CASE STUDY

A Tool to Visualise and Interact with Probability Density Functions – Development and Case Study

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Abstract

This article is an overview of the design, implementation and testing of a tool to visualise and interact with probability density functions. The tool is a desktop application implemented entirely in Python using the tkinter library for the graphical user interface. The project was undertaken as part of a collaboration between Mathematics and Computer Science. The goal of the application is to provide a simple user interface for teaching staff and students to visualise and interact with probability density functions, and we investigate whether such a simple visualisation tool can improve staff and student engagement. The application should help improve students' understanding of the concepts involved and its simple design should reduce the complexity barrier that often faces users when using technology in the classroom. Following initial testing, a variety of teaching staff were involved with trialling the tool, together with student volunteers from a first-year and second-year statistics module at Swansea University. Feedback was obtained and evaluated from all participants. For the teaching staff group, we found that all four participants strongly agreed that the application is easy to use and that the user interface was not distracting. Furthermore, all teaching staff stated that they would consider using the application in their own teaching and all would recommend using the application to a colleague/friend. For the student volunteer group, all twelve participants either agreed or strongly agreed with the statements that the application is easy to use, useful and not distracting. Similar to the teaching staff group, all the student participants stated that they would consider using the application in their own learning and all would recommend the application to a friend. A full analysis of the survey results is provided in the Feedback section.

Keywords: Teaching application, visual learning, statistics, probability density functions, python.

1. Introduction

This application was developed as part of a multi-disciplinary project between Mathematics and Computer Science, which was undertaken as part of an MSc dissertation in Computer Science. In particular, the first-named author, who is based in the Mathematics Department at Swansea University, supervised the MSc dissertation of the second-named author, who was studying MSc Computer Science at Swansea University. This was part of a wider initiative to promote collaboration between Mathematics and Computer Science at Swansea University. The main aim of this project was to create a user-friendly application which helped visualise and perform simple functions on continuous probability density functions. Due to time limitations of the project, we focussed on three distributions, namely the normal, gamma and student's t distributions.

Using visualisation in mathematics contributes to the development of abstract thinking (Yilmaz and Argun, 2018), with some research indicating that using visualisations is a vital component of teaching mathematics (Boaler et al., 2016). Visualisation in mathematics involves the creation and formation of models that reflect mathematical information (Van Garderen and Montague, 2003). There are many examples of using visualisation in mathematics, e.g., graphical representation of functions and histograms to spot patterns in a data set. Using technology to aid in the visualisation process can provide additional opportunities for students to explore and interact with mathematical concepts, which in turn leads to better student outcomes (Scharaldi, 2020). The potential of the use of technology to aid the visualization process has been noticed from as far back as the 1990s e.g., (Thomas et al., 1996). Despite recognition of the potential of digital tools for visualisation, specific research into effective design choices which influence adoption and usage for such applications is limited.

Next, we consider research that has been conducted on the use of software packages in mathematics to justify the need for digital visualization tools from a pedagogical perspective. Geogebra is one example of such a software package that is available as a web application and desktop application, which is an interactive application for learning and teaching mathematics. Students using the Geogebra software had a better understanding of geometry concepts than those who did not use the software (Jelatu et al., 2018) and also had improved reasoning skills (Bhagat and Chang, 2015). These findings were also confirmed by (Liburd and Jen, 2021), who also found that using technology in the classroom improved students' attitudes to learning. In another study (Fraij and Al-Mahadeen, 2012) which focused on the whether a digital tool assisted students in understanding one-to-one functions, two groups of students were asked to identify one-to-one functions, one group only had the definition whilst the other had the definition and access to a digital visualisation tool. The group with access to the visualisation tool outperformed the group that did not have access to the visualization tool, indicating that the visualisation tool had a positive impact on their learning (Fraij and Al-Mahadeen, 2012). While these results provide an argument in favour of incorporating digital tools in the classroom, they are based on small-case studies and there is a lack of variety of different applications being investigated.

