisfies otherwise all the conditions of a perfectly competitive market.

Our paper is also related to the psychological literature on panic buying (e.g. Baddeley, 2020; Bentall et al., 2021). Broadly speaking, this literature argues that it is indeed "panic" (i.e., anxiety, fear, or stress) that causes excessive stockpiling purchases. Apart from the different mechanisms in our paper (i.e., demand interdependencies), our model allows us to predict under which circumstances (e.g., depending on the storability of a product and the expected size of a negative supply shock) episodes of excess demand are expected to emerge. Finally, there is a link between the consumer demand cascade studied in this paper and the demand cascades that were already studied in the context of producer supply chains (e.g. Kahn, 1987; Lee et al., 1997). Despite this similarity, the underlying mechanisms are entirely different.

2 Model

There is a mass one of consumers who desire to consume one unit of a storable good in each out of infinitely many time periods, $t = 0, 1, 2, ...$ Consumers forego one unit of utility for

[3]For instance, the German government paid an average of 2.31 Euro per FFP2 mask during its massive (5.986 bn Euro) purchase program to fight the Corona pandemic in 2020–21 (Federal Audit Office, 2021). The free market price for an FFP2 mask was, at the same time, only around 0.50–1.00 Euro.

[4]The anticipation effect is, of course, present in many of these models. Yet, it is attenuated by the fact that the suppliers take advantage of it by incorporating it into their pricing decisions.

every t where they fail in doing so. Consumers can, however, prepare for this eventuality by buying up to two units of the good per period: one unit for immediate consumption and one unit on inventory. At the end of each period, a consumer can hold at most one unit of the good in her inventory until the next period.

Unlike consumers, suppliers (shops) cannot store the good. Shops are, moreover, capacity-constrained. In particular, in "normal times" (without an anticipated or real negative supply shock), the aggregated supply by all shops, ς_t, just matches consumers' per-period demand, that is, $\varsigma_t = 1$.[5]

By contrast, in period T, there is a potential negative supply shock so that the aggregate supply is limited to $\varsigma_T \equiv s \in [0,1]$ which is revealed to all shops and consumers at the beginning of period T. The shock might, for instance, be triggered by critical input shortages, supply chain disruptions, or mobility restrictions that deter consumers from going to a shop.[6] In either case, we assume that the shock only occurs with a certain probability so that the aggregate supply is a random variable with an expected value of $\mathbb{E}[s] \in (0,1)$.

The expected supply in period T thus falls short of the demand in the same period. Moreover, because the supply shock is also anticipated by all shops and consumers from period $t = 0$ onward, the supply may also fall short of the demand in earlier periods (before T) when consumers build up inventories to prepare for it.[7] Whenever this occurs, we assume random rationing so that all consumers attempting to enter a shop are turned away with the same probability σ_t.

Let $d_{it} \in \{0,1\}$ denote consumer i's attempt to enter a shop in period t, $b_{it} \in \{0,1,2\}$ the number of goods bought by i, $q_{it} \in \{0,1\}$ the number of goods consumed, and $\iota_{it} \in \{0,1\}$ the number of goods stored for period $t+1$. The realized utility of a consumer i can then be written as

$$u_i = \sum_{t=0}^{\infty} \delta^t \left[(q_{it} - 1) - \varepsilon \cdot d_{it} - p \cdot b_{it} - c \cdot \iota_{it} \right], \tag{1}$$

where $\delta \in (0,1]$ denotes the common discount factor, ε the fixed cost of visiting a shop, p the price of the good, and c the consumer's storage cost that accrues when one unit of

[5]The assumptions of constrained supplier capacities and no storability are discussed in Section 7.

[6]One might alternatively think of a demand shock in period T so that a random share of consumers has excess demand for the product (e.g., medical masks or sanitizers).

[7]In Section 7, we discuss what happens when a certain share of consumers does not anticipate the arrival of the shock but just imitates the shopping behavior of the other consumers in the same period.

the good is stored for the next period. More precisely, as we normalized the utility loss incurred in a period where a consumer fails to consume the good to one, c measures the storage cost per unit relative to the cost of non-consumption. Hence, a small value of c may either reflect small storage costs per unit, or a large cost of non-consumption.

Regarding the shopping cost ε, we assume throughout the paper that this cost is marginally small ($\varepsilon \to 0$). Hence, periodic shop visits are optimal, but only those consumers visit a shop who seriously attempt to buy. Moreover, we assume that the sales price p is the same across all shops and all periods of time. Such a fixed price may, for instance, be rationalized by shop owners' menu costs, reputational concerns, or a price cap implemented by the government (see Section 7 for a more detailed discussion of this and other modeling choices).

To simplify the exposition, we normalize this price to $p = 0$. The price is thus net of the (constant) marginal production cost in our model. Nevertheless, we add the assumption that a purchase must either be consumed or stored by a consumer (but is never thrown away) to rule out the possibility of excessive shopping. Finally, we assume that $c < \delta$ to ensure that storage costs are not so high that no consumer would ever consider storing a good for the next period. However, as we will show right at the beginning of Section 3, storage costs are also never so low (even for $c = 0$) that a consumer would want to keep a unit for the next period that she could otherwise consume immediately.

Let $h_t \equiv \int_{j=0}^{1} \iota_{j,t-1}$ denote the share of consumers who are holding one unit in their inventory at the beginning of period t. Henceforth, we will refer to the consumers who attempt to buy at least one unit in period t as the "shoppers". Moreover, let x_t (resp. y_t) denote the mass of shoppers who succeed in buying 1 unit (2 units) in period t, and let z_t be the mass of shoppers turned away. The assumption of constrained capacities implies that

$$x_t + 2y_t \leq \varsigma_t \qquad \forall t. \tag{2}$$

Moreover, random rationing implies a probability of rejection at a shop given by

$$\sigma_t = \frac{z_t}{x_t + y_t + z_t}. \tag{3}$$

In our analysis of this setting, the aggregate variables will play a key role, while the consumer-specific variables are of minor importance. They were introduced just for reasons of clarity. Table 1 summarizes the central variables and parameters of our model.

We will focus on Markov perfect equilibria in pure strategies. This means that each

Table 1: Main variables and parameters

Variable/parameter	Interpretation
x_t	mass of shoppers who buy 1 unit in period t
y_t	mass of shoppers who buy 2 units in period t
z_t	mass of shoppers turned away in period t
h_t	mass of consumers who enter t with a stored unit
σ_t	fraction of shoppers turned away in period t
c	storage cost for one unit of the good stored
δ	discount factor
s	supply in period T (random variable)

consumer conditions her actions on (i) her own inventory at the beginning of period t, $\iota_{i,t-1}$, (ii) the *aggregated* inventory h_t of all consumers, and (iii) the period number t, but not on the full history of the game until t.[8] By assumption, all consumers start the game without a unit in the first period, that is, $h_0 = 0$.

3 Equilibrium

We first characterize the properties of consumers' shopping behavior. We then turn to the general patterns of aggregate demand, which follow from here and yield a panic buying episode under certain parameter constellations.

Individual decisions. We first argue that any consumer i who *can* consume a unit in a period t, *will* consume it. To see this, note that even if all other consumers wish to buy two units in $t+1$, i is still able to enter a shop in $t+1$ with the positive probability $1 - \sigma_{t+1} \geq \varsigma_{t+1}/2 > 0$. Thus, the certain loss that is avoided by consuming a unit today strictly exceeds the *expected* loss that can be avoided by storing the good for $t+1$. Moreover, because of our focus on Markov strategies, i's decision has no impact on the shopping behavior of any other consumer in any future period $\tau > t$. Hence, when i consumes a unit in t, instead of keeping it for $t+1$, she can always reproduce (at negligible costs since $\varepsilon \to 0$) the same storage decision in $t+1$ that she deemed optimal under the alternative strategy where she kept the unit for $t+1$.

Next, we characterize the optimal shopping behavior of a consumer who is already in a shop. Note first that, by the same argument used above, buying an additional unit for

[8] The period number is payoff-relevant as the supply shock takes place in a specific period T.

$t+1$ and not buying it leads to the same equilibrium continuation payoff at the end of period $t+1$. Comparing the cost of stockpiling today, that is, the storage cost c, with the expected cost of non-consumption tomorrow thus leads to the following conclusion: A consumer weakly prefers to buy a unit on storage in period t if and only if

$$c \leq \begin{cases} \delta \cdot \mathbb{E}_s[\sigma_T] & \text{if } t+1 = T \\ \delta \cdot \sigma_{t+1} & \text{otherwise}. \end{cases} \tag{4}$$

Market dynamics. Condition (4) has some immediate implications for the dynamics of the market. This is because the converse of this condition states that *no* consumer is willing to build up inventory in any period t when the probability of rejection in the next period $t+1$ is sufficiently small. Hence, we get:

Lemma 1. *If $\sigma_{t+1} < \frac{c}{\delta}$ for $t+1 \neq T$ or $\mathbb{E}_s[\sigma_T] < \frac{c}{\delta}$ for $t+1 = T$, no consumer stockpiles in the preceding period t so that $h_{t+1} = 0$. If $t \neq T$ in addition, then demand in period t does not exceed supply so that $\sigma_t = 0$. As a result, consumers also do not stockpile in period $t-1$ so that $h_t = 0$.*

Another implication for market dynamics follows from our assumption that no goods are thrown away. As a result, demand can only exceed supply in any given period $t \neq T$ when so many consumers build up inventory in that period that the aggregate inventory grows in the next period. In other words, when the probability of rejection is positive in a period t, $\sigma_t > 0$, then the reason must be that $h_{t+1} > h_t$. The following lemma (proven in the appendix) shows that there is, in fact, a lower bound for h_{t+1}:

Lemma 2. *If $\sigma_t > 0$ for $t \neq T$, then $h_{t+1} \geq h_t + (1-h_t)\sigma_t$.*

The combination of Lemmas 1 and 2 leads to our first main result regarding the market dynamics: The aggregate inventory is weakly increasing over the periods $t = 0, ..., T$. The reason is that if $h_t < h_{t-1}$ for some period t, then there would be so little demand in period $t-1$ that, by the converse of Lemma 2, no consumer would be rejected in that period. By Lemma 1, it would then follow, however, that no consumer would stockpile for period $t-1$, which means that $h_{t-1} = 0$. This, in turn, leads to a contradiction to $h_t \geq 0$.

