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Validating DSGE Models Through SVARs Under Imperfect Information

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ABSTRACT

We study the ability of SVARs to match impulse responses of a well-established DSGE model where the information of agents can be imperfect. We derive conditions for the solution of a linearized NK-DSGE model to be invertible given this information set. In the absence of invertibility, an approximate measure is constructed. An SVAR is estimated using artificial data generated from the model and three forms of identification restrictions: zero, sign and bounds on the forecast error variance. We demonstrate that a VAR may not recover a subset of structural shocks when imperfect information causes the underlying model to be non-invertible.

JEL Classification: C11, C18, C32, E32

1 | Introduction

Following a precedent set by Christiano et al. [1], researchers often try to compare the impulse response functions of an estimated structural VAR (SVAR) with a dynamic stochastic general equilibrium (DSGE) model, but do not directly test whether the two types of models are compatible.[1] The question we pose in this paper is whether SVAR methods can indeed be employed to recover the structural shocks and impulse responses if the data-generating process (DGP) is a DSGE theoretical model. In principle this may be possible since the rational expectations (RE) solution of a linearized DSGE model is a VARMA which may be approximated by a finite order VAR representation in which the reduced-form prediction errors are linear functions of the structural shocks. A necessary and sufficient condition for such a representation is that the VARMA is invertible (or, almost equivalently,[2] satisfies fundamentalness).

The invertibility-fundamentalness problem is often described in the macroeconometrics literature as one of "missing information" when the econometrician does not have all the information that agents in the DGP have. We refer to the econometrician's problem as "E-invertibility". The non-E-invertibility of the RE solution of a DSGE model is ubiquitous. The problem for the econometrician occurs when faced with a number of observed variables that is less than the number of shocks; or with some variables of the system observed with a lag; or in models featuring anticipated shocks with a delayed effect on the system such

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as news shocks; and even with square systems when a particular choice of observed variables is observed with neither delayed effects nor a lag.

However, in our paper, missing information of this form is not at the heart of the problem, but rather imperfect information (II) on the part of both agents and the econometrician takes centre stage. Indeed, we assume that the (possibly imperfect) information sets are the same for both. We contrast II with perfect information (PI) – the standard informational assumption in the RE solutions of DSGE models. We refer to the agents' problem as "A-invertibility", and in its absence, the PI and II RE solutions of the model differ. Agents then cannot recover the current and past structural shocks and face a signal extraction problem. For an econometrician observing this model economy, this "contamination" of aggregate dynamics by filtering errors has crucial implications.

Thus, the resulting macroeconomic time-series cannot contain the necessary information to recover the structural shocks in an SVAR estimation: A-non-invertibility results in E-non-invertibility. As pointed out by Leeper et al. [5] and Blanchard et al. [6], if the agents in the DGP are unable to back out structural shocks then, faced with either the same data or a subset (for instance, in a framework that identifies news shocks), neither can the econometrician. In the absence of invertibility, the econometrician estimates an SVAR that recovers the one-period ahead prediction errors (the "innovations process"), *not* the structural shocks. Consequently, validation procedures for DSGE models comparing impulse responses with those based on estimated SVARs can be seriously misleading.

Much of the empirical literature using SVARs for estimating shocks that are identified by means of some of the DSGE restrictions is relatively silent on the invertibility (fundamentalness) issue and focuses on identification, that is, the recovery of structural shocks from the SVAR. Inappropriate identification of the latter coupled with E-non-invertibility can result in impulse response functions that deviate from the true responses to structural shocks predicted by the theoretical DSGE model. Separating out these two issues for a model that highlights the incomplete information sets of agents is the focus of our paper. In fact, we show that the missing information problem cannot be solved through identification techniques applied in SVARs.

1.1 | Main Results

The paper focuses on the potential ability of an SVAR to match impulse response functions (henceforth IRFs) of a well-established DSGE model [7] under different information sets. Failure to do so originates from both non-invertibility and a poor choice of identification restrictions. We estimate an SVAR(1) by generating artificial data from the theoretical model. Based on the SVAR representation of the DSGE model, we compare three forms of SVAR-identification restrictions: zero, sign and theory-driven bounds on the forecast error variance (henceforth BoundsFEV), for mapping the reduced-form residuals of the empirical model to the structural shocks of interest. For the estimated non-invertible (in both E- and A-senses) DSGE models, we assume II on the part of both agents and the econometrician. We utilise the II measures of approximate fundamentalness and assess the ability of these measures to predict the non-invertibility of the estimated model.

The results have strong implications for the researcher using an SVAR to compare IRFs with those generated by a structural model. First, we can actually report some good news for the estimated Smets and Wouters [7] model. For the original square case where the number of observations (data sets) equals the number of structural shocks, there is no invertibility problem. The RE solution is both A- and E-invertible and the PI and II solutions coincide. In this case the divergence between the estimated DSGE and SVAR is entirely due to a combination of the finite order VAR assumption and the choice of identification strategies. Regarding the latter, we find that, of the identification schemes, it is very clear that BoundsFEV of Volpicella [8] delivers the best estimation precision, removing the implausible responses and outperforming the VARs with zero and sign restrictions in replicating the IRFs of the assumed DGP.

Our second finding reports more good news even for the non-square non-invertible case where the number of structural shocks, which include a shock to the inflation target and measurement errors, is greater than the number of observations. Although the PI and II solutions of the model now differ, the monetary policy and government spending shocks are *approximately* fundamental as indicated by the IRFs and our approximate fundamentalness measures. This is encouraging as many empirical researchers only focus on these two shocks.[3] These results are very robust to our alternative identification strategies, but again BoundsFEV delivers the best fit. Our positive argument is that we can actually validate the Smets and Wouters [7] model by carrying out a comparison between the IRFs of the theoretical model and an empirical model for these two shocks, linking our procedure to practical macroeconomic implications.

However, our third finding is that non-invertibility-fundamentalness does matter in general and a comparison of our approximate fundamentalness measure with the actual IRFs of the DGP demonstrates its usefulness. For the non-square case, specific results are that four shocks – investment, preference, price mark-up and inflation target – are not approximately fundamental and this is confirmed by the poor matching of the IRFs of the SVARs with those of the DGP even with our preferred identification scheme. We show that II of agents in DSGE models can be a source of shock contamination and non-fundamentalness. Consequently, validation of a DSGE model by means of a VAR that even employs model-consistent identifying assumptions can be a major problem.

These, to the best of our knowledge, are novel results for both the SVAR and DSGE literature. While there is a large literature devoted to understanding theory-driven identification strategies for the VAR shocks, no previous studies have linked this literature to the informational assumptions in the DGP in the context of constructing data-SVARs that are compatible with the theory-based DSGE restrictions.

1.2 | Literature Background and Contributions

We discuss several strands of literature closely related to our paper. Additionally, we discuss the contrast between these relevant pieces of literature and our approach to further clarify the contributions of our paper.

The first strand is a largely econometrics literature on the invertibility issue of the VARMA representation that summarises the DSGE restrictions. Two seminal papers on the invertibility-fundamentalness problem are Lippi and Reichlin [9] that introduces Blaschke matrices and Fernandez-Villaverde et al. [10] that examines the conditions for a solution of a RE model to have a VAR representation. A popular example of the invertibility or missing information problem comes from "news shocks" observed by agents but not by the econometrician – see, for example, Sims [11] and Leeper et al. [5]. "Noisy" news papers by Blanchard et al. [6] and Forni et al. [12] study models closely related to our II general framework.