Despite the numerous digital tools that have been developed in the past, teacher adoption of technology has been limited (Niu, 2018). Numerous factors have been suggested as to why teachers are reluctant to adopt the use of technology in the classroom, some of these include lack of training, the time commitment to learn how to use the tool and confidence in their own and their students' ability level in using technology (Ertmer et al., 2012). Perceived ease of use plays an important role in the adoption of digital tools in the mathematics classroom (Arthur, 2022), therefore one of the key focuses of this application is making it easy to use. It is worth noting that wider issues such as lack of resources are also limiting factors, which cannot be addressed by this project.

In (Bansilal, 2015), student teachers' perception of use of technology in the classroom was investigated from the perspective of their own learning and their own teaching. The study focuses on a broad range of technologies, however some reference is made to tools for mathematics teaching. There were some contrasting views on the use of technology in the context of sketching functions. Some students highlighted how technology is beneficial to sketch complex functions rather than sketching by hand, yet others found this to be a negative point and that learners should be taught how to sketch by hand. These skills are expected to be developed in secondary education, see (Gov, 2016) and (Department for Education, 2013), and whilst we do not dispute that students need to develop skills in sketching by hand, it is important to consider the practicalities of taking this approach all the time. Mainly, it is the time element; a fairly accurate hand drawn graph of a function requires several calculations at different points. Using a computer for this purpose is a much faster process, allowing the efficient use of time, and it enables learners to focus on investigating properties of the functions rather than on several calculations (Forster, 2006). In such cases, an important consideration for teachers to consider is the appropriateness of using such tools within the context of the lesson. (Ochkov and Bogomolova, 2015) goes further to suggest that some teachers only hold this negative perception of digital tools because they do not know how to use them. While this is a bold claim, it does raise the question whether those who did have negative perception towards the use of digital tools in the classroom would change their viewpoint if the tools have a minimal learning curve and are relevant to the curriculum.

In section 2, an overview of various existing tools will be provided. This section informed the development of the application, in particular with respect to the user interface design and features. This theme will continue into section 3, where the application will be discussed in more detail. In this section, the requirements of the application will be set out, and the design and main features of the application will also be discussed. Section 4 will focus on the feedback we received from both teaching staff and students. Details of the questionnaires used will be provided, together with a thorough evaluation of the results. Section 5 is dedicated to concluding remarks. This section will include the main points highlighted from the feedback questionnaires and remarks about future improvements to the application.

2. Existing Tools

Desmos and the Normal Distribution applet are examples of tools which exist for visualising probability density functions and performing an evaluation of these tools aided the design of this application. This was particularly the case when looking at some design choices which we may have over-looked in the initial design phase of the proposed application. For the purpose of these evaluations, the focus was based on visualising the normal distribution. Please note that the existing tools considered are not necessarily designed for probability density functions alone, but do offer the capability of performing the visualisations.

Desmos

Desmos graphing calculator is an advanced graphing calculator that is available as a web application and a mobile application that was launched in 2011 (Empson, 2011). At the time of writing, Desmos does not have full support of all Greek letters without installing additional plug-ins. Desmos is very responsive and provides a smooth experience for the user, which includes a feature to display easy-to-use sliders. Due to the potential need to manually impose restrictions on the parameters (for example, the standard deviation) and manually inputting the formula (if the built-in function is missed), there is a steep learning curve to using Desmos to fully utilise all of its features and it could be unsuitable for users with limited technological background. Due to the vast capability of Desmos, it can be used in other areas such as geometry, making the time investment in learning about its capabilities worthwhile. Finally, it is important to note that Desmos has some excellent accessibility features, in particular, sonification of graphs that provide visually impaired users with the ability to explore visualisation through sound.

The Normal Distribution Applet

The second tool that was considered was the Normal Distribution applet (Bognar, 2021a). The applet is free to use and has been specifically designed for the normal distribution. The applet allows the user to enter values for the mean “μ” and standard deviation “σ” and it also can be used for calculating probabilities. The colour scheme used in the applet is visually pleasing and there are no distracting features. The help button is visible and provides easy to follow instructions on how to use the applet and it also provides some information about the distribution and the constraints on the parameters. Overall, the applet is very easy to use. It is important to note that Bognar has also created similar applets for other distributions, see (Bognar, 2021b).