Our second main result is summarized in the following proposition. To formulate it, let $\bar{s}$ ($\underline{s}$) denote the maximal (minimal) s in the support of the distribution function, $F(s)$, of the aggregate supply. We then get:

Proposition 1. *In period T, it is $h_T < 1 - \underline{s}$ so that there is a strictly positive probability that supply falls short of demand, i.e., $\mathbb{E}_s[\sigma_T] > 0$. Prior to period T, stockpiling can only occur during a non-interrupted sequence of consecutive periods. From period T onward, consumers just buy for immediate consumption so that $\sigma_t = 0$ and $h_t = 0$ for all $t > T$.*

According to part one of the result, the aggregate inventory at the beginning of period T can never be so large that no consumer is ever rejected regardless of the realization of s. That is, it must be $h_T < 1 - \underline{s}$ at the beginning of period T so that some consumers cannot consume in that period. The reason can be found in Lemma 1. It implies that no consumer would see a reason to stockpile for period T when sufficiently many others have already done so, that is, when $h_T \geq 1 - \underline{s}$. Hence, there is a contradiction between the incentives to stockpile for period T and a large inventory at the beginning of period T.

Second, the result states that consumers only stockpile for a finite number of consecutive periods before T and no longer stockpile from period T onward. The intuition builds on Lemmas 1 and 2. By Lemma 1, stockpiling purchases can only occur in a non-interrupted sequence of periods. This is because if consumers would stop stockpiling in some period $t < T$ (or $t > T$), then successive application of this lemma would imply that consumers also do not stockpile in any period $\tau < t$ because $\sigma_{\tau'} = 0$ for all $\tau < \tau' \leq t$. By Lemma 2, it follows, in turn, that a non-interrupted sequence of stockpiling purchases cannot last forever because at some point in time, the accumulated inventory, h_t, is so large that it rules out the possibility of excess demand that is needed to maintain this sequence.

In sum, thus, Proposition 1 suggests that there is a limit to the number of periods during which stockpiling purchases occur. Nevertheless, the length of the sequence and the amount of stockpiling purchases can be significant under certain parameter constellations, particularly when the supply shock is large and storage costs are small. This is what the following result shows.

Proposition 2. *Suppose that $\bar{s} < \frac{T}{(T+1)^2}$ and $c < \frac{\delta}{T+1}$. Then, stockpiling purchases and thus rejection of shoppers occur in all periods prior to T (i.e., from $t = 0$ onward). When $\underline{s} > \frac{1}{2(T+1)}$ and $T > 1$ in addition, then the probability of rejection in some $t < T$ is even higher than the expected rejection probability in period T.*

Hence, for any finite T, there is an expected capacity $\mathbb{E}[s]$ and a storage cost c small enough such that all consumers are stockpiling from $t = 0$ onward. The intuition lies in the interdependencies in the demand of different consumers who "compete" for the scarce product even in periods prior to T. When the expected supply shock is large (i.e., $\mathbb{E}[s]$ is small) and storage costs are small, consumers are already preparing for the supply shock in period $T-1$ by building up inventory. This creates excess demand in that period. When this excess demand is anticipated, it triggers further rounds of stockpiling purchases in periods $T-2$, $T-3$, and so on. Hence, consumers' efforts to prevent running out of the good can generate a "panic buying" episode, that is an extended cascade of excess demands.

As the result also shows, the demand cascade might even peak in some period prior to T so that a larger number of consumers is rejected from the shops in some period $t < T$ than in period T itself. The intuition can be found in the two countervailing effects of stockpiling purchases. Stockpiling creates excess demand in the period when it occurs because some of the consumers who start that period without a unit in store will attempt to buy two units. At the same time, stockpiling lowers the demand in the period thereafter because additional consumers enter the period with a unit in stock. Stockpiling thus raises σ_t and lowers σ_{t+1}.

Such a peaking demand prior to period T occurs in particular when the supply shock is of intermediate size. The intuition for this is as follows. When the supply shock is small (i.e., $\mathbb{E}[s]$ is large), stockpiling for period T does not pay off so that there is also no excess demand in any period $t < T$. By contrast, when the shock is large (i.e., $\mathbb{E}[s]$ is small), consumers are already buying as much as they can in periods $t < T$, so any further increase in the shock's size just materializes in higher excess demand in period T.

In sum, Proposition 2 suggests that the rational anticipation of a supply shock can create an extreme form of a panic buying episode where all consumers stockpile as much as they can from the time on when they anticipate the shock. Nevertheless, as we show in the following section, a panic buying episode may also emerge in some less extreme forms in our model so that consumers only build up inventory after some period $t > 0$.

4 Example

Here, we present a complete characterization of the special case of our model with $T = 2$ and a binarily distributed supply shock with $s \in \{\underline{s}, 1\}$ and $Prob[s = \underline{s}] = \theta$. Based on Proposition 1, we proceed by backward induction starting from period T:

Period T**.** Because there is no excess demand from period $T+1$ onward, all consumers entering period T with a stored unit (mass h_T) stay at home. Consumers entering period

T without a stored unit (mass $1 - h_T$), by contrast, visit a shop to buy one unit. Hence, we get $y_T = 0$. Each one of the shoppers is successful if the total mass of shoppers $(1 - h_T)$ is smaller than the supply in T, that is, $x_T = \min\{1 - h_T\,;\, s\}$.

However, we know from Proposition 1 that demand must exceed supply for at least one realization of s. For our binarily distributed shock, this means that there is excess demand if and only if $s = \underline{s}$. In this case, the share of shoppers turned away is given by $z_T = \max\{1 - h_T - \underline{s}\,;\, 0\}$ and the probability of rejection by

$$\sigma_T = \begin{cases} \frac{1-h_T-\underline{s}}{1-h_T} & \text{if } s = \underline{s} \\ 0 & \text{if } s = 1\,. \end{cases} \tag{5}$$

Period $T - 1$. All consumers entering period $T - 1$ *without* a stored unit will try to visit a shop to buy at least one unit for immediate consumption. The interesting question is how many units they buy and what the consumers *with* a stored unit will do?

Below, we distinguish between "interior" and "corner" solutions. An interior solution is an equilibrium where some consumers (either those who start period $T - 1$ with a unit or those who do not) are *indifferent* between buying an extra unit for period T and not. A corner solution is, by contrast, an equilibrium where no consumer is indifferent so that either all consumers try to build up inventory for period T (such as in the scenario used for Proposition 2) or no one does (such as in the extreme case of Proposition 1). In particular, a consumer *without* a stored unit at the beginning of period $T - 1$ is *indifferent* between buying an extra unit for T and just for immediate consumption if and only if

$$c = \delta \cdot \mathbb{E}_s[\sigma_T]\,. \tag{6}$$

If this condition is met, then all consumers *with* a stored unit at the beginning of $T - 1$ stay at home because they have their desired unit for the current period, while stockpiling does not pay off because of the additional (small) shopping cost ε. Combined with expression (5) and the probability of the shock, θ, condition (6) gives the following expression for the mass of consumers entering period T with a stored unit:

$$h_T \;=\; \frac{\delta\theta(1 - \underline{s}) - c}{\delta\theta - c} \;\equiv\; H\,. \tag{7}$$

In the second type of interior solution, it is the consumers *with* a stored unit at the beginning of period $T - 1$ who are *indifferent* between buying another unit for period T

and staying at home. Because these consumers need to make an extra trip to a shop, their indifference results in the condition $\delta \cdot \mathbb{E}_s[\sigma_T] = c + \varepsilon$. In the limit of $\varepsilon \to 0$, this becomes just indifference condition (6). Hence, also in this interior solution, the share of consumers entering period T with a unit on store, h_T, is just the value defined in (7).

Adding the two corner solutions, we arrive at the following five equilibrium types:

- Solution type A ($\underline{s}$ is large): If $\underline{s} > \frac{\delta\theta - c}{\delta\theta}$, only consumers *without* a stored unit visit a shop and nobody stockpiles for period T (corner solution).
- Solution type B: If $\frac{(1-h_{T-1})(\delta\theta-c)}{\delta\theta} \leq \underline{s} \leq \frac{\delta\theta - c}{\delta\theta}$, only consumers *without* a stored unit visit a shop. They are indifferent between stockpiling and not so that some buy one and others buy two units (interior solution). Nobody is turned away.
- Solution type C: If $\frac{\delta\theta - c}{2\delta\theta} \leq \underline{s} < \frac{(1-h_{T-1})(\delta\theta-c)}{\delta\theta}$, the outcome is the same as in Solution type B, except that some consumers are turned away (interior solution).
- Solution type D: If $\frac{(1-h_{T-1})(\delta\theta-c)}{(2-h_{T-1})\delta\theta} \leq \underline{s} \leq \frac{\delta\theta - c}{2\delta\theta}$, all consumers *without* a stored unit try to stockpile. Consumers *with* a stored unit are indifferent between going to a shop and staying at home (interior solution).
- Solution type E ($\underline{s}$ is small): If $\underline{s} < \frac{(1-h_{T-1})(\delta\theta-c)}{(2-h_{T-1})\delta\theta}$, everybody visits a shop and attempts to stockpile (corner solution).

All parameter conditions are derived in the appendix.

Period $T-2$. By extension of the above arguments, the same five solution types can emerge in period $T-2$ depending on the expected benefits, $\delta \cdot \sigma_{T-1}$, and costs, c, of stockpiling. The only additional complication lies in the fact that *different* solution types may co-exist in periods $T-2$ and $T-1$.