A vast econometrics literature is also devoted to understanding the relationship between SVAR and DSGE models and the implications of E-non-invertibility. Methods for assessing non-invertibility in DSGE models when the VAR dimension and number of structural shocks are not equal have been initiated by Sims and Tao [13]. A very influential paper in this literature is Ravenna [14] which sets out the conditions under which a DSGE model has a finite order VAR representation and shows that identification strategies consistent with the theoretical model can perform poorly in the truncated VAR approximation. Surveys of this strand of literature and the invertibility-fundamentalness problem are provided by Alessi et al. [15], Sims [11] and Giacomini [16]. Beaudry et al. [17] and Forni et al. [18] propose a method for assessing approximate invertibility for non-invertible (non-fundamental) linear approximations of DSGE models.

A more recent and related literature, initiated by Chahrour and Jurado [19], generalises identification and invertibility to the notion of recoverability. It argues that structural shocks may be recoverable from a non-invertible representation as a combination of past, contemporaneous and future observables, thus providing an alternative means of using VARs for deriving IRFs of shocks. Drawing on Forni et al. [18], Pagan and Robinson [20] develop a simple Kalman-smoother based test for identifying the recoverable shocks. Plagborg-Møller and Wolf [21] and Canova and Ferroni [22] provide another test on the VAR residuals for recoverability when invertibility fails and the number of structural shocks exceeds the number of observables.[4] However, in common with the literature, these studies explore the issue without examining the main focus of our paper – the informational assumptions of agents in the underlying structural model.[5]

Despite a growing literature on the important impact of II on DSGE models, many (indeed most) models of the macroeconomy are still solved and/or estimated on the assumption that agents are simply provided with PI, effectively as an *endowment* rather than the consequence of A-invertibility. Based on the earlier studies of information frictions, II in representative agent (RA) models was initiated by Minford and Peel [25] and generalised by Pearlman [26] and Pearlman et al. [27] – henceforth PCL – with major contributions by Woodford [28] and Collard and Dellas [29]. These papers show that II can act as an endogenous persistence mechanism in business cycles. More recently, applications with estimation were made by Collard et al. [30], Neri and Ropele [31] and Levine et al. [32]. Levine et al. [33] describes a toolkit that implements the procedures for II in a RA environment and provides, as one of a number of examples, the application in this paper.

A related literature studies II in a heterogeneous agent framework: See, for example, Pearlman and Sargent [34], Nimark [35], Angeletos and La'O [36], Graham and Wright [37], Rondina and Walker [38], Huo and Takayama [39], Huo and Pedroni [40], Angeletos and Huo [41], Angeletos and Huo [42] and Levine et al. [24]. Angeletos and Lian [43] provide a recent comprehensive survey of what they refer to as the incomplete information literature. Section 2.2 provides a link between the RA framework of our paper and this literature.

Our analysis also relates to a rapidly growing empirical literature that studies the role of II in expectations formation and shock propagation in the context of economic policy. Empirical evidence initiated by Melosi [44] and recent studies, such as Melosi [45], Nakamura and Steinsson [46], Andrade et al. [47], Bauer and Swanson [48], Gambetti et al. [49], Melosi et al. [50], and Okuda et al. [51], delves into the macroeconomic effects of II. Notably, using expectations obtained from surveys, Andrade et al. [47] and Okuda et al. [51] investigate the formation of inflation expectations with II on the underlying fundamentals based on firm-level survey data, while others such as Bauer and Swanson [48] and Melosi et al. [50] reveal substantial evidence of signalling effects of announcements about monetary and fiscal policy. Indeed, information frictions have important implications for the propagation mechanisms to explain fluctuations.

If the RE solution of a DSGE model is not invertible in both E- and A-senses, all is not lost in the ability of IRFs from an SVAR to replicate those in the assumed DGP, the DSGE model. As noted above, the solution may be *approximately* fundamental, at least for some shocks, in the sense described by Forni and Gambetti [52], Beaudry et al. [17], Canova and Sahneh [53] and Forni et al. [18]. Section 3 develops a new metric on assessing invertibility which generalises the approximation measures of Forni et al. [18] to the II case.

Turning to the identification issue, early SVAR studies employ short-run or long-run restrictions for the identification of structural shocks. However, recent research has relaxed controversial restrictions and has identified structural shocks with model-implied zero and sign restrictions on either the IRFs or the structural parameters,[6] and informational restrictions broader than sign information (see, for example, [54]).[7] Furthermore, Volpicella [8] provides an up-to-date review and a novel identification tool for estimation and inference in SVARs that are set-identified through bound restrictions on the forecast error variance decomposition. These restrictions complement the standard sign restriction approach which are also employed in our paper.

In the light of this review, our paper makes the following three main contributions to the literature. The first two are methodological and the third is an application.

First contribution: Our paper emphasises the crucial importance of the information problems of agents and the econometrician when validating a DSGE model by comparing its IRFs with those of an estimated SVAR. We distinguish invertibility from the viewpoint of the econometrician and agents, E- and A-invertibility, respectively. An application of the "Poor Man's Invertibility Condition" of Fernandez-Villaverde et al. [10] then states that E-invertibility only holds if additional conditions for A-invertibility hold, in which case the agents' information problem under II replicates that under PI. We show both generally, and in an illustrative example, the presence of Blaschke factors in model solutions where A-invertibility does not hold.

Second contribution: When A- and therefore E-invertibility fails, we construct novel measures of approximate fundamentalness which generalise results to the II case in the literature that explicitly or implicitly assumes PI on the part of agents in the assumed DGP.

Third Contribution: In our application to a well-established estimated DSGE model, we use this measure and the identification scheme of Volpicella [8] to demonstrate why a VAR may not recover some of the impulse responses to structural shocks when the underlying model assuming II is non-invertible.

1.3 | Illustrative Example

We illustrate the implications of informational assumptions for the VAR econometrician using the following linearized tractable RBC model with an inelastic labour supply

$$\begin{aligned}
\text{End-of-Period Capital}:&\ k_{t+1} = \kappa_1 k_t + \kappa_2 a_t + (1-\kappa_1-\kappa_2)c_t\\
\text{Consumption}:&\ \mathbb{E}_t c_{t+1} = c_t + \frac{1}{\sigma} r_t\\
\text{Output}:&\ y_t = (1-\alpha)a_t + \alpha k_t = c_y c_t\\
&\ + (1-c_y)i_t\\
\text{Investment}:&\ i_t = (k_{t+1} - (1-\delta)k_t)/\delta\\
\text{Real Interest Rate}:&\ r_t = \mathbb{E}_t r^K_{t+1}\\
\text{Gross Return on Capital}:&\ r^K_t = (1-\beta(1-\delta))\upsilon_t\\
\text{Rental Rate (Observation)}:&\ \upsilon_t = (1-\alpha)(a_t - k_t)\\
\text{TFP Shock Process}:&\ a_t = \epsilon_{a,t} \sim \text{n.i.i.d}(0,\sigma_a^2)
\end{aligned}$$

where $\kappa_1 = \frac{1}{\beta}$, $\kappa_2 = \frac{(1-\alpha)}{\alpha\beta}(1-\beta(1-\delta))$, β is the discount factor, α is the capital share of output in a Cobb-Douglas production function, δ is the depreciation rate, σ is the risk aversion parameter in the single-period utility function and, for the II case, the rental rate υ_t is assumed to be observed.[8]