One of the main issues found with this applet was the lack of immediate response to the user when parameters are changed. There are no sliders and all of the inputs must be manually typed.

Additionally, it is difficult to determine the visual effect that changing the parameter has on the distribution. Another identified issue is the small size of the applet.

There is no option to vary its size and when viewed on a screen with resolution of 1920x1080 pixels, it is small (in fact, the visible plot section only measures 620x200 pixels). In a teaching environment, using the applet while projecting to a screen could be problematic as it could be difficult for the audience to see.

Other Tools

Further tools are available for probability density functions, for example Matlab and R. With both of these tools there is a steep learning curve involved since programming is required to create visualisations. On one hand, the steep learning curve involved in learning the required programming skills just for the purpose of creating simple visualisations of probability density functions would not seem to be a viable option, and it could be very discouraging to beginner users. However, on the other hand, as they are both very powerful languages with advanced capability, it could be beneficial as an introduction to using the tools for future work.

From trialling the existing tools highlighted above, it was decided that it is important for this application to be user-friendly, featuring sliders and for it not to have too many distracting features. In particular, when the sliders are adjusted, the plot should automatically be updated to reflect the changes. Furthermore, it should be visually pleasing, of a suitable size and it should contain a useful help function.

3. The Application

The application considers three continuous distributions, namely the normal distribution, the Student’s t distribution and the gamma distribution. The application was created using Python (version 3.10) and there is a Windows-compatible version and a Mac-compatible version. There were initial concerns regarding performance as Python is slower than other high-level languages such as Java and C (Brihadiswaran, 2020). The suitability of Python was assessed during the build of the prototype by building a working visualizer for the normal distribution with sliders for the parameters, and it appeared to be a suitable choice as there was no noticeable delay when the parameters were changed. Numerous principles have been written in the past to guide the development of user interfaces. Examples of these include the usability heuristics, see (Nielsen, 2005), and the golden rules of interface design, see (Shneiderman et al., 2016). This is very important for a successful application since a well-designed user interface provides the user with ease of use of the application which in turn allows the user to naturally and intuitively interact with the application (Christensson, 2009). The Graphical User Interface (GUI) is built using Python's standard GUI library, tkinter, which is cross-platform (Amos, 2022). For plotting the probability density functions, the matplotlib library is used which is a "comprehensive library for creating static, animated, and interactive visualizations in python" (matplotlib, 2022). For calculations of the cumulative distribution function, the stats module from Scipy is used which covers a large number of probability distributions and other topics of statistics (Scipy, 2022). The documentation for Scipy were very clear and easy to navigate and understand, making it a convenient module to use for development.

In order for end users to use the application, PyInstaller is used to bundle the application and its dependencies into a single package. The benefit of using PyInstaller is that there is no requirement for the end user to install a Python interpreter or any of the modules (Cortesi, 2022), making this a suitable option for the beginner user. Unfortunately, PyInstaller is not a cross-compiler and does not support backward compatibility. Building the application on a Windows OS will not be compatible with Mac, and vice-versa, however the same code can be used for both applications.

Full details of the coding and the steps taken to finalise the functioning of the application are too detailed to be considered here, but further details can be provided on request.

The application needs to be downloaded for use. In order to make this accessible to Swansea University students, the application was made available on the University computer system (in the same way that other applications are made available). The user interface maintains a consistent appearance across all distributions, and it is straightforward for the user to switch between these distributions, see Figure 1 below.