By assumption, consumers start the game without inventories,

$$h_{T-2} = 0\,.$$

This means that only those consumers who buy two units in period $T-2$ enter period $T-1$ with a stored unit,

$$h_{T-1} = y_{T-2}. \tag{8}$$

It furthermore implies that only solutions of type A, C, and E can occur in period $T-2$, as the solution types B and D require $h_{T-2} > 0$ (see the results for period $T-1$). Let us indicate the feasible solution types for period $T-2$ by A′, C′, and E′, while we continue to use A–E for the solutions for period $T-1$. We then get the following combinations of solutions:

- Solution type A′–A ($\underline{s}$ is large): If $\underline{s} > \frac{\delta\theta - c}{\delta\theta}$, nobody stockpiles in any period.
- Solution type A′–C: If $c < \frac{\delta}{2}$ and $\frac{(\delta\theta-c)(\delta-c)}{\delta^2\theta} < \underline{s} \leq \frac{\delta\theta-c}{\delta\theta}$, or $c \geq \frac{\delta}{2}$ and $\frac{\delta\theta-c}{2\delta\theta} \leq \underline{s} \leq \frac{\delta\theta-c}{\delta\theta}$, then nobody stockpiles in period $T-2$, and an interior solution emerges in $T-1$.
- Solution type A′–E: If $c > \frac{\delta}{2}$ and $\underline{s} < \frac{\delta\theta-c}{2\delta\theta}$, nobody stockpiles in period $T-2$, but everybody tries to stockpile in $T-1$.
- Solution type C′–C/D: If $\frac{(\delta\theta-c)(\delta-c)}{2\delta^2\theta} \leq \underline{s} \leq \frac{(\delta\theta-c)(\delta-c)}{\delta^2\theta}$, then Solution type C'–C arises if it additionally holds that $\underline{s} \geq \frac{\delta\theta-c}{2\delta\theta}$, resp. Solution type C'–D arises if $\frac{c(\delta\theta-c)}{\delta^2\theta} \leq \underline{s} \leq \frac{\delta\theta-c}{2\delta\theta}$. In both cases, an interior solution is obtained in both periods.
- Solution type C′–E: If $\frac{\delta}{3} \leq c \leq \frac{\delta}{2}$ and $\underline{s} < \frac{c(\delta\theta-c)}{\delta^2\theta}$, then an interior solution emerges in period $T-2$ and a corner solution in which all consumers try to stockpile in $T-1$.
- Solution type E′–D: If $\frac{\delta\theta-c}{3\delta\theta} \leq \underline{s} < \frac{(\delta\theta-c)(\delta-c)}{2\delta^2\theta}$, all consumers try to stockpile in period $T-2$, whereas an interior solution arises in $T-1$ in which the consumers entering period $T-1$ with a stored unit are indifferent between stockpiling for T and not.
- Solution type E′–E: If $c < \frac{\delta}{3}$ and $\underline{s} < \frac{\delta\theta-c}{3\delta\theta}$, all consumers try to stockpile in both periods.

It is straightforward to verify that the above parameter conditions for c and $\underline{s}$ cover the entire parameter space.

Figure 2 summarizes the feasible solution types and their parameter conditions. A general observation is that the smaller $\underline{s}$ and c are, the tighter the markets in both periods $T-1$ and $T-2$. Of particular interest are the parameter ranges where a demand cascade emerges with a peak of demand that is even larger than in period T. Figure 3 plots the rejection probabilities for different values of $\underline{s}$. As can be seen, when the supply shock is sufficiently large ($\underline{s} < 0.5$), stockpiling can drive up the demands in periods $T-1$ and $T-2$ to the point where the probability of rejection is even higher in these periods than in period T itself.[9]

[9] Even though there are some striking similarities between Figure 3 and the empirical panic buying episode illustrated in Figure 1, there is one important difference. In our model, the realized demand is just one in all periods, except for maybe period T where the realized demand may be lower.

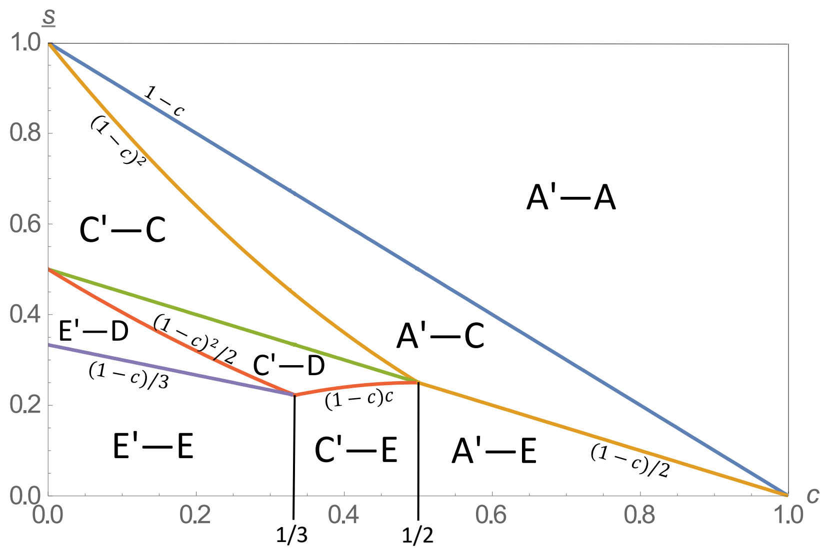


Figure 2: Markov perfect equilibria for $T = 2$ and $\theta = \delta = 1$

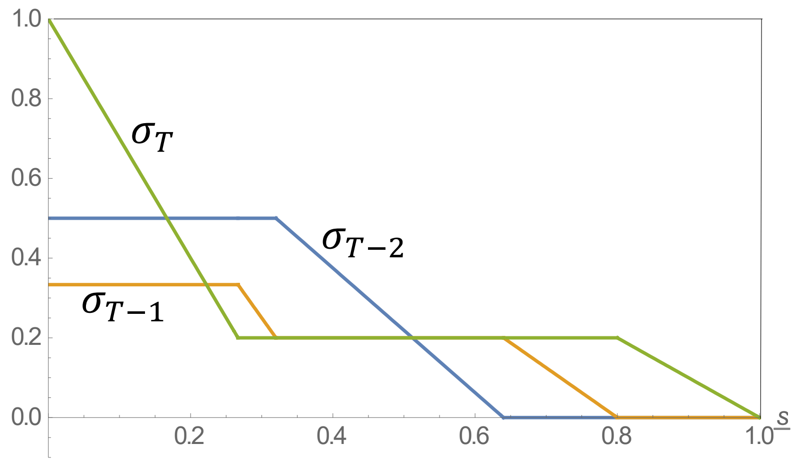


Figure 3: Rejection probabilities in period $T-2$ (blue), $T-1$ (orange), and T (green), for $T = 2$, $c = 0.2$, and $\delta = \theta = 1$

5 Welfare

One of the main findings of the empirical literature on earlier panic buying episodes is that stockpiling was never really justified from an ex-post perspective because there was never really a substantive supply shortage in the observed product categories (e.g. Hori and Iwamoto, 2014; O'Connell et al., 2021). However, stockpiling seemed to make sense from an individual perspective because it gave consumers a feeling of control over the contingencies created by the crisis.

The same trade-off between the individual and the welfare perspective is also present in our model: Stockpiling makes sense from an individual viewpoint because it prepares a consumer for the upcoming period of a supply shortage. Yet, since stockpiling does not create the additional units that would be needed to cushion the actual supply reduction (if it occurs), it just shifts the problem of non-consumption from one consumer to the next. In the following, we identify several different channels through which stockpiling damages welfare in our model and subsequently illustrate the potential magnitude of the welfare losses.

Stockpiling creates three potential sources of inefficiency in our model: First, it entails storage costs. Second, it can create avoidable consumption losses for consumers who fail to enter a shop in periods $t = 0, ..., T-1$ because other consumers have bought on stock. In particular, if the actual supply reduction in period T turns out to be smaller than expected, any inventory built up in excess of the required number of units to cushion the shock is inefficient because it entails avoidable consumption losses. Finally, stockpiling shifts the problem of consumption losses to periods earlier than T, which is detrimental to welfare when $\delta < 1$.

To quantify these different channels, let us abstract from the third channel and assume that $\delta = 1$. Compared to the situation without a supply shock in period T, ex-ante welfare can then be written as

$$\begin{aligned}\mathbb{E}_s\big[W|h_1, ..., h_T\big] \;\; = \;\; & -c\sum_{t=0}^{T-1} h_{t+1} \; - \; \mathbb{E}_s\big[1-s\,|\,1-s > h_T\big]\cdot Prob\big(1-s > h_T\big) \\ & - h_T \cdot Prob\big(1-s \leq h_T\big)\,. \end{aligned} \tag{9}$$

The first term measures the obvious cost, the total storage cost accumulated over the periods $t = 0, ..., T-1$. The other two terms, in turn, measure the consumption losses in response to the shock. First, the total consumption outages in the case where the actual supply shock, $1-s$, is larger than expected (i.e., when $1-s > h_T$). Specifically, since all consumers start with empty inventories ($h_0 = 0$) but want to consume one unit per period, the total mass of missing units over all periods is simply $1-s$. This is just the same number of units that would be missing when nobody stockpiles in any period, that is, when $h_t = 0$ in every period t. Compared to this case, the welfare costs of this channel are thus zero when $\delta = 1$ but they are positive when $\delta < 1$ because the consumption losses arise earlier when consumers stockpile.

In addition, stockpiling creates another welfare cost when the supply shock turns out to be smaller than expected (i.e., when $1 - s \leq h_T$). This cost is shown in line two of (9). It emerges from the fact that more units are bought than necessary to cushion the shock, and it even accrues when $\delta = 1$. Specifically, when the supply shock is small, the bottleneck does not lie in period T but in the periods prior to it because, for every one of the h_T consumers who enter period T with a unit on stock, we can find one other consumer who could not buy a unit in some earlier period $t < T$ and who thus suffers a loss in consumption utility.

It is obvious from above that stockpiling generates several avoidable welfare costs, regardless of whether we take in an ex-ante or an ex-post welfare perspective and regardless of whether the supply shock is small or large. When the shock is small, stockpiling creates avoidable consumption outages because of excess inventories. When the supply shock is large, by contrast, any inventory built up until period T is socially wasteful as well because the associated consumption outages could have been equally well deferred to period T, while inventory building creates additional storage costs. Hence, we get

Proposition 3. *From an ex-ante and an ex-post perspective, welfare is maximal when no consumer stockpiles in any period, that is, when $h_t = 0$ in every period t.*

For illustration of the size of the welfare costs of stockpiling, let us assume, as in our example of the previous section, a binary distributed supply shock so that the total supply is limited to $\underline{s} < 1$ with probability θ. Furthermore, let us assume that the supply shock is so large (i.e., $1 - \underline{s}$ is so small) that all consumers attempt to build up inventories already from period $t = 0$ onward.[10] This is obviously an extreme scenario that nevertheless illustrates how large the welfare costs of stockpiling can be.