The details of the general solution procedure are provided below in Section 2. Saddle path properties are identical under both PI and II with the stable root given by $\mu \in (0,1)$. Using these procedures, the reduced-form ARMA solution for the single aggregate observable, υ_t, can be written, for the PI and II cases, as

$$\upsilon_t = -\left(\frac{1-\psi_s L}{1-\mu L}\right)\underbrace{\left(\frac{L-\lambda_s}{1-\lambda_s L}\right)}_{\text{Blaschke Factor}}\frac{\alpha}{\lambda_s}\epsilon_{a,t} \tag{1}$$

for $s \in \{\text{II,PI}\}$ where

$$\lambda_{II} = \beta\frac{\kappa_1}{\kappa_1+\kappa_2} \tag{2}$$

$$\lambda_{PI} = \psi_{PI} = \frac{1}{\mu}\frac{\kappa_1}{\kappa_1+\kappa_2} > \lambda_{II} \tag{3}$$

$$\psi_{II} = \beta^2\frac{\kappa_1}{\kappa_1+\kappa_2} < \psi_{PI} \tag{4}$$

We can show (see Supporting Information Appendix A.4) that for plausible parameter values of σ, $-1 < \psi_s < 1$ and $-1 < \lambda_s < 1$. Both cases therefore have the common characteristic that the second ARMA factor in the solution is a Blaschke factor, implying that the structural shock $\epsilon_{a,t}$ is non-fundamental in a time-series sense, that is, it cannot be recovered from the history of υ_t alone. As a result, both cases imply that the fundamental representation is an ARMA(1,1), each with a different fundamental innovation.

However, for the PI case, $\lambda_{PI} = \psi_{PI}$, so two of the terms cancel out. But since $\lambda_{PI} = \psi_{PI} < 1$, the relationship between υ_t, the structural shock, $\epsilon_{a,t}$, and the innovations process, $e_{PI,t}$, is given by

$$\begin{aligned}
\upsilon_t &= -\frac{(L-\lambda_{PI})}{(1-\mu L)}\frac{\alpha}{\lambda_{PI}}\epsilon_{a,t}\ (\textbf{non-fundamental})\\
&= -\left(\frac{1-\lambda_{PI}L}{1-\mu L}\right)\underbrace{\left(\frac{L-\lambda_{PI}}{1-\lambda_{PI}L}\right)}_{\text{Blaschke Factor}}\frac{\alpha}{\lambda_{PI}}\epsilon_{a,t}\\
&\equiv \left(\frac{1-\lambda_{PI}L}{1-\mu L}\right)e_{PI,t}\ (\textbf{fundamental})
\end{aligned} \tag{5}$$

where μ and λ_{PI} are the outcome of a standard estimation. $\epsilon_{a,t}$ can formally be recovered by applying a Blaschke factor to $e_{PI,t}$, as in Lippi and Reichlin [9], using the parameter λ_{PI}.

For the II case, $s \in \{II\}$, we have

$$\upsilon_t = -\left(\frac{1-\psi_s L}{1-\mu L}\right)\underbrace{\left(\frac{L-\lambda_s}{1-\lambda_s L}\right)}_{\text{Blaschke Factor}}\frac{\alpha}{\lambda_s}\epsilon_{a,t}\ (\textbf{non-fundamental}) \tag{6}$$

$$\equiv \left(\frac{1-\psi_s L}{1-\mu L}\right)e_{II,t}\ (\textbf{fundamental}) \tag{7}$$

Since $\psi_s \neq \lambda_s$ for these two II solutions, (6) is now an **ARMA(2,2)** process in the structural shock $\epsilon_{a,t}$. Since (again for plausible parameter values) $-1 < \psi_s < 1$ and $-1 < \lambda_s < 1$, (6) has one root ($L = \lambda_s$) less than unity and is therefore non-fundamental in $\epsilon_{a,t}$. But again the **ARMA(1,1)** process (7) is fundamental in the innovation e_t and can be estimated giving the parameter ψ_s. But in this case the VAR econometrician will have no inference about the value of λ_s, so this is now an *identifiability problem* arising from non-fundalmentalness.[9]