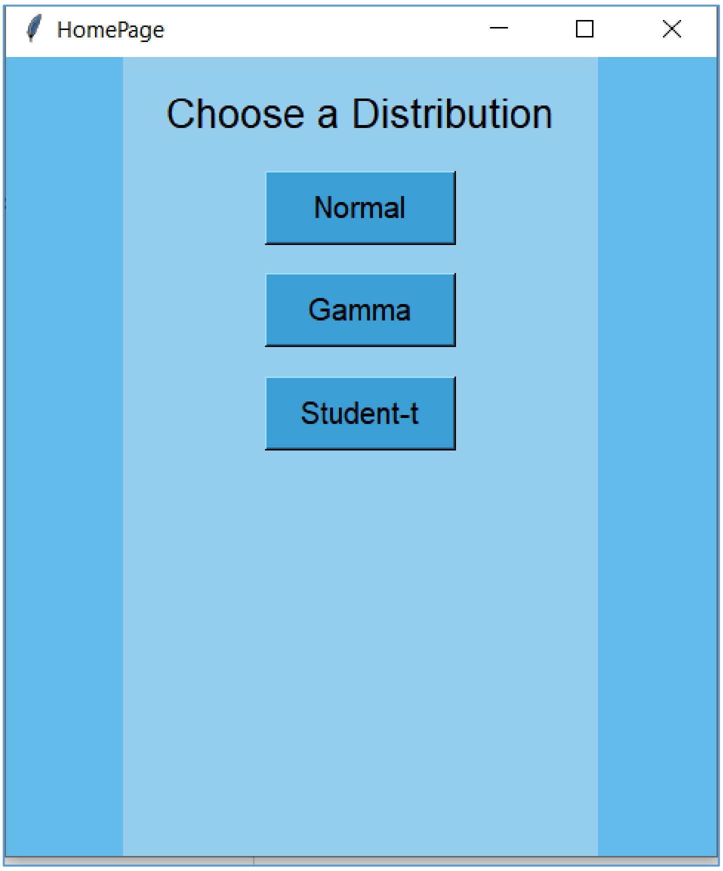

Figure 1. Application home page indicating the choice of distributions.

If users want to return to the home page after selecting a distribution in order to select a different distribution, this can be simply done by using the **Back** button. For all three distributions, the application displays a plot of the distribution according to the parameter(s) input by the user. The parameters can be adjusted by the use of sliders or they can be manually inputted by using a text box. When the user changes the parameter(s) the application updates the plot automatically so that the user can immediately see the impact of any changes on the properties of the plot. Further features can be found under the **Show** option on the application. In particular, there is a zoom option for the plots to help with visualisations and the user may choose to add gridlines to the plot. It is also possible to calculate probabilities for the chosen distribution and the area under the curve that represents the probability is automatically highlighted on the corresponding plot. For example, see Figure 2 below for the probability $P(-1 \leq X \leq 3)$ for $X \sim N(1.54, 1.64^2)$.