The share of consumers with a unit on stock at the beginning of any period t, $0 \leq t \leq T$, can then be written as $h_t = t/(t+1)$ (see the proof of Proposition 2). Plugging this expression into (9) gives a total welfare cost of stockpiling of

$$-c \sum_{t=0}^{T} \frac{t}{t+1} \; - \; (1-\theta)\frac{T}{T+1} \, .$$

The first summand measures the accumulated storage costs; the second summand the avoidable consumption outages due to excessive stockpiling. These outages accrue with probability $1 - \theta$, and they result in a total foregone consumption utility over all periods $t < T$ that is as large as the aggregated inventory $h_T = T/(T+1)$ in period T.

[10] This requires that $\underline{s} < (\theta(T+1) - 1)/(T+1)^2$ and $c < 1/(T+1)$.

Table 2: Welfare costs of stockpiling

Period (T)	2		3		10		20	
Probability (θ)	stockpiling	direct	stockpiling	direct	stockpiling	direct	stockpiling	direct
0.5	-.39	-.50	-.47	-.50	-.83	-.50	-1.30	-.50
0.1	-.66	-.10	-.77	-.10	-1.20	-.10	-1.68	-.10

NOTES: The remaining parameters are set to $\delta = 1$, $c = \frac{1}{21}$, and $\underline{s} = 0$.

Table 2 compares the welfare total cost of stockpiling with the direct, unavoidable welfare cost of the supply shock, $-\theta(1 - \underline{s})$. Clearly, the indirect costs of stockpiling can grow very large, in particular when the risk of an actual supply shock (θ) is small and consumers learn about the potential shock long before it takes place (i.e., T is large).

6 Policy

One policy that many governments pursued during times of crisis was the free provision of essential consumer products. Considering that such policies can be very costly,[11] we propose here an alternative supply policy that extends on our earlier findings. In particular, we address the following questions: how many units does a benevolent government need to inject into the market in period T in order to limit the remaining supply shortage to a point where no consumer finds it worthwhile to build up any inventory?[12]

It should be clear from what we said before that in order to be effective, the government needs to announce such a "zero-panic-buying" policy already when the shock is expected to arrive. Otherwise, it would be too late because consumers have already adjusted their consumption plans. Let us, therefore, assume that the government is able to credibly commit to the following policy at time $t = 0$: It promises to inject an amount of k units in period T into the market where k is chosen in a way to deter any stockpiling purchases in periods $t < T$.[13]

[11] During the COVID-19 pandemic, for instance, the German government purchased face masks and medical protection suits for a total of 5.986 bn Euro for free provision to hospitals and health care organizations (Federal Audit Office, 2021).

[12] A full analysis of the optimal supply policy, also considering the costs of provision, is much more complex than the analysis pursued here due to the various possible equilibrium configurations that can arise in our model (see Section 4). As we believe that this offers little additional insight, we refrain from such an analysis here.

[13] The potential welfare gains of our suggested policy could be even larger when the government makes the amounts contingent on the actual realization of the supply shock in period T. In this way, wasteful over-provision of the good can be avoided when the supply shock is smaller than expected. Still, even

Table 3: A zero-panic-buying policy

Period T	2		3		10		20	
Prob. θ	Required units k	WTP per unit	Required units k	WTP per unit	Required units k	WTP per unit	Required units k	WTP per unit
0.5	.90	.93	.90	1.02	.90	1.42	.90	1.94
0.1	.52	1.35	.52	1.56	.52	2.39	.52	3.31

NOTES: The remaining parameters are set to $\delta = 1$, $c = \frac{1}{21}$, and $\underline{s} = 0$. WTP = Average willingness to pay per unit given by $(c\sum_{t=0}^{T}\frac{t}{t+1} + (1-\theta)\frac{T}{T+1} + \theta k)/k$.

To determine the required amount k, note that according to Lemma 1, no consumer finds it worthwhile to build up inventory for period T (or for any earlier period) when $\delta\,\mathbb{E}_s[\sigma_T] < c$. Hence, it suffices to supply an amount k such that $\delta\,\mathbb{E}_s[\sigma_T|\, h_T = 0,\, k \geq 0] = c$ to deter any stockpiling purchases in $t = 0, ..., T-1$.[14] This means that the government can afford to commit to a smaller intervention the more costly it is for consumers to store the good and the smaller the expected size of the supply shock is.

For illustration, let us return to the example in Section 5. We get for a binary distributed supply shock

$$\mathbb{E}_s[\sigma_T] \;=\; \theta\frac{1-h_T-\underline{s}}{1-h_T}\,.$$

The above condition can thus be rewritten as $\delta\theta(1-\underline{s}-k) = c$, giving

$$k = 1 - \underline{s} - \frac{c}{\delta\theta}\,. \tag{10}$$

Table 3 summarizes the required amount k for different values of T and θ. Moreover, it computes the average price per unit (WTP) that a benevolent government would be willing to pre-commit to in period $t = 0$ to have the required amount available in period

our simple policy can be very effective because it deters socially wasteful stockpiling purchases in periods $t < T$ and helps avoid possible consumption losses in T in case the supply shock is large.

Another alternative for the government might be to supply even more units (beyond the quantity k) to avoid any consumption losses in period T with certainty. Such a "zero-shortage" policy pays off if the extra costs per unit beyond k are smaller than the discounted consumption losses which every extra unit aims to prevent: $\delta^T(1-\mathbb{E}[s])$. Note that this value is (much) smaller than the WTP per unit in the illustrations of our preferred policy in Table 3.

By contrast, another obvious alternative policy, namely to banish stockpiling purchases altogether, is less effective. In our model, consumers could just buy one unit at multiple shops. In line with this, the empirical evidence reported in O'Connell et al. (2021) suggests that the nationwide purchase limits implemented in the UK during the first weeks of the Corona pandemic have had hardly any measurable impact.

[14]We assume here that indifference is enough for consumers to abstain from stockpiling purchases.

T. As can be seen from the table, the average WTP can be very large and even exceed the original consumption value of the good (which is one). This may justify the sometimes very expensive purchasing programs in times of crisis. The German government paid, for instance, an average of 2.31 Euro per FFP2 mask during the Corona pandemic, while the market price for the same mask was only 0.50–1.00 Euro (Federal Audit Office, 2021).

The average WTP is particularly high when the indirect welfare costs of stockpiling are high, that is, when the risk of an actual supply shock (θ) is small and consumers anticipate the shock long before it is expected to take place (large T). The reason lies in the combination of two factors. First, if the supply shock is unlikely, the government just needs to supply a few extra units to convince consumers to refrain from stockpiling (see the expression for k in (10)). The impact of every unit injected is thus large. Second, even though the actual arrival of a supply shock may be unlikely, the welfare costs in the run-up to this event can be very high because of the certain consumption losses that accrue in every period prior to the shock when consumers build up inventories. Thus, in sum, our suggested policy is particularly effective in the case of an unlikely, but potentially sizable, negative supply shock that is long anticipated.

7 Discussion

In this section, we review some of our key modeling assumptions and briefly sketch several model extensions to rationalize them. Further details on these extensions can be obtained from the authors upon request.

Fixed prices. First, consider our assumption that the suppliers of the good keep their prices fixed during $t = 0, ..., T$ in spite of the anticipated negative supply shock. Clearly, if prices were fully flexible, then they could simply increase their prices in period T until there is no excess demand in this period. And if consumers start stockpiling the good in some periods $t < T$ in anticipation of the price increases, then the suppliers may also raise their prices in these periods until each period's market clears.

However, the empirical facts point in a different direction. During all the crises investigated in the empirical studies reviewed in our introduction, the prices of a large range of consumer products remained very much on the same level despite the large swings in demands (Cavallo et al., 2014; Gagnon and López-Salido, 2020; Hansman et al., 2020). This can be explained in different ways. One possibility is direct price controls. Governments may prohibit price increases during times of crises to avoid exploitation of consumers. Another explanation is based directly on suppliers' incentives. They may fear a loss in customer goodwill, and it is easily imaginable that the resulting losses in future demand weigh more heavily than the temporary profit increases due to a higher price during the crisis.

Limited capacities. Next, consider our assumption that the suppliers have just enough capacity to serve the per-period demand without the shock. Without that assumption, any additional demand during $t = 0, ..., T$ could be matched with the suppliers' inventories so that no "panic buying" episode would arise in our model.[15]

To motivate our assumption, we developed an extension of our basic model where in the beginning of the game the suppliers first build up capacities, not anticipating the arrival of the shock. In this extension, the suppliers install a capacity that is, on aggregate, just large enough to serve the constant per-period demand during these "normal times". At the same time, the market price is just high enough for suppliers to recoup their investment costs over infinitely many time periods. Now, the news of the supply shock arrives in period $t = 0$ that, when it occurs, reduces the suppliers' capacities in period T by the same amount $1-s$. To prepare for the shock, the suppliers could then build additional capacities in period $t = 0$. However, given that the market price is fixed at the pre-crisis level, the suppliers would not be able to recoup their *additional* investment costs. The reason is that the market price was just high enough to recoup the costs of an investment over infinitely many periods. Yet, as the additional capacities would remain idle after the crisis is over, their costs could be recouped over the limited number of time periods during the crisis.