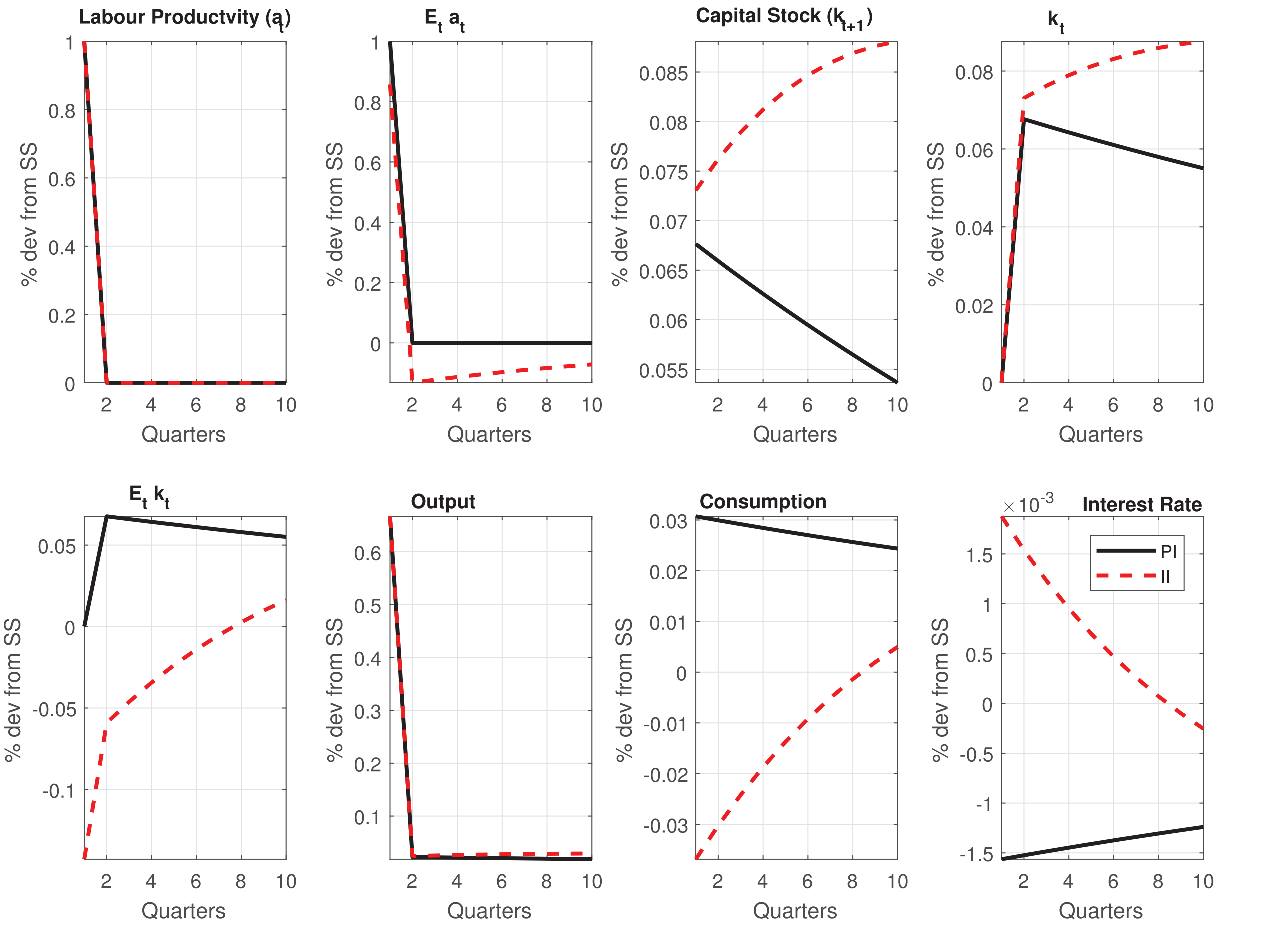


FIGURE 1 | Impulse Responses to a Temporary Technology Shock for PI and II. Parameter Values: $r = 0.01$, $\alpha = 0.333$, $\delta = 0.025$, $\sigma = 2$, $\rho_a = 0$. [Colour figure can be viewed at wileyonlinelibrary.com]

To summarise, for an econometrician observing this economy under II, the presence of a Blaschke factor in (6) where λ_{II} cannot be inferred from the estimation of (7) is crucial and provides the link to Lippi and Reichlin [9] on invertibility-fundamentalness. The VAR estimated will recover the innovations e_t, but not the structural shock $\epsilon_{a,t}$, leading to misleading comparisons of IRFs.

One measure of the non-invertibility problem we later employ is the difference between IRFs for the PI and II cases. In Figure 1, a temporary technology shock, $\epsilon_{a,t}$, raises the gross return r_t^K, proportional to v_t. Comparing the PI and II trajectories, the agent with II then underestimates $\epsilon_{a,t}$ with $\mathbb{E}_t a_t < a_t$ and confuses this with an underestimate of the capital stock ($\mathbb{E}_t k_t < k_t$). She therefore expects the return to increase in the future and therefore overestimates the real interest rate $r_t = \mathbb{E}_t r_{t+1}^K$. Consumption falls, savings increase thus crowding in more investment and capital stock under II.

Figure 1 illustrates another crucial feature of the II case: the initial errors in interpreting the productivity shock have prolonged impacts on capital accumulation, and thus induce additional dynamics in response to a productivity shock that are absent under PI.

We see then that the II case also differs in a crucial way from the PI case, since the true (non-fundamental) structural reduced-form is in both cases an ARMA(2,2), whereas in the PI case, since $\lambda_{PI} = \psi_{PI}$, it is an ARMA(1,1). Thus, the nature of the structural ARMA reduced-form captures the property noted above, that II changes the dynamics of the macroeconomy. But, crucially, the non-fundamentalness of the representation implies that the history of v_t alone can provide the econometrician with *neither* information about the structural shock *nor* these additional dynamics. In the terminology of Lippi and Reichlin [9], the structural ARMA representation is both non-fundamental and "non-basic" since it is of higher order than the observable representation unless the MA parameter $\mu = \psi_s$ which they correctly describe as a "fluke". They go on to limit non-fundamental representations to be basic. But in our framework, that focuses on the information sets of agents, non-basic (higher order) representations arise endogenously from the agents' signal extraction problem under II. These features of this simple example illustrate our general results.

1.4 | Structure of Paper

The rest of the paper is organised as follows. Section 2 links the invertibility (fundamentalness) issue with the informational assumptions in the model. Section 3 develops measures of approximate fundamentalness where invertibility fails. Section 4 estimates the log-linearized Smets and Wouters [7] model. We consider two forms of the model: a square system as in the original paper which is E- and A-invertible and for which the PI and II RE solutions coincide; and a modified non-square system which is no longer invertible in both E- and A-senses. Subsection 4.2 then provides measures of invertibility-fundamentalness for each shock and confirms the effectiveness of our measure. Section 5 compares the IRFs of the estimated model with those from the SVAR estimated from artificial data comparing the identification schemes described above. To further examine the performance of IRF comparisons, Section 6 computes a metric to measure the cumulative distance for the IRF divergence over the response horizon. Up to this point the paper chooses an SVAR(p) with lag $p = 1$; a final robustness check in Supporting Information Appendix J confirms our main results for lags up to $p = 5$. Section 7 carries out an empirical application using real data and compares the IRFs of the estimated model to a monetary policy shock (shown to be approximately fundamental) with those of a data-SVAR(1). Section 8 conducts some further investigation of the monetary policy shock through a direct estimation of IRFs using external instruments. Section 9 provides concluding remarks.

2 | Implications of Information Solutions for Invertibility

This section describes the econometrician's problem and the implications of the informational assumptions for invertibility. The general linear set-up and the PI and II solution procedures are described in Supporting Information Appendix B.

2.1 | From the Perspective of the Econometrician

This section shows how the econometrician's problem relates to the solution of the agents' problem presented in Supporting Information Appendix B.

Informational Assumptions. Throughout the paper, we assume under II that the agents have the same information set for the aggregate economy as the aggregate information set available to the econometricians; thus $m_t^A = m_t^E$. The notation is consistent with that in Supporting Information Appendix B.

A-Invertibility: When II Replicates PI. It is evident that II introduces non-trivial additional dynamics into the responses to structural shocks – a contrast which is crucial to much of our later analysis. However, there is a special case of the general problem under II, which asymptotically replicates PI, and hence where $P^A = BB'$.

Definition 1 (**A-Invertibility**). The RE solution is A-invertible if agents can infer the true values of the structural shocks ϵ_t (and hence ϵ_{it}) from the history of their observables, or equivalently $P^A = BB'$ is a stable fixed point of the agents' Riccati equation, (B.66) in Supporting Information Appendix B.

E-Invertibility: The ABCD (and E) of VARs. Corresponding to A-invertibility, we now define the corresponding concept from the viewpoint of the econometrician:

Definition 2 (**E-Invertibility**). The RE solution is E-invertible if the values of the shocks ϵ_t can be deduced from the history of the econometrician's observables, $\left\{m_s^E : s \le t\right\}$.