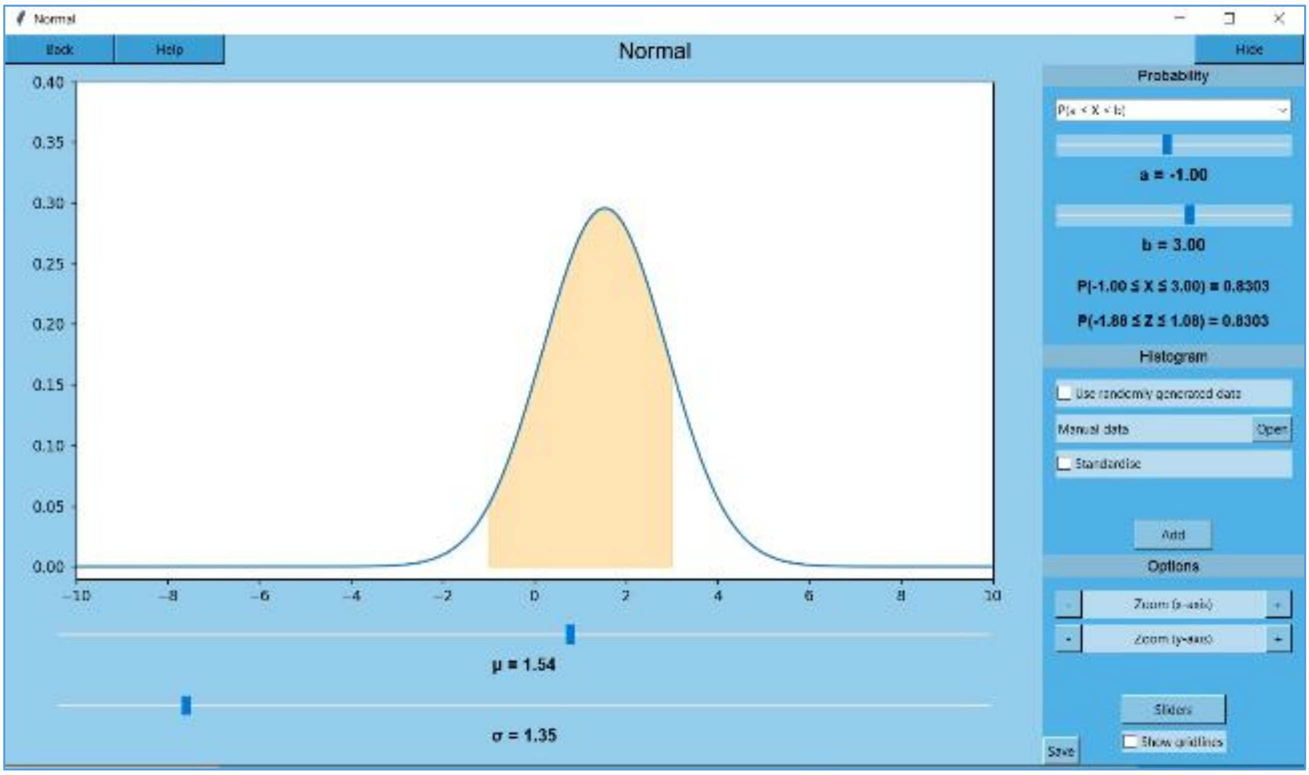

Figure 2. An illustration of an example using the normal distribution option of the application.

For the normal and gamma distributions, histograms can be produced from randomly generated data or from a data set – either manually entered or by uploading a csv or txt file. The Student's t-distribution also has a confidence interval calculator. Users have a number of options for producing these – they can enter the required parameters to produce the interval, or they can enter the data directly (either manually or by uploading a csv or txt file). See Figure 3 below for an illustration. The **Raw data** option is used for entering data.

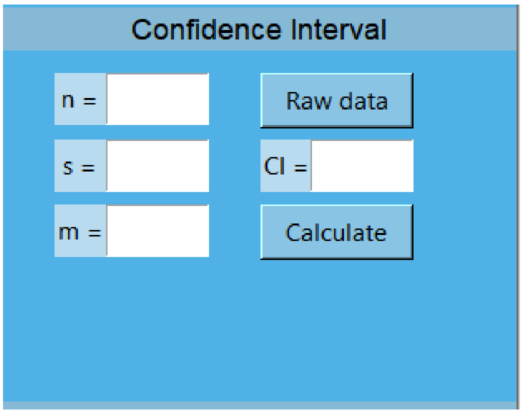

Figure 3. An illustration of the application's confidence interval calculator.

Another feature of the student's t-distribution case is the **Show Normal** option. This allows the user to see how the Student's t-distribution compares with the standard normal distribution, in particular as the degrees of freedom become larger. See Figure 4 below for an illustration of this feature where we have the Student's t-distribution with 4 degrees of freedom (in blue) and the standard normal distribution (in orange).