Marginal shopping costs. Now, consider our assumption that shopping costs are marginally small during all time periods $t = 0, ..., T$. Our justification is that for many goods, shopping costs are indeed negligible. For example, consumers typically combine their purchases of specific goods (such as toilet paper) with the purchases of other goods; or they combine their shop visits with other trips, such as the return trip from work. All this makes the average shopping cost per product appear negligible compared to the sometimes substantial storage costs for certain product types, such as fruits or vegetables. For this reason, we neglect shopping costs altogether in our model and focus on the more

[15] In fact, the empirical pattern in Figure 1 suggests that supermarkets had at least some goods on stock during the first weeks of the Corona pandemic.

important storage costs.[16]

Rational consumers. Finally, we motivate our assumption of rational consumers that correctly anticipate the arrival of the shock already in $t = 0$. Our main motivation is the remarkable feature of our model that even under these conditions, it predicts a pattern of demand that has much in common with the "panic buying" episodes observed in real markets. Our model is thus the first to provide an economic explanation to a phenomenon that was so far only approached in psychology (e.g. Baddeley, 2020; Bentall et al., 2021).

Nevertheless, we do not want to deny that anxiety, fear, or stress play an important role in consumer decision-making, particularly in crises. Yet, it turns out that the results of our simple model are also robust with regard to an extension in this direction. In particular, we developed an extension where a certain exogenous share of consumers does not shop rationally in any period $t < T$ but instead imitates the shopping behavior of the majority of other consumers in the same period. More concretely, while lining up in front of a shop, these imitators observe the shopping baskets of all the shoppers already leaving a store. An imitator then decides to buy two units if the share of shoppers with two units in their baskets (other imitators and rational consumers included) is strictly larger than 1/2. Otherwise, an imitator just buys for immediate consumption.

The most remarkable result of this extension is that its equilibrium predictions are just the same as in our basic model where all consumers are fully rational. This means more concretely that the equilibrium values of all the important variables (i.e., the aggregate inventories h_t and the rejection probabilities σ_t) as well as all the conditions leading to these values are just the same. The only requirement attached to this result is that the share of rational consumers surpasses a certain threshold, which is 1/2 in our specific case.

Even though the critical share might be different depending on the specific model setup, in particular the decision rules of the imitators, our extension thus makes clear an important point: The predictions of our simple model go through even though a substantive share of consumers dos not choose their optimal consumption basket, but is instead guided by emotions like anxiety, fear, or stress that makes them imitate the behavior of others.

[16] As explained in Section 4, the only role for a positive shopping cost ε in our model is to serve as a tie-breaker when several types of consumers (with/without a unit of the good in their inventory) would otherwise be indifferent between going to a shop or not. The marginal cost helps us to break the indifference tie for each group of consumers at a time.

8 Conclusion

We analyze a simple dynamic model of a consumer market where an expected negative supply shock triggers a potential cascade of excess demands that sets in well before the shock is expected to arrive. Excess demand in periods prior to the shock is due to consumers' stockpiling purchases as consumers try to prepare for the shock. This drives up the demands and, thus, shifts the risk of non-consumption from the period of the shock to the periods before. We refer to this pattern as "panic buying".

Panic buying can result in substantial welfare losses on top of the direct welfare costs of the supply shock. First of all, stockpiling entails storage costs for consumers. These costs are socially wasteful because stockpiling does not help to produce any of the extra units that would be needed to undo the supply shortage. Second, when the actual supply shock is smaller than expected, more consumers may hold inventories than what is needed for the shock. However, because these consumers competed with other consumers for scarce supplies in the periods prior to the shock, the latter may suffer severe consumption losses. We show that these consumption losses may even be larger than the losses triggered by the shock itself.

Finally, we use the insights from our model to develop a suitable, low-cost supply policy that can nevertheless eliminate the entire welfare costs of stockpiling. What is important for this policy to work is that the government credibly commits to this policy early on, when consumers hear about the shock for the first time, so that they can adjust their consumption plans accordingly. Less important is whether the government neutralizes the negative supply shock altogether when it arrives. All that must be achieved is that the risk of a consumption outage becomes sufficiently small such that consumers do not see a reason to build up stocks.

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Appendix

Proof of Lemma 2

We prove that for any given $0 \leq h_t < 1$ and $0 < \sigma_t \leq \frac{1}{2}$, h_{t+1} is bounded from below by $\underline{h}(h_t, \sigma_t) = h_t + (1 - h_t)\sigma_t$. In order to determine this lower bound, we need to make a case distinction:

Suppose first that $h_t < 1/2$. Throughout the remainder of the appendix, let us refer to the group of consumers who do not have a unit on storage at the beginning of a period t as the "nh-consumers", and the consumers who do have a unit on storage as the "h-consumers".

Then, a lower bound for h_{t+1} is determined by the situation where only the "nh-consumers" (mass $1 - h_t$) go shopping in period t and sufficiently many of them buy two units so that $0 < \sigma_t \leq \frac{1}{2}$ holds.[17] Hence, in this situation, the mass of shoppers is equal to the mass of nh-consumers, that is, $x_t + y_t + z_t = 1 - h_t$. Moreover, because only the nh-consumers store for the next period, $h_{t+1} = y_t$. Finally, because $z_t > 0$, capacity constraint (2) is binding, that is, $x_t + 2y_t = 1$. Together, this implies that $z_t = h_{t+1} - h_t$ and thus

$$\sigma_t = \frac{z_t}{x_t + y_t + z_t} = \frac{h_{t+1} - h_t}{1 - h_t}\,.$$

Rewriting this identity leads to our lower bound for h_{t+1}:

$$\underline{h}(h_t, \sigma_t) = (1 - h_t)\sigma_t + h_t \qquad \text{if} \quad 0 \leq h_t < 1/2 \quad \text{and} \quad 0 < \sigma_t \leq \frac{1}{2}\,. \tag{A.1}$$

Suppose next that $h_t \geq 1/2$. Then, $\sigma_t > 0$ requires that at least some h-consumers go shopping as well, in addition to all the nh-consumers. Capacity constraint (2) is thus binding ($x_t + 2y_t = 1$) and the mass of consumers going to a shop ($x_t + y_t + z_t$) satisfies $1 - h_t < x_t + y_t + z_t \leq 1$. Furthermore, because all the nh-consumers attempt to buy two units (see explanation in footnote 17), the mass of consumers with a unit on store at the beginning of period $t + 1$ is given by $h_{t+1} = x_t + y_t$.

We now calculate the probability of rejection in period t. As the mass of consumers sums up to one, we start with the identity

$$x_t + y_t + z_t + (h_t - x_t - \frac{x_t}{x_t + y_t} \cdot z_t) = 1.$$

[17] When $h_t < 1/2$ and the h-consumers go shopping as well, h_{t+1} will be (weakly) larger than when only the nh-consumers go shopping. This is because the h-consumers only go to a shop when the net benefit of attempting to buy a second unit exceeds the (small) shopping cost ϵ. However, when this condition is met, then all the nh-consumers will attempt to buy two units because these consumers go to a shop anyhow to satisfy their immediate demand. Hence, for our lower bound for h_{t+1}, consider a situation where only the nh-consumers go shopping.

The first three summands are self-explanatory: x_t is the mass of h-consumers and y_t is the mass of nh-consumers who are able to enter a shop. The third summand, z_t, denotes the rejected consumers. The final summand shows the h-consumers staying at home. To determine this mass, first note that no nh-consumer stays at home. Next, subtract the consumers who buy one unit, x_t, from the h-consumers, h_t, because only h-consumers buy one unit. Finally, subtract the h-consumers who are rejected. More specifically, because all nh-consumers attempt to buy two units, the share of h-consumers in the total mass of rejected consumers, z_t, is proportional to the share of x_t in the total mass of shoppers who succeed to enter a shop, $x_t + y_t$.

Combining the above identities leads to the following expressions:

$$\begin{aligned} x_t &= 2h_{t+1} - 1, \\ y_t &= 1 - h_{t+1}, \\ z_t &= \frac{(h_{t+1} - h_t)h_{t+1}}{1 - h_{t+1}}, \end{aligned}$$

and thus

$$\sigma_t = \frac{z_t}{x_t + y_t + z_t} = \frac{z_t}{h_{t+1} + z_t} = \frac{h_{t+1} - h_t}{1 - h_t}. \tag{A.2}$$

Rewriting this identity gives our lower bound for h_{t+1}, again:

$$\underline{h}(h_t, \sigma_t) = h_t + (1 - h_t)\sigma_t \qquad \text{if} \quad 1/2 \leq h_t < 1 \quad \text{and} \quad 0 < \sigma_t \leq \frac{1}{3}.$$

Q.E.D.

Proof of Proposition 1

We first prove the claim for T. Suppose that, contrary to the statement, sufficiently many consumers have a unit on store at the beginning of period T so that no one faces the risk of rejection for no s in the support of $F(s)$, that is, suppose that $h_T \geq 1 - \underline{s}$ so that $\mathbb{E}_s[\sigma_T] = 0$. By Lemma 1, this means that no consumer would see a reason to stockpile for period T, i.e., $h_T = 0$, and that, therefore, the demand in period T must be met by the current supply. Yet, this is obviously a contradiction to any $s \in S$ with $s < 1$.

We next prove the claim for $t > T$. Suppose, to the contrary, that some consumers buy on storage in some subgame starting in period $t \geq T$. It follows from consumers' optimal purchasing decisions (see inequality (4)) that the reason for this must be that $\sigma_{t+1} \geq \frac{c}{\delta}$. In fact, successive application of this argument leads to the conclusion that $\sigma_\tau \geq \frac{c}{\delta}$ must hold for all $\tau > t$ because, by Lemma 1, $\sigma_{\tau'} < \frac{c}{\delta}$ for some $\tau' > t$ would imply that $\sigma_\tau = 0$ for all $t < \tau < \tau'$. Hence, demand must exceed supply forever after t when $\sigma_{t+1} \geq \frac{c}{\delta}$.

It remains to be seen that such a never-ending period of excess demand is inconsistent with itself. The reason lies in Lemma 2, according to which $\sigma_\tau \geq \frac{c}{\delta}$ for all $\tau > t$ implies a monotonically increasing sequence of stocks, $(h_\tau)_{\tau \geq t}$. In fact, it holds at any step of this sequence that $h_{\tau+1} - h_\tau \geq (1 - h_\tau)\sigma_t$. Together with $\sigma_\tau \geq \frac{c}{\delta}$, this means that the sequence converges to one. As the mass of inventory holders increases, we must however also have that $\lim_{\tau \to \infty} y_\tau = 0$. Yet, by the definition of the rejection probability (3), this means that there must be a $\tau' > t$ with $\sigma_{\tau'} < \delta^{-1}c$ and, thus, there must be an end to the sequence of excess demands. Combined with the first argument, this leads us to our initial statement that *no* consumer will stockpile in any subgame starting at $t \geq T$. By the same argument, stockpiling will only occur in finitely many periods before T. Q.E.D.