To see how the two concepts of A- and E-invertibility relate to each other, consider an econometrician's state space representations of the aggregate economy of the type that arise naturally from our solution method in Supporting Information Appendix B, of the general form

$$s_t = \tilde{A}s_{t-1} + \tilde{B}\epsilon_t \qquad m_t^E = \tilde{E}s_t \equiv \tilde{C}s_{t-1} + \tilde{D}\epsilon_t \tag{8}$$

where $\tilde{C} \equiv \tilde{E}\tilde{A}$ and $\tilde{D} \equiv \tilde{E}\tilde{B}$ and where the tildes over each of the matrices distinguish this state space representation from the particular form (without tildes) under PI. It is straightforward to show that both the PI and II representations of the previous two sections are in the ABE form of (8).[10]

For the PI case, given the informational assumptions set out above, we have, straightforwardly, $s_t = z_t, \tilde{A} = A, \tilde{B} = B, \tilde{E} = E$. For the II case, we have

$$s_t = \begin{bmatrix} z_{t,t-1} \\ \tilde{z}_t \end{bmatrix} \tag{9}$$

$$\tilde{A} \equiv \begin{bmatrix} A & A\mathcal{K}J \\ 0 & Q^A \end{bmatrix} \tag{10}$$

$$\tilde{B} \equiv \begin{bmatrix} 0 \\ B \end{bmatrix} \tag{11}$$

$$\tilde{E} \equiv \begin{bmatrix} E & E\mathcal{K}J \end{bmatrix} \tag{12}$$

where A, $\mathcal{K}$, J, Q^A and E are as defined after (B.60) to (B.63) in Supporting Information Appendix B.

Theorem 1 (**Poor Man's Invertibility Condition (PMIC)**). *The conditions for the RE solution to be E-invertible which we exploit below in Theorem 2 is then an application of the "Poor Man's Invertibility Condition" of Fernandez-Villaverde et al.* [10].[11] *The necessary and sufficient conditions are*:

Condition 1 A "square system" with $m = k$ (an assumption we relax when we consider the innovations representation and when we come to Section 3 on measures of approximate invertibility/fundamentalness).

Condition 2 $\tilde{D}$ (now a square matrix) is non-singular.

Condition 3 $\tilde{E}\tilde{B}$ is invertible and that $\tilde{A}(I - \tilde{B}(\tilde{E}\tilde{B})^{-1}\tilde{E})$ has stable eigenvalues.

Proof. See Supporting Information Appendix C. □

E-invertibility When Agents Have Perfect Information. The conditions for E-invertibility under PI are straightforward, and are identical to the original PMIC, derived from the ABCD representation, in Fernandez-Villaverde et al. [10], with $\tilde{A} = A$, $\tilde{B} = B$, $\tilde{E} = E$, $s_t = z_t$. Hence we immediately have: if agents have PI, the conditions for E-invertibility (as in Definition 2) are: the square matrix EB is of full rank and $A(I - B(EB)^{-1}E)$ is a stable matrix.

E-Invertibility When Agents Have Imperfect Information. We now consider the more general case of E-invertibility under II. The result is straightforward, but powerful:

Theorem 2. **E-invertibility under II**. *Assume that the number of observables equals the number of shocks ($m = k$). Assume further that the PMIC under PI holds (so the system would be E-invertible under PI), but agents do not have PI. Then E-invertibility under II holds if and only if A-invertibility holds, and this requires that the square matrix JB is of full rank, and $Q_A = F(I - B(JB)^{-1}J)$ is a stable matrix.*

Proof. See Supporting Information Appendix D. □

2.2 | Explaining Imperfect Information in a Representative Agent Model

While there has been a substantial literature that assumes II in a RA model, building on the foundations developed by PCL, any such model is subject to the critique that it cannot explain *why* information is imperfect. Drawing on the recent heterogeneous agent II literature outlined in Section 1.2, this question is addressed in Levine et al. [24]. There it is shown that if, in a heterogeneous agent framework, agent i observes a composite aggregate plus idiosyncratic shock and we solve the model for the limiting case of extreme heterogeneity, as a general result, the solution for the aggregate economy turns out to have the same finite state space form as for a parallel economy with a RA with II. But the aggregate dynamics of this parallel economy are affected in important ways by the underlying heterogeneity. The RA-II solution can then be rationalised as partial equilibrium symmetric heterogeneous agent economy where idiosyncratic far outweighs aggregate uncertainty (empirically plausible) and agent i fails to take into account the fact that she is a representative agent. Alternatively, Pearlman and Sargent [34] show that the RA-II solution is a "pooling" (of information) solution of the heterogeneous agent II case where expectations of the forecasts of others becoming *knowing* the forecasts of others.

3 | Exact and Approximate Fundamentalness (Invertibility)

But suppose that the PMIC fails? Then there exists an invertible *innovations representation* for the one-period ahead prediction errors

$$e_t = m_t^E - \mathbb{E}_{t-1} m_t^E \tag{13}$$

where e_t is the innovation found by solving another filtering problem.[12] The resulting VAR in e_t is what the econometrician estimates so she does *not* recover the structural shocks ϵ_t. But when the system is invertible $e_t = D\epsilon_t$ where D is a matrix of structural parameters in the ABC and D state space representation of the model's RE solution so the two shock processes are perfectly correlated.

Forni et al. [18] suggest that one can use VARs as well for "short systems", where the number of observables is smaller than the number of shocks.[13] Utilising the underlying VARMA model, they suggest regressing the structural shocks against the innovations process, that is, for the structural shock i, choose the least-squares vector m_i by minimising the sum of squares of $\epsilon_{i,t} - m_i' e_t$. Clearly, the theoretical value of this is

$$\hat{m}_i = \text{cov}(e_t)^{-1}\text{cov}(e_t, \epsilon_{i,t}) = (EP^E E')^{-1}(EB)_i \tag{14}$$

where $(EB)_i$ denotes the ith column of EB. A measure of goodness of fit is then

$$\begin{aligned} \mathbb{F}_i^{PI} &= \text{cov}(\epsilon_{i,t}) - \text{cov}(\epsilon_{i,t}, e_t)\text{cov}(e_t)^{-1}\text{cov}(e_t, \epsilon_{i,t}) \\ &= 1 - (EB)_i'(EP^E E')^{-1}(EB)_i \end{aligned} \tag{15}$$

Thus one can as usual define a linear transformation of Me_t (where M is made up of the rows m_i') as representing the structural shocks, but only take serious note of those shocks where the goodness of fit is close to 0. Once again, one can use the multivariate measure of goodness of fit[14]

$$\mathbb{F}^{PI} = I - B'E'(EP^E E')^{-1}EB \tag{16}$$

where the diagonal terms then correspond to the terms $\mathbb{F}_i$ of (15). In (16), we note that $EP^E E' = \text{cov}(e_t)$ from the steady state of the Riccati matrix and $(EB)_i = \text{cov}(e_t, \epsilon_{i,t})$.

If the number of measurements is equal to the number of shocks, and if $\mathbb{F}_i = 0$ for all i, then, since $\mathbb{F}^{PI}$ is by definition a positive definite matrix, it must be identically equal to 0. Of course, it may be the case that none of the $\mathbb{F}_i$ are zero, but that a linear combination of the structural shocks is exactly equal to a linear combination of the residuals. In addition, we might specify a particular value of the R^2 (e.g., $R_s^2 = 0.9$) fit of residuals to fundamentals such that we are happy to approximate the fundamental by the best fit of residuals.[15]

The maximum eigenvalue of $\mathbb{F}^{PI}$ then provides a measure of overall fundamentalness. It must of course be emphasised that none of these measures can be obtained directly from the data. The papers cited above all provide details of how simulations on the underlying VARMA models can indicate how to make appropriate inferences on the structural shocks using just the data and a VAR estimation.