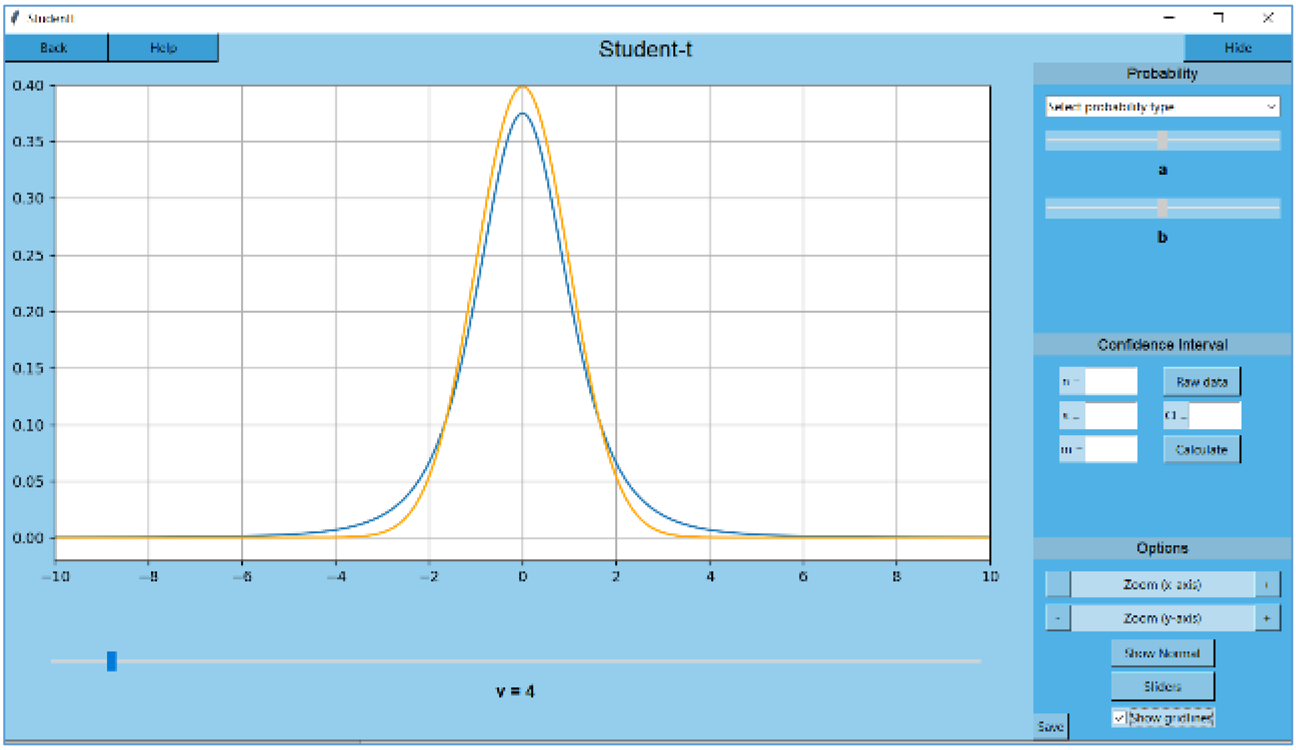

Figure 4. An illustration of the application's Student's t-distribution option.

Finally, all three distributions feature a **Help** function. The help function contains a description of all of the options along with worked examples. Users can try to perform the examples themselves and check their answers within the help function. For example, Figure 5 below is an illustration of a practice question from within the help function for the normal distribution:

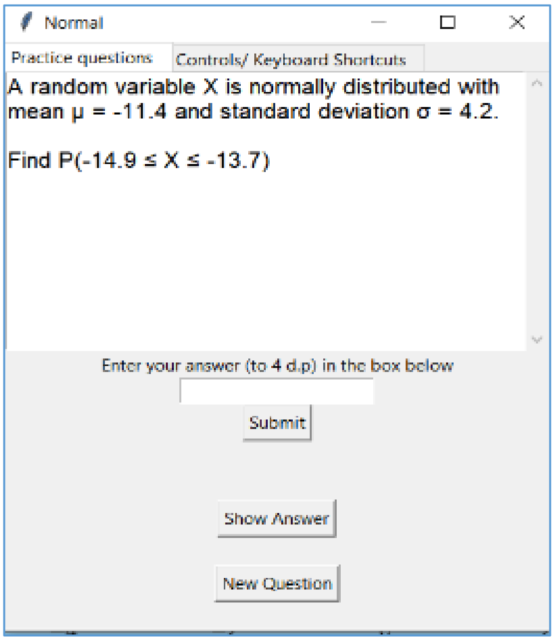

Figure 5. An illustration of the help function for the normal distribution.

4. Feedback

As mentioned in the abstract, feedback was received from both a teaching perspective and from a student perspective. The participants from the teaching perspective were two school teachers who taught A-level mathematics and two members of staff involved with university-level teaching. The students who provided feedback were volunteers from first-year and second-year statistics modules at Swansea University. In particular, four first-year students volunteered to provide feedback, while eight second-year students volunteered to provide feedback. All participants provided suitable consent to take part in the case study.