Proof of Proposition 2

We will demonstrate that (I) for any period number T, there is an upper bound $\bar{\bar{s}}$ for the maximum supply $\bar{s}$ in the support of $F(s)$ and an upper bound $\bar{c}$ for the storage cost c such that for any parameter constellation with $\bar{s} < \bar{\bar{s}}$ and $c < \bar{c}$, all consumers attempt to (re-)fill their storage in every period $t < T$ (panic buying). Subsequently, we will show that (II) when the lowest $\underline{s}$ in the support of $F(s)$ is larger than some $\underline{\underline{s}}$, then the rejection probability in some period $t < T$ is even larger than the expected rejection probability in period T itself, that is, $\sigma_t > \mathbb{E}_s[\sigma_T]$ for some $t < T$.

Part (I). By inequality (4), all consumers try to stockpile from $t = 0$ onward when $c < \delta \cdot \sigma_t$ for all $t < T$ and $c < \delta \cdot \mathbb{E}_s[\sigma_T]$ for $t = T$. Since this means that capacity constraint (2) is binding in all periods, we get $x_t + 2y_t = 1$ for every $t \leq T$. Moreover, as everybody goes to a shop, it additionally holds $x_t + y_t + z_t = 1$ in every $t \leq T$. Finally, the mass of consumers entering period $t+1$ with a unit on store is equal to the mass of consumers successfully entering a shop in period t, that is $h_{t+1} = x_t + y_t$.

Random rationing, in turn, implies that the ratio between the masses of shoppers who succeed to buy one and two units respectively is identical to the ratio between the masses of consumers entering the period with and without a unit on store,

$$\frac{x_t}{y_t} = \frac{h_t}{1 - h_t}.$$

Altogether, this gives

$$
\begin{aligned}
x_t &= \frac{h_t}{2-h_t} \\
y_t &= z_t = \sigma_t = \frac{1-h_t}{2-h_t} \qquad \text{(A.3)} \\
h_{t+1} &= \frac{1}{2-h_t}\,.
\end{aligned}
$$

Moreover, inserting these values into (3) gives a rejection probability in period T of

$$\sigma_T = \frac{1-h_T-s}{1-h_T}, \qquad \text{(A.4)}$$

for any realization of s with $s < 1-h_T$, and $\sigma_T = 0$ for any $s \geq 1-h_T$.

As the sequence of stockpiling purchases starts at $h_0 = 0$, we can deduct all remaining values for h_t, which become

$$h_t = \frac{t}{t+1}\,. \qquad \text{(A.5)}$$

Inserting (A.5) into (A.3) and (A.4), finally yields for all $t < T$

$$
\begin{aligned}
\sigma_t &= \frac{1}{t+2} \\
\mathbb{E}_s[\sigma_T] &= \left(1 - \frac{1}{1-h_T}\cdot \mathbb{E}\big[s\,|\,s<1-h_T\big]\right)\cdot Prob\big(s<1-h_T\big) \qquad \text{(A.6)} \\
&= \left(1-(T+1)\cdot \mathbb{E}\big[s\,|\,s<\frac{1}{T+1}\big]\right)\cdot Prob\big(s<\frac{1}{T+1}\big)\,.
\end{aligned}
$$

For such a sequence of persistent stockpiling purchases to exist, we just need to check that the condition $c < \delta\cdot\sigma_t$ is satisfied for the smallest value of σ_t in this sequence. By (A.6), this is at $t = T-1$ because σ_t is declining in t. This yields an upper bound for the storage cost:

$$c < \frac{\delta}{T+1} \equiv \bar{c}\,. \qquad \text{(A.7)}$$

An upper bound for $\bar{s}$ follows from the other condition, $c < \delta\cdot\mathbb{E}_s[\sigma_T]$. Suppose that

$$\bar{s} < \frac{T}{(T+1)^2} \equiv \bar{\bar{s}}\,. \qquad \text{(A.8)}$$

Then, (A.8) implies that $Prob(s < \frac{1}{T+1}) = 1$. This in turn implies that $\mathbb{E}_s[\sigma_T] = 1-(T+1)\cdot\mathbb{E}[s]$. Thus, $c < \delta\cdot\mathbb{E}_s[\sigma_T]$ is equivalent to

$$\mathbb{E}[s] < \frac{1-\frac{c}{\delta}}{T+1}. \qquad \text{(A.9)}$$

Now, because $\mathbb{E}[s] \le \bar{s}$ and $c < \delta/(T+1)$, condition (A.9) is certainly satisfied when

$$\bar{s} < \frac{1 - \frac{\delta}{T+1}\delta^{-1}}{T+1} = \frac{T}{(T+1)^2}$$

and, hence, when $\bar{s}$ has the upper bound in (A.8).

Part (II). Suppose the conditions (A.7) and (A.8) are satisfied. Then, we have

$$\sigma_0 > \mathbb{E}_s[\sigma_T] \;\Leftrightarrow\; \mathbb{E}[s] > \frac{1}{2(T+1)}\,.$$

Since $\bar{s} \ge E[s] \ge \underline{s}$, the outcome $\sigma_0 > \mathbb{E}_s[\sigma_T]$ is compatible with conditions (A.7) and (A.8) if and only if $T > 1$ and

$$\frac{1}{2(T+1)} \;<\; \underline{s} \;\le\; \bar{s} \;<\; \frac{T}{(T+1)^2}\,.$$

Hence, when $\underline{s}$ and $\bar{s}$ satisfy this constraint, we have $\sigma_t > \mathbb{E}_s[\sigma_T]$ for some $t < T$. Q.E.D.

Example

Here, we present the complete characterization of the equilibrium in periods $T-1$ and $T-2$ of the example in Section 4.

Throughout the following, we denote the group of consumers who do *not have* a unit on storage at the beginning of some period $t \in \{T-2, T-1, T\}$ as the "nh-consumers", and the consumers who *do have* a unit on storage as the "h-consumers".

Equilibrium in $T-1$:

Solution type A. Here, the supply shock is so small that all h-consumers stay at home in $T-1$. Moreover, the n-th-consumers buy one unit for immediate consumption, so nobody is prepared for the shock.

By Lemma 1, this means that it must be $\delta \cdot \mathbb{E}_s[\sigma_T] < c$. Because nobody stockpiles, we get $y_{T-1} = h_T = 0$. Furthermore, because the demand is smaller than or equal to (if $h_{T-1} = 0$) one, we have $z_{T-1} = 0$. Combined with the identity

$$x_{T-1} + y_{T-1} + z_{T-1} + h_{T-1} \;\;=\;\; 1,$$

we thus get

$$\begin{aligned} x_{T-1} &= 1 - h_{T-1}, \\ \sigma_{T-1} &= 0, \\ \mathbb{E}_s[\sigma_T] &= \theta \cdot (1 - \underline{s}). \end{aligned}$$

Extending on the inequality $\delta \cdot \mathbb{E}_s[\sigma_T] < c$ and expression (5), this solution type thus emerges in equilibrium if

$$\underline{s} > \frac{\delta\theta - c}{\delta\theta}. \tag{A.10}$$

Solution type B. Here, the supply shock is still small, so only the nh-consumers prepare for it. They are indifferent between buying just for consumption and another unit in stock. No consumer is rejected from the shops.

Because only the nh-consumers prepare for the shock (more precisely those who buy two units), we get $h_T = y_{T-1}$. Combined with the identity

$$x_{T-1} + y_{T-1} + z_{T-1} + h_{T-1} = 1\,,$$

all remaining variables follow (see below). Furthermore, because nobody is rejected, h_{T-1} must be sufficiently large so that (a) $x_{T-1} \geq 0$, (b) $y_{T-1} \geq 0$, (c) $z_{T-1} = 0$, and (d) capacity limit (2) is not binding.

Expression (7) and $z_{T-1} = 0$ lead to the following equilibrium expressions:

$$\begin{aligned} H = h_T &= \frac{\delta\theta(1 - \underline{s}) - c}{\delta\theta - c}, \\ x_{T-1} &= 1 - H - h_{T-1}, \\ y_{T-1} &= h_T = H, \\ z_{T-1} &= \sigma_{T-1} = 0, \\ \mathbb{E}_s[\sigma_T] &= \frac{c}{\delta}. \end{aligned}$$

The boundary conditions (a) and (b) (see above) require that

$$\frac{\delta\theta - c}{\delta\theta} \cdot h_{T-1} \leq \underline{s} \leq \frac{\delta\theta - c}{\delta\theta}.$$

Furthermore, condition (c) requires that $h_{T-1} \geq h_T = H$. This can be written as

$$\underline{s} \geq \frac{\delta\theta - c}{\delta\theta} \cdot (1 - h_{T-1}).$$

Now, because the game starts with $h_{T-2} = 0$ and because at most half of all consumers can purchase on stock in period $T-2$, we have $h_{T-1} \leq 1/2$. It thus follows that $1 - h_{T-1} \geq h_{T-1}$. Hence, the lower bound for $\underline{s}$ that is relevant for this solution type is $\frac{\delta\theta - c}{\delta\theta} \cdot (1 - h_{T-1})$, and not $\frac{\delta\theta - c}{\delta\theta} \cdot h_{T-1}$. Therefore, a Solution type B emerges in equilibrium if

$$\frac{\delta\theta - c}{\delta\theta} \cdot (1 - h_{T-1}) \leq \underline{s} \leq \frac{\delta\theta - c}{\delta\theta}. \tag{A.11}$$

Solution type C. This solution is similar to solution type B except that now, h_{T-1} is so small that (a) $x_{T-1} \geq 0$ and (b) $y_{T-1} > 0$ and (c) $z_{T-1} > 0$.