Likewise under II, utilising the solution in Section 2, the multivariate Forni et al. [18] measure can, after some algebra, be written[16]

$$\mathbb{F}^{II} = I - B'J'(JP^A J')^{-1}JP^A E'(EZE')^{-1}EP^A J'(JP^A J')^{-1}JB \tag{17}$$

where, analogously to the PI case, $EZE' = \text{cov}(e_t)$, with $EP^A J'(JP^A J')^{-1}JB = \text{cov}(e_t, \epsilon_t)$. The latter follows firstly because, from (B.63) in Supporting Information Appendix B and the innovations representation, we can write $e_t = E(z_{t,t-1} - \bar{s}_{1t}) + EP^A J'(JP^A J')^{-1}J\tilde{z}_t$. The first term is clearly independent of ϵ_t, while the covariance of the second term with ϵ_t is obtained by calculating $\mathbb{E}[\tilde{z}_{t+1}\epsilon_{t+1}']$ in (B.60) in Supporting Information Appendix B.

4 | The Empirical Model

For our application, we use an industry standard DSGE model, Smets and Wouters [7]. The model has at its core as our motivating RBC model set out in Supporting Information Appendix A. To save space, we refer to the original article for full details of the micro-foundations. The notation is consistent with the illustrative example of Section 1.3 and the Smets and Wouters [7]–henceforth SW–paper.[17]

4.1 | Bayesian Estimation

The model is estimated by Bayesian methods. The data sample (1966Q1-2004Q4) and the corresponding measurement equations for the observables are the same as in SW. When we assume that this exactly coincides with the agents' II set so in effect the number of measurements is equal to the number of shocks and EB is non-singular. This we refer to as **Case 1**: the original SW with 7 shocks and 7 observables (the data sets). In the modified versions of the model, the only changes we make are that (1) we add an inflation target shock so the number of shocks exceeds the number of observables; (2) we further add measurement errors (MEs)[18] to the observations of real variables and inflation. We refer to this non-square system as **Case 2**: SW with 13 shocks and again 7 observables.[19] In terms of fitting the model empirically, we show that, for Case 1 the PI and II cases coincide. From Case 2, including the additional shocks under II leads to a small improvement in fitting the data including the second order empirical moments.[20]

4.2 | Invertibility and Perfect Versus Imperfect Information

Table 1 first presents the key invertibility results from the estimated models and the test for non-fundamentalness, based on Section 3, as useful measures to show the (mis-)use of VARs to validate DSGEs. We find that Case 1 is completely invertible according to the eigenvalue measures and indeed produces exactly the same simulated moments (including the IRFs). The model is E- and A-invertible. When we add the additional shocks in Case 2, this introduces non-invertibility-fundamentalness into the model, driving a bigger wedge between PI and II, in the sense that the fundamentalness problem worsens for the performance of VARs (suggesting larger differences in IRFs). We plot the posterior IRFs based on the estimates and compare them for Case 2 which is non-square and therefore has a non-invertible RE solution.

To illustrate the effectiveness of our measure of approximate fundamentalness (invertibility) for individual shocks for Case 2, Figure 2 compares IRFs for the technology, monetary policy and preference shocks. For the former two, the relevant II measures are $\mathbb{F}_a^{II} = 0.0004$ and $\mathbb{F}_r^{II} = 0.0036$ which are close to indicating a good approximation of the structural shock to the innovation whereas for the latter, $\mathbb{F}_b^{II} = 0.9526$, indicating a poor approximation. From Section 2.1, invertibility requires A-invertibility and the ability of agents to back out the shocks. Then in this case the PI and II equilibria coincide. Thus a wedge between PI and II IRFs is also a measure of non-fundamentalness. However, one needs to take into account that the estimates for the PI and II cases are different so part of the wedge arises for this reason. So we also compare PI and II in the model where both simulations apply to the SW model estimated under our preferred II case (i.e., the latter generates a better fit in the Bayesian comparison in [33]).

TABLE 1 | Fundamentalness and invertibility measures for estimated SW model.

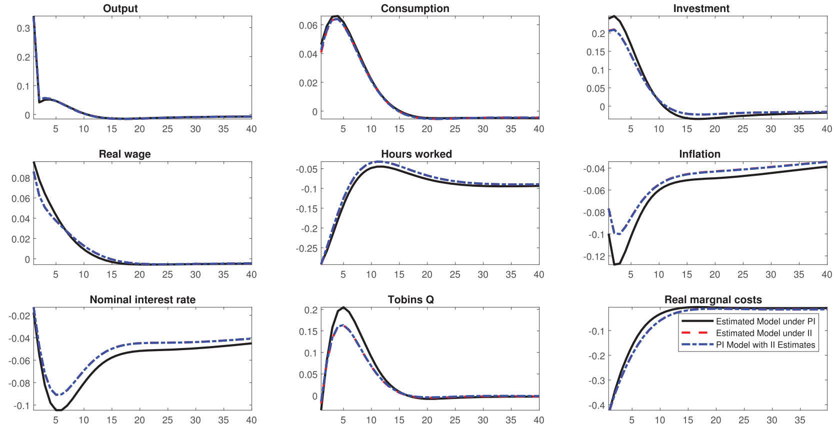
	Case 1: original SW	Case 2: SW with MEs	
	Measurements = shocks = 7	Measurements = 7 < 13 shocks	
PMIC	Holds	Fails	
Goodness of fit	$\mathbb{F}^{PI} = \mathbb{F}^{II} = 0$	$\mathbb{F}^{PI}_{(13\times13)}$	$\mathbb{F}^{II}_{(13\times13)}$
Diagonal values	All zero	0.0003	0.0004
		0.2904	0.9526
		0.2020	0.0194
		0.1211	0.5085
		0.0405	0.0036
		0.1680	0.6655
		0.0344	0.0111
		0.9904	0.9989
		0.9996	1
		0.4551	0.1287
		1	1
		0.9994	1
		0.8671	0.4968

Note: Order of shocks: technology (e_t^a), preference (e_t^b), government spending (e_t^g), investment-specific (e_t^i), monetary policy (e_t^r), price (e_t^p) and wage mark-up (e_t^w), inflation objective (e_t^π) and measurement errors for output growth, consumption growth, investment growth, real wage growth and inflation. The results are based on the posterior estimates.