Teacher Feedback

The questionnaire provided to the teachers was as follows:

The application is easy to use. (1 = Strongly disagree, 5 = Strongly agree)	1	2	3	4	5
The user interface is not distracting. (1 = Strongly disagree, 5 = Strongly agree)	1	2	3	4	5
The application is responsive. (1 = Strongly disagree, 5 = Strongly agree)	1	2	3	4	5
I would consider using this application in my own teaching/ learning.	No			Yes	
Would you recommend this application to a colleague/ friend?	No			Yes	

Figure 6 – The questionnaire provided to the teaching staff participants.

The following open-ended questions were also asked to try to determine if any design choices/considerations had been missed (and which would not have been picked up in the questionnaire):

- *What did you like the most about the application?*
- *What did you dislike the most about the application?*
- *Additional comments:*

Figure 7 below contains the results from the questions in Figure 6 for the teaching staff group. The entries in the cells in Figure 7 denote the number of participants who provided that particular answer for each question.

In summary, all four participants from this group strongly agreed that the application is easy to use and that the user interface was not distracting, indicating that the design choices made for the application have been successful. Whether the application was responsive was positively rated with three participants strongly agreeing and the other agreeing. In addition, all four participants stated that they would consider using the application in their own teaching and all would recommend using the application to a colleague/friend.

The application is easy to use. (1 = Strongly disagree, 5 = Strongly agree)	0	0	0	0	4
The user interface is not distracting. (1 = Strongly disagree, 5 = Strongly agree)	0	0	0	0	4
The application is responsive. (1 = Strongly disagree, 5 = Strongly agree)	0	0	0	1	3
I would consider using this application in my own teaching/ learning.	0			4	
Would you recommend this application to a colleague/ friend?	0			4	

Figure 7 – Results of the teaching staff questionnaire.

With respect to the open-ended questions, three out of the four participants commented that they liked the simple functionality of the application. Furthermore, two out of the four participants commented that they liked the practice questions. One participant did comment that they would use the application on the condition that it was free, and further commented that the application would be particularly beneficial for new teachers because of its ease of use. Finally, one participant liked most the links to histograms and the visualisations.

The participants were also asked to include what they did not like about the application – this was to gather an idea of any oversights in the design or features of the application. The ability to manually configure parameter values was not clear for one user, and they were unsure whether this was possible when using the application. Another participant commented that the normal distribution did not link to z-values, but in fact the application does include z-values when the probability is selected. This indicates that this feature is not immediately obvious to the user. A further comment was that not being able to input a probability value to calculate the corresponding z-value was something they did not like about the application, therefore this is a potential feature implementation for a future release. A further comment made by one participant was that the application looked outdated because everything was too square/rectangular. Unfortunately, this is a limitation of using tkinter, and while efforts were made to give the application a more modern look, it does not match up to modern applications. Finally, one participant mentioned that locating the questions in the help section does not make it immediately obvious that there is a question generator. This comment is understandable, and perhaps a better option would be to have a button specifically for the question generator.

Student Feedback

Feedback was obtained from student volunteers during designated lab sessions, which took place in-person during the second semester of the 2022/23 academic year. During these sessions, student participation and consent information was provided to all students both verbally and in writing.

The teaching staff questionnaires took place before the student ones, and in light of the feedback received from the teaching staff group, we decided to include additional questions in the student questionnaire in order to try to obtain more feedback on the usefulness of the application and on the help function. In addition, we asked the student participants for feedback specifically about potential changes or additional features of the application – these comments will be taken into account in any future work in this area.

The questionnaire provided to students was as follows:

The application is easy to use. (1 = Strongly disagree, 5 = Strongly agree)	1	2	3	4	5
The application is useful. (1 = Strongly disagree, 5 = Strongly agree)	1	2	3	4	5
The user interface is not distracting. (1 = Strongly disagree, 5 = Strongly agree)	1	2	3	4	5
The application is responsive. (1 = Strongly disagree, 5 = Strongly agree)	1	2	3	4	5
The in-built help function is useful. (1 = Strongly disagree, 5 = Strongly agree)	1	2	3	4	5
I would consider using this application in my own teaching/ learning.	No			Yes	
Would you recommend this application to a colleague/ friend?	No			Yes	

Figure 8 - The questionnaire provided to the student participants.