Capacity constraint (2) is thus binding, that is, $x_{T-1} + 2H = 1$. Moreover, based on the identities for solution type B, we get

$$\begin{aligned}
x_{T-1} &= 1 - 2h_T = 1 - 2H, \\
y_{T-1} &= h_T = H, \\
z_{T-1} &= h_T - h_{T-1} = H - h_{T-1}, \\
\sigma_{T-1} &= \frac{z_{T-1}}{1 - h_{T-1}} = \frac{H - h_{T-1}}{1 - h_{T-1}}, \\
\mathbb{E}_s[\sigma_T] &= \frac{c}{\delta}.
\end{aligned}$$

This solution type requires the following parameter conditions: Boundary conditions (a) and (b) are equivalent to

$$\frac{\delta\theta - c}{2\delta\theta} \leq \underline{s} < \frac{\delta\theta - c}{\delta\theta}.$$

Moreover, while an equilibrium of type B required that $z_{T-1} = 0$ and, thus, that $h_{T-1} \geq h_T = H$, the converse condition is required for solution type C:

$$h_{T-1} < h_T = H.$$

As we saw earlier, this can be rewritten as

$$\underline{s} < \frac{\delta\theta - c}{\delta\theta}(1 - h_{T-1}).$$

Since $1 - h_{T-1} \leq 1$, the above three conditions combined thus yield the following condition for this solution type:

$$\frac{\delta\theta - c}{2\delta\theta} \leq \underline{s} < \frac{\delta\theta - c}{\delta\theta} \cdot (1 - h_{T-1}). \tag{A.12}$$

Solution type D. Here, all h-consumers (i.e., those who enter period $T-1$ with a unit) are indifferent between purchasing another unit for stock and staying at home. As a result, all nh-consumers attempt to buy two units because they do not incorporate the shopping cost.

Since both h-consumers and nh-consumers go shopping, we get

$$x_{T-1} + y_{T-1} = h_T.$$

Moreover, some consumers are necessarily rejected, i.e., $z_{T-1} > 0$, due to the competition between consumers.

We now calculate the share z_{T-1}: As the mass of consumers sums up to one, we start from the identity

$$x_{T-1} + y_{T-1} + z_{T-1} + (h_{T-1} - x_{T-1} - \frac{x_{T-1}}{x_{T-1} + y_{T-1}} \cdot z_{T-1}) = 1.$$

The first three summands are self-explanatory: x_{T-1} is the mass of h-consumers resp. y_{T-1} the mass of nh-consumers who are able to enter a shop in period $T-1$. The third summand, z_{T-1}, in turn, is the total mass of rejected consumers. The most interesting term is the final summand, representing the mass of h-consumers staying at home. To determine this number, first note that no n-th-consumer stays at home. Next, subtract the mass of consumers who buy one unit, x_{T-1}, from the mass of h-consumers, h_{T-1}, because only h-consumers buy one unit. Finally, subtract the mass of h-consumers who are rejected. More specifically, because all nh-consumers attempt to buy two units, the share of h-consumers in the total mass of rejected consumers, z_{T-1}, is proportional to the share of x_{T-1} in the total mass of shoppers who are able to enter a shop, $x_{T-1} + y_{T-1}$.

Combining the above identities with the binding capacity constraint (2), we arrive at the following expressions:

$$\begin{aligned}
x_{T-1} &= 2H - 1, \\
y_{T-1} &= 1 - H, \\
z_{T-1} &= \frac{(h_T - h_{T-1}) \cdot h_T}{1 - h_T} = \frac{(H - h_{T-1})H}{1 - H}, \\
\mathbb{E}_s[\sigma_T] &= h_T = H,
\end{aligned}$$

and

$$\sigma_{T-1} = \frac{z_{T-1}}{x_{T-1} + y_{T-1} + z_{T-1}} = \frac{z_{T-1}}{h_T + z_{T-1}} = \frac{H - h_{T-1}}{1 - h_{T-1}}.$$

This solution type emerges in equilibrium if (a) $x_{T-1} \geq 0$ and (b) $y_{T-1} \geq 0$ and (c) $(h_{T-1} - x_{T-1} - \frac{x_{T-1}}{x_{T-1}+y_{T-1}} z_{T-1}) \geq 0$ hold true.[18] Constraint (a) leads to $\underline{s} \leq \frac{\delta\theta - c}{2\delta\theta}$ and constraint (b) to

[18] An additional constraint $z_{T-1} \geq 0$ (as in the first interior solution) is not needed here because it is always satisfied.

$\underline{s} \geq 0$. Constraint (c) demands that the mass of h-consumers staying at home is non-negative. This leads to the following lower bound for $\underline{s}$:

$$\underline{s} \geq \frac{\delta\theta - c}{\delta\theta} \cdot \frac{1 - h_{T-1}}{2 - h_{T-1}}.$$

This latter condition is stronger than constraint (b). Solution type D, thus, emerges in equilibrium if

$$\frac{\delta\theta - c}{\delta\theta} \cdot \frac{1 - h_{T-1}}{2 - h_{T-1}} \leq \underline{s} \leq \frac{\delta\theta - c}{2\delta\theta}\,. \tag{A.13}$$

Solution type E. Here, every consumer tries to store a unit for period T because $c < \delta \cdot \mathbb{E}_s[\sigma_T]$.

A more general version of this solution type has already been analyzed in the proof of Proposition 2. There, we derived the equilibrium expressions for an entire sequence of solution types E from period $t = 0$ to period T. Based on the terms presented there, we get

$$x_{T-1} = \frac{h_{T-1}}{2 - h_{T-1}},$$

$$y_{T-1} = z_{T-1} = \sigma_{T-1} = \frac{1 - h_{T-1}}{2 - h_{T-1}},$$

$$h_T = \frac{1}{2 - h_{T-1}},$$

$$\mathbb{E}_s[\sigma_T] = \theta\,\frac{1 - h_T - \underline{s}}{1 - h_T} = \theta\left(1 - \frac{2 - h_{T-1}}{1 - h_{T-1}}\underline{s}\right).$$

Recall that $c < \delta \cdot \mathbb{E}_s[\sigma_T]$ is a necessary condition for a solution of type E to occur in $T - 1$. This results in the following upper bound for $\underline{s}$:

$$\underline{s} < \frac{\delta\theta - c}{\delta\theta}\frac{1 - h_{T-1}}{2 - h_{T-1}}\,. \tag{A.14}$$

Equilibrium in $T - 2$:

By extension of the arguments for period $T - 1$, the same five solution types may emerge in period $T - 2$ depending on the expected benefits, $\delta \cdot \sigma_{T-1}$, and costs, c, of stockpiling. The only additional complication lies in the fact that *different* solution types may co-exist in periods $T - 2$ and $T - 1$.

Nevertheless, certain combinations of solutions can be ruled out ex-ante:

1. There cannot exist an equilibrium with a Solution type B in period $T - 1$ because this would require that $\sigma_{T-1} = 0$ and $h_{T-1} > 0$ (see our results for $T - 1$). Yet, by Lemma 1, this is impossible when the game starts in $T - 2$.

2. Solution type A in period $T-1$ can only be combined with Solution type A in $T-2$. The reason lies, again, in Lemma 1.

3. Solution type A in period $T-2$ rules out Solution type D in period $T-1$. This is because a solution of type A requires that $h_{T-1}=0$, but Solution D is just valid for $h_{T-1}>0$.

4. Solution types C and D in period $T-1$ isomorphic because σ_{T-1} is identical in both cases so that also the benefits from stockpiling in period $T-2$ are the same. Therefore, we will merge solutions C and D into a joint type C/D.

For the sake of clarity, let us denote the feasible solution types for period $T-2$ by A′, C′, and E′, while we continue to use A–E for the feasible solutions for period $T-1$. The following solution combinations then exist:

Solution type A′–A. Here, the supply shock is so small that nobody stockpiles in periods $T-2$ and $T-1$.

We therefore get $x_{T-2}=1$ and $y_{T-2}=z_{T-2}=h_{T-1}=\sigma_{T-1}=0$. Furthermore, from the equilibrium expressions for $T-1$, we get $x_{T-1}=1$ and $y_{T-1}=0$. The rejection probabilities in this solution type are thus given by

$$\sigma_{T-2}=\sigma_{T-1}=0, \text{ and } \mathbb{E}_s[\sigma_T]=\theta\cdot(1-\underline{s}).$$

For this solution to indeed arise in period $T-1$, it must hold that $\delta\cdot\mathbb{E}_s[\sigma_T]<c$. Hence, inequality (A.10) must be satisfied, that is,

$$\underline{s}>\frac{\delta\theta-c}{\delta\theta}.$$

The respective condition for period $T-2$, $\delta\cdot\sigma_{T-1}<c$, is automatically satisfied by Lemma 1.

Solution type A′–C. Here again, nobody stockpiles in period $T-2$.

This means that $\sigma_{T-2}=0$. Moreover, by Lemma 1, it must hold that $\sigma_{T-1}<\frac{c}{\delta}$ for a solution type A′ in period $T-2$, whereby $\sigma_{T-1}=\frac{H-h_{T-1}}{1-h_{T-1}}=H$ since $h_{T-1}=0$. Hence, using (7), we obtain the condition $\frac{1-\frac{c}{\delta\theta}-\underline{s}}{1-\frac{c}{\delta\theta}}<\frac{c}{\delta}$. This can be rewritten as $\underline{s}>\frac{(\delta\theta-c)(\delta-c)}{\delta^2\theta}$. Furthermore, for solution type C to indeed emerge in period $T-1$, it must be that $\frac{\delta\theta-c}{2\delta\theta}\leq\underline{s}\leq\frac{\delta\theta-c}{\delta\theta}$ (see A.12), where we have used $h_{T-1}=0$. Hence, in sum, a solution type A′–C arises in equilibrium if

$$\frac{\delta\theta-c}{2\delta\theta}\leq\underline{s}\leq\frac{\delta\theta-c}{\delta\theta}\text{ and }\underline{s}>\frac{(\delta\theta-c)(\delta-c)}{\delta^2\theta}.$$

Notice that the first inequality is weaker than the third inequality if and only if $c<\frac{\delta}{2}$. Hence, we can rewrite the above conditions as follows:

$$\frac{(\delta\theta - c)(\delta - c)}{\delta^2\theta} < \underline{s} \leq \frac{\delta\theta - c}{\delta\theta} \quad \text{if} \quad c < \frac{\delta}{2}, \quad \text{and}$$

$$\frac{\delta\theta - c}{2\delta\theta} \leq \underline{s} \leq \frac{\delta\theta - c}{\delta\theta} \quad \text{if} \quad c \geq \frac{\delta}{2}.$$

Finally, as can be easily confirmed, the rejection probabilities are given by

$$\sigma_{T-2} = 0, \;\; \sigma_{T-1} = H, \;\; \mathbb{E}_s[\sigma_T] = \frac{c}{\delta},$$

with $\sigma_{T-1} < \sigma_T$.