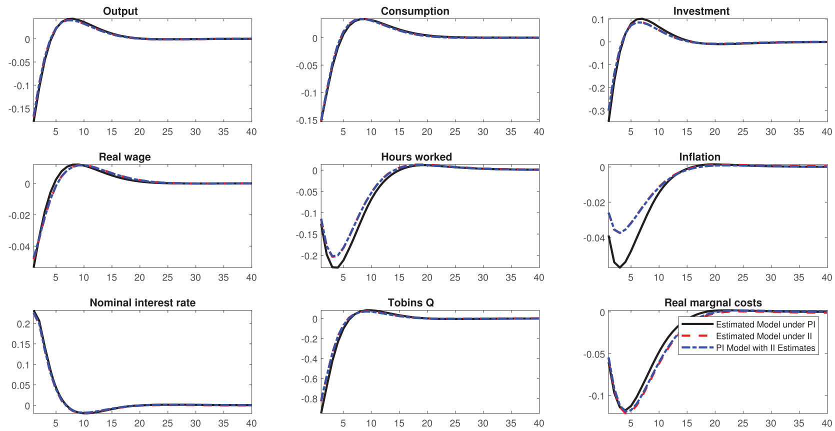
In Figure 2, the wedge between the blue and red lines arises solely from the different informational assumptions (i.e., an indication of pure A-invertibility). From the figure for the approximately fundamental technology shock, these are impossible to discern whereas for the preference shock with a high $\mathbb{F}_i^{II}$ measure, the wedge is considerable.[21]

As noted, contrasting IRFs of II and PI depends not just on the information solutions, but also on the different estimated parameters which include different estimates of persistence. The latter in particular might drive the IRF differences (e.g., for the price mark-up shock). When we compare the wedge between the black and blue lines (i.e., the IRFs under PI computed with different posterior estimates) in Figure 2, it is useful to know that the effect from the estimated parameters is very small for the investment-specific, government spending and preference shocks, suggesting that the divergence in this case is almost entirely owing to non-A-invertibility.[22]

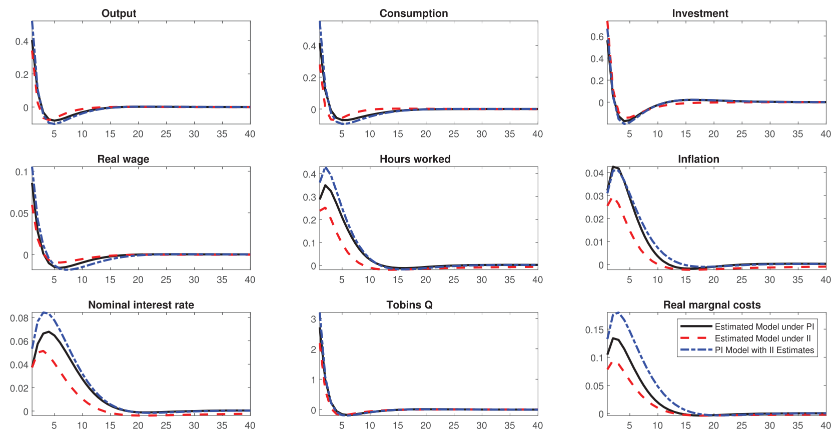
Further insight into the differences between PI and II solutions can be obtained by comparing the agents' expectations of shock



(a) Technology: $\mathbb{F}_a^{II} = 0.0004$



(b) Monetary Policy: $\mathbb{F}_r^{II} = 0.0036$



(c) Preference: $\mathbb{F}_b^{II} = 0.9526$

FIGURE 2 | Estimated SW Model Non-Invertible Case 2. Where red lines are invisible they coincide with the blue lines and therefore PI is equivalent to II based on the same estimates. Each panel plots the mean response corresponding a positive one standard deviation of the shock's innovation. Each response is level deviation of a variable from its steady-state value. (a) Technology: $\mathbb{F}_a^{II} = 0.0004$ (b) Monetary Policy: $\mathbb{F}_r^{II} = 0.0036$, (c) Preference: $\mathbb{F}_b^{II} = 0.9526$. [Colour figure can be viewed at wileyonlinelibrary.com]

process from actual outcomes. Under PI (as an endowment) they are the same of course. But under II (in the absence of A-invertibility) agents need to solve a signal extraction problem and learn about the shocks using the Kalman filter. Thus, for each shock process x_t, where $x_t = \{e_t^a, e_t^b, \dots\}$, $\mathbb{E}_t[e_t^a] = e_t^a$ under PI but not under II. IRFs give plots for each shock at a time, so with e_t^a we have $e_t^b = e_t^g = 0$, etc. But under II $\mathbb{E}_t[e_t^a] \neq e_t^a$ and nor are $e_t^b = e_t^g = 0$. Then the difference between $\mathbb{E}_t[x_t]$ and x_t is a measure of the II of the shock process.

Figure 3 shows this Kalman learning process about the shocks that do occur and the misperceptions regarding those that do

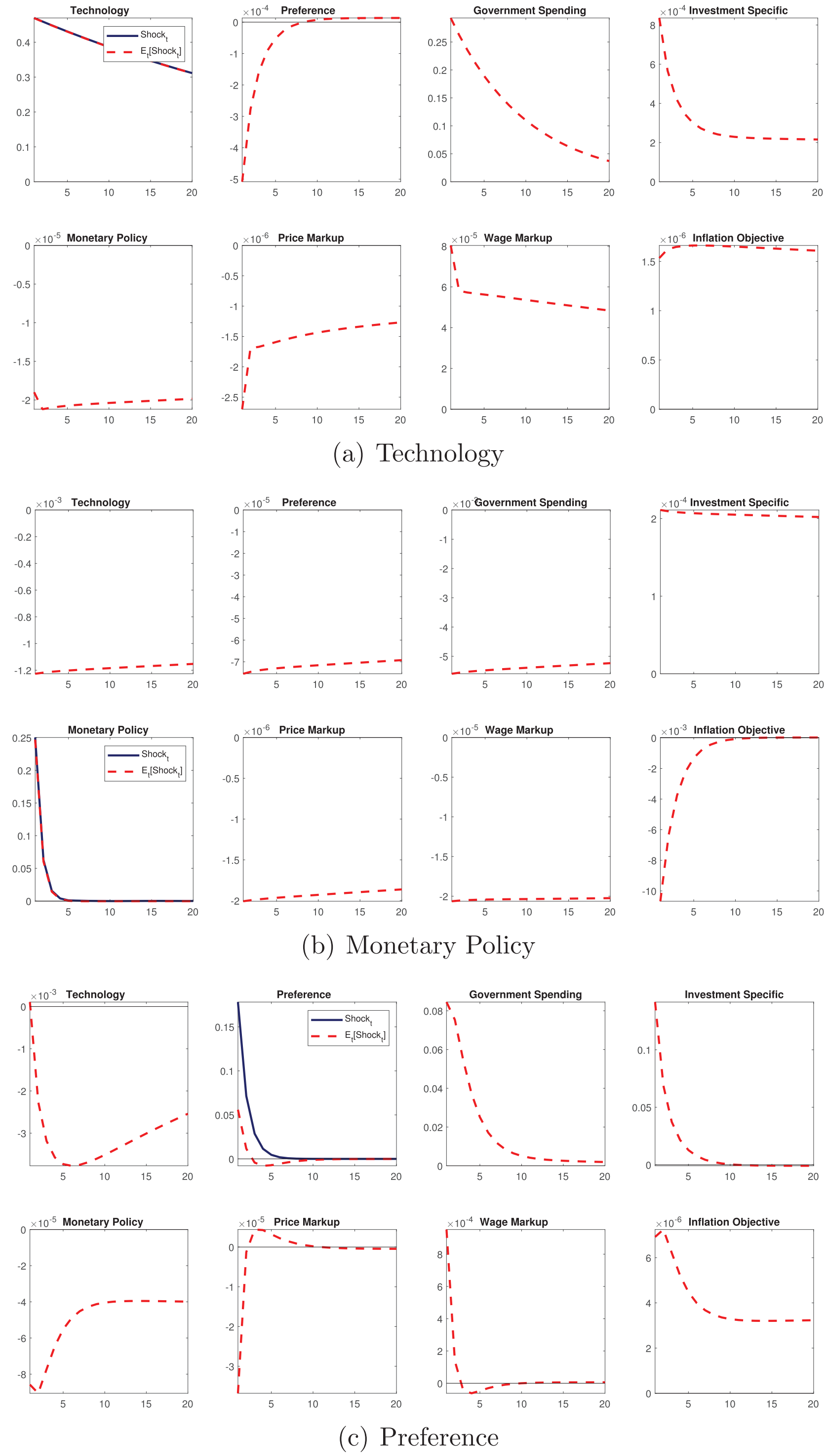


FIGURE 3 | Misperceptions About the Shocks under II. Estimated SW non-invertible Case 2. The graphs compare the actual structural unobserved shock process x_t with the agents' belief $\mathbb{E}_t[x_t]$. (a) Technology, (b) Monetary Policy, (c) Preference. [Colour figure can be viewed at wileyonlinelibrary.com]