The following open-ended questions were also asked:

- *What did you like the most about the application?*
- *What did you dislike the most about the application?*
- *Would you like to see any changes or additional features to the application?*
- *Additional comments:*

Figure 9 below contains the results from the questions in Figure 8 for first-year students, and Figure 10 provides the results for second-year students. The entries in the cells in Figure 9 and Figure 10 denote the number of participants who provided that particular answer for each question.

The application is easy to use. (1 = Strongly disagree, 5 = Strongly agree)	0	0	0	3	1
The application is useful. (1 = Strongly disagree, 5 = Strongly agree)	0	0	0	2	2
The user interface is not distracting. (1 = Strongly disagree, 5 = Strongly agree)	0	0	0	1	3
The application is responsive. (1 = Strongly disagree, 5 = Strongly agree)	0	0	1	1	2
The in-built help function is useful. (1 = Strongly disagree, 5 = Strongly agree)	0	1	0	0	3
I would consider using this application in my own teaching/ learning.	0			4	
Would you recommend this application to a colleague/ friend?	0			4	

Figure 9 – Results of the first-year student questionnaire.

The application is easy to use. (1 = Strongly disagree, 5 = Strongly agree)	0	0	0	3	5
The application is useful. (1 = Strongly disagree, 5 = Strongly agree)	0	0	0	0	8
The user interface is not distracting. (1 = Strongly disagree, 5 = Strongly agree)	0	0	0	2	6
The application is responsive. (1 = Strongly disagree, 5 = Strongly agree)	0	0	1	2	5
The in-built help function is useful. (1 = Strongly disagree, 5 = Strongly agree)	0	0	1	2	5
I would consider using this application in my own teaching/learning.	0			8	
Would you recommend this application to a colleague/friend?	0			8	

Figure 10 – Results of the second-year student questionnaire.

In summary, all students either agreed or strongly agreed with the statements that the application is easy to use, useful and not distracting. Ten out of the twelve student participants either agreed or strongly agreed with the statements that the application is responsive and that the in-built help function is useful. All the student participants would consider using the application in their own learning and all would recommend the application to a friend.

With respect to the open-ended questions, three students commented on the simplicity of the application as being what they liked most, while five students stated that it was the ease of use of the application that they liked most. Two students commented that they liked most the fact that the application provides good visualisations and one student stated that they liked the examples within the help function most.

With respect to what students disliked most about the application, three students commented about the use of the sliders being difficult to choose specific values – as mentioned above, it is possible for specific values to be inputted without using the slides, therefore this feature could be made clearer. One student also commented that they would like to see all features accessible in one window and another student stated the location of the practice questions as being what they disliked most.

With respect to changes or additional features to the application, many of the comments reflected the comments from the previous question. In particular, five students stated that they would like to be able to input specific values instead of using the sliders – this clearly relates to the remarks provided in the previous question above, therefore this needs to be made clearer to users. One student explained that they would like to see all functions of the application available in one window, while another student stated that they would like to see the practice questions more clearly labelled. These remarks will be taken into account in any future developments of the application.

5. Conclusion

Having identified the need for simplifying applications for use in education through background work, an application for the purpose of visualising and interacting with probability density functions was created. The application works as expected and have met the requirements initially set out.

Future work on the application would involve acting on the feedback gathered from the test participants. Implementations to make features more apparent would improve the application, such as question generator, manually entering parameter values etc. These are minor implementations but making these features more apparent to the user would be beneficial in further simplifying the application and providing a better user experience.

One of the non-functional requirements that could have been improved is the communication of errors to users. At the moment, when users enter invalid values when manually setting slider ranges, for invalid values the submit button is disabled but does not offer much guidance to the user in what constitutes a valid range. In this respect, improvements could be made to communicate to the user what errors they have made when entering values.

In addition, using the application and seeing if student understanding of probability density functions is improved in comparison to not using any technological aids could be the basis of future research that could contribute to research in the effectiveness of digital visualisation tools in the mathematics classroom.

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