Solution type A′–E. Here again, nobody stockpiles in period $T-2$ so that we have $\delta\sigma_{T-1} < c$ and $y_{T-2} = z_{T-2} = h_{T-1} = 0$.

This solution type emerges in equilibrium if $\delta\sigma_{T-1} < c$, giving $c > \delta/2$. Furthermore, by (A.14), it must be $\underline{s} < \frac{\delta\theta - c}{2\delta\theta}$. Hence, we get the following conditions:

$$\underline{s} < \frac{\delta\theta - c}{2\delta\theta} \qquad \text{and} \qquad c > \frac{\delta}{2}.$$

Extending on the expression $\sigma_{T-1} = (1 - h_{T-1})/(2 - h_{T-1})$ (see solution type E above), we finally get the following rejection probabilities:

$$\sigma_{T-2} = 0, \;\; \sigma_{T-1} = \frac{1}{2}, \;\; \mathbb{E}_s[\sigma_T] = \theta \cdot \left(1 - \frac{\underline{s}}{2}\right),$$

whereby $\sigma_{T-1} < \mathbb{E}_s[\sigma_T]$.

Solution type C′–C/D. A solution of type C′ in period $T-2$ requires that the following indifference condition is satisfied: $\frac{c}{\delta} = \sigma_{T-1}$. For both solution types C and D in period $T-1$, we require that $\sigma_{T-1} = \frac{H - h_{T-1}}{1 - h_{T-1}}$ (see solution type C above), where H is given in (7).

Solving the indifference condition for h_{T-1}, we get

$$h_{T-1} = \frac{H - \frac{c}{\delta}}{1 - \frac{c}{\delta}} = \frac{(\delta\theta - c)(\delta - c) - \underline{s}\delta^2\theta}{(\delta\theta - c)(\delta - c)} \equiv H',$$

where we define H' for notational convenience (similar to the combined parameter H). Next, using the same steps we used in the derivation of solution type C, we obtain for type C′ in period $T-2$:

$$x_{T-2} = 1 - 2H' \;,\;\; y_{T-2} = z_{T-2} = H'.$$

Such a solution emerges in period $T-2$ if $x_{T-2} \geq 0$ and $y_{T-2} \geq 0$. This yields $0 \leq H' \leq \frac{1}{2}$, or equivalently

$$\frac{(\delta\theta - c)(\delta - c)}{2\delta^2\theta} \leq \underline{s} \leq \frac{(\delta\theta - c)(\delta - c)}{\delta^2\theta}.$$

However, we must also verify that solution type C resp. D indeed arises in $T-1$. First, consider a solution type C. Using (A.12) and $h_{T-1} = H'$, this requires that

$$\frac{\delta\theta - c}{2\delta\theta} \leq \underline{s} < \frac{\delta\underline{s}}{\delta - c}.$$

But the right inequality is always satisfied, leaving us with the condition

$$s \geq \frac{\delta\theta - c}{2\delta\theta}.$$

Now, consider a solution type D. Using (A.13) and again $h_{T-1} = H'$, this is a valid solution if

$$\frac{\underline{s}\delta(\delta\theta - c)}{(\delta\theta - c)(\delta - c) + \underline{s}\delta^2\theta} \leq \underline{s} \leq \frac{\delta\theta - c}{2\delta\theta}.$$

The left inequality simplifies to $\underline{s} \geq \frac{c(\delta - c)}{\delta^2\theta}$ and, thus, solution type D solves period $T-1$ if

$$\frac{c(\delta - c)}{\delta^2\theta} \leq \underline{s} \leq \frac{\delta\theta - c}{2\delta\theta}.$$

Recall that each of these conditions (for solution type C and type D) for period $T-1$ must hold simultaneously with our above condition for C′. This pins down the overall parameter range that is necessary for the solution types C′–C or C′–D.

The rejection probabilities under both these solution types are given by

$$\sigma_{T-2} = H', \ \sigma_{T-1} = \mathbb{E}_s[\sigma_T] = \frac{c}{\delta}.$$

Solution type C′–E. Here, we again have the indifference condition $\frac{c}{\delta} = \sigma_{T-1}$ in period $T-2$, but now it is $\sigma_{T-1} = \frac{1-h_{T-1}}{2-h_{T-1}}$ (see solution type E above) with $h_{T-1} = y_{T-2}$ (see (8)).

Solving the indifference condition for h_{T-1}, we get

$$h_{T-1} = \frac{\delta - 2c}{\delta - c}.$$

Using similar steps as in our derivation of a solution of type C in period $T-1$, this yields

$$x_{T-2} = \frac{3c - \delta}{\delta - c}\ , \ y_{T-2} = z_{T-2} = \frac{\delta - 2c}{\delta - c}.$$

Solution type C′ emerges in period $T-2$ if $x_{T-2} \geq 0$ and $y_{T-2} \geq 0$, yielding

$$\frac{\delta}{3} \leq c \leq \frac{\delta}{2}.$$

Moreover, a solution type E emerges in period $T-1$ when condition (A.14) is satisfied in addition. Using $h_{T-1} = \frac{1-2\frac{c}{\delta}}{1-\frac{c}{\delta}}$, this yields the condition

$$\underline{s} < \frac{c(\delta\theta - c)}{\delta^2\theta}.$$

The rejection probabilities are finally given by

$$\sigma_{T-2} = h_{T-1} = \frac{\delta - 2c}{\delta - c},\ \sigma_{T-1} = \frac{c}{\delta},\ \text{and },\ \mathbb{E}_s[\sigma_T] = \theta\frac{c - \delta\underline{s}}{c}.$$

Solution type E′–C/D. For solution type E′ to emerge in period $T-2$, it must hold that $\frac{c}{\delta} < \sigma_{T-1}$.

Using the expression for σ_{T-1} from solution type C (see above), this yields the condition $\frac{c}{\delta} < \frac{H-h_{T-1}}{1-h_{T-1}}$. Now, because all consumers would like to buy on storage in period $T-2$, only half of them can enter a shop so that $h_{T-1} = y_{T-2} = \sigma_{T-2} = \frac{1}{2}$, while $x_{T-2} = 0$. Applying $h_{T-1} = \frac{1}{2}$ and H (from (7)) to the above condition for E′ and rearranging the term, yields the condition

$$\underline{s} < \frac{(\delta - c)(\delta\theta - c)}{2\delta^2\theta}.$$

Furthermore, for solution type C to emerge in $T-1$, condition (A.12) must be satisfied. However, with $h_{T-1} = \frac{1}{2}$, the lower and upper bounds for $\underline{s}$ coincide in (A.12) . Therefore, a solution of type E′–C cannot occur, and we are left with the solution type E′–D. Now, for a solution of type D to emerge in period $T-1$, we get the condition (using (A.13) as well as $h_{T-1} = \frac{1}{2}$):

$$\frac{\delta\theta - c}{3\delta\theta} \leq \underline{s} \leq \frac{\delta\theta - c}{2\delta\theta}.$$

The right inequality is always satisfied if our above condition for solution type E′ is satisfied. We are thus left with the following condition:

$$\frac{\delta\theta - c}{3\delta\theta} \leq \underline{s} < \frac{(\delta - c)(\delta\theta - c)}{2\delta^2\theta}.$$

Moreover, as the upper and lower bounds for $\underline{s}$ in this condition coincide for $c = \frac{\delta}{3}$, we require $c \leq \delta/3$ in addition.

The rejection probabilities are finally given by

$$\sigma_{T-2} = \frac{1}{2},\ \sigma_{T-1} = 2H - 1 = \frac{\delta\theta - c - 2\delta\theta \underline{s}}{\delta\theta - c},\ \mathbb{E}_s[\sigma_T] = \frac{c}{\delta}.$$

Because $c < \delta\sigma_{T-1}$, it is $\sigma_{T-1} > \mathbb{E}_s[\sigma_T]$. Moreover, $\sigma_{T-2} > \sigma_{T-1}$ if $\underline{s} > \frac{\delta\theta - c}{4\delta\theta}$. That condition is always satisfied because it is weaker than our earlier condition $\underline{s} \geq \frac{\delta\theta - c}{3\delta\theta}$ (see above), so that we have $\sigma_{T-2} > \sigma_{T-1} > \mathbb{E}_s[\sigma_T]$.

Solution type E′–E. Just as for solution type E′–C/D, we again have $h_{T-1} = y_{T-2} = \sigma_{T-2} = \frac{1}{2}$ and $x_{T-2} = 0$.

Now, for solution type E′ to emerge in $T-2$, we require that $\frac{c}{\delta} < \sigma_{T-1}$. Combined with the expression $\sigma_{T-1} = \frac{1-h_{T-1}}{2-h_{T-1}}$ from solution type E (see above) and $h_{T-1} = \frac{1}{2}$, this yields the condition

$$c < \frac{\delta}{3}. \tag{A.15}$$

Moreover, for solution type E to indeed emerge in period $T-1$, it must also hold that (using $h_{T-1} = \frac{1}{2}$ in (A.14)):

$$\underline{s} < \frac{\delta\theta - c}{3\delta\theta}. \tag{A.16}$$

In combination, (A.15) and (A.16) determine the range of parameter values for a solution of type E′–E. The rejection probabilities are then given by

$$\sigma_{T-2} = \frac{1}{2},\ \sigma_{T-1} = \frac{1}{3},\ \mathbb{E}_s[\sigma_T] = \theta(1 - 3\underline{s}).$$