not occur for the approximately fundamental technology shock and the very strongly non-fundamental preference shock.[23] For the approximately fundamental technology and monetary policy shocks, both types of misperception are very small with the exception of the government spending shock in the presence of only a technology shock. The reason for this is simple: namely, the inclusion of the latter in the AR(1) process for government spending: $e_t^g = \rho_g e_{t-1}^g + \epsilon_t^g + \rho_{ga}\epsilon_t^a$. As expected, for the technology shock ($\mathbb{F}_a^{II} = 0.0004$) and monetary policy shock again ($\mathbb{F}_r^{II} = 0.0036$), the responses between $\mathbb{E}_t[x_t]$ and x_t clearly overlap, showing no divergence driven by the learning process.[24]

5 | Impulse Responses From the Estimated SVAR and DSGE Models

In this section, we contrast the invertible Case 1 with the non-invertible Case 2 and compare IRFs from the RE solution of the estimated model with those of the SVAR estimated on artificial data simulated from the RE solutions of the model under PI and II (the DGP). Our procedure for simulating the data under II is described in Supporting Information Appendix F.[25] We estimate and compare our SVAR using the following identification schemes: zero short-run restrictions (Supporting Information Appendix G), sign restrictions (Section 5.2 and Supporting Information Appendix H) and restrictions with BoundsFEV (Section 5.2).

Note that sign restrictions and BoundsFEV deliver set-identified IRFs, leading to some challenges for estimation and inference. In particular, i) sign-restricted IRFs come from different, potentially contrasting, models ([58, 59]) and ii) the prior for the orthonormal (rotation) matrix transforming reduced-form into structural shocks cannot be updated, not even asymptotically ([60, 61]). With respect to i), BoundsFEV, consistently with Fry and Pagan [58] and Kilian and Murphy [62], aims at reducing the identification uncertainty by adding quantitative information to the standard sign restrictions. Furthermore, we avoid reporting the posterior mean or median as a measure of central tendency because this is likely to be a combination of different structural models ([58, 62]). With respect to ii), for estimation and inference of IRFs under sign and BoundsFEV restrictions, we use a distribution-free approach robust to prior specification on the rotation matrix echoing the spirit of Giacomini and Kitagawa [63].[26]

5.1 | The SVAR(P) Approximation to the DGP

We first recall the ABC and D form of a RE solution

$$\begin{aligned} \epsilon_t &= \tilde{D}^{-1} m_t^E - \tilde{D}^{-1}\tilde{C}\sum_{j=1}^{\infty}(\tilde{A} - \tilde{B}\tilde{D}^{-1}\tilde{C})^j \tilde{B}\tilde{D}^{-1} m_{t-j}^E \\ \Rightarrow m_t^E &= \tilde{C}\sum_{j=1}^{\infty}(\tilde{A} - \tilde{B}\tilde{D}^{-1}\tilde{C})^j \tilde{B}\tilde{D}^{-1} m_{t-j}^E + \tilde{D}\epsilon_t \end{aligned} \tag{18}$$

where, for $t = 1, \cdots, T$, m_t^E is a $n \times 1$ vector of endogenous observed variables (the data) and ϵ_t is a $n \times 1$ vector of structural white noise processes.

An invertible RE solution of a linearized model is of form (18) if the following PMIC holds: $\tilde{D}$ is non-singular and $(\tilde{A} - \tilde{B}\tilde{D}^{-1}\tilde{C})$ has stable eigenvalues. Both the state space s_t and the $\tilde{A}$, $\tilde{B}$ $\tilde{C}$ and $\tilde{D}$ matrices *differ for PI and II*.

For a possibly non-square system, the econometrician estimates an SVAR(p) model in structural shocks from reduced-form VAR(p)

$$m_t^E = \sum_{j=1}^{p} A_j m_{t-j}^E + P\epsilon_t \tag{19}$$

where P is the (structural) impact matrix. A_j and $u_t = P\epsilon_t$ are the reduced-form lags matrices and the vector of reduced-form errors, respectively.

The IRFs stem from the MA representation

$$m_t^E = \sum_{j=0}^{\infty} C_j(\bar{A}) P\epsilon_{t-j} \tag{20}$$

where $\bar{A} = A_1, \ldots, A_p$, each $C_j(\bar{A})$ is a matrix of $(I_n - \sum_{j=1}^{p} A_j L^j)^{-1}$. Identification then comes down to the choice of matrix P that satisfies $\Sigma_u = PP'$, where Σ_u is the variance-covariance matrix of u_t.

In the absence of any identifying restrictions with an invertible system

$$\Sigma_u \equiv \mathbb{E}[u_t u_t'] = \Sigma_{tr}^u \Sigma_{tr}^{u\,\prime} = P\Sigma_\epsilon \Sigma_\epsilon' P' = PP' = \Sigma_{tr}^u Q Q' \Sigma_{tr}^{u\,\prime} \tag{21}$$

$$P = \Sigma_{tr}^u Q \tag{22}$$

where Σ_ϵ is the diagonal variance-covariance matix of ϵ, Σ_{tr}^u is lower triangular of Cholesky factor of Σ_u and Q is an orthonormal (rotation) matrix. Columns of Q are denoted by $q_1, q_2, \ldots$. In particular, the IRF of variable z to shock s at some horizon h can be written as $c_{zh}'(\bar{A}, \Sigma_u) q_s$, where $c_{zh}'(\bar{A}, \Sigma_u)$ is the zth row vector of $C_h(\bar{A})\Sigma_{tr}^u$.

But if the PMIC fails and the model RE solution is **not A-invertible**, then the a-theoretical econometrician may think that the reduced-form VAR representation of the DGP is (19) whereas in fact it is given by

$$m_t^E = \sum_{j=1}^{p} \tilde{A}_j m_{t-j}^E + \tilde{P} e_t \tag{23}$$

where, we recall from (13), e_t is an $n \times 1$ vector of one-period ahead prediction errors (with variance-covariance matrix Σ_e) and *not* the structural shocks. Let $\bar{\tilde{A}} = \tilde{A}_1, \ldots, \tilde{A}_p$. If the model RE solution **is invertible**, then e_t is a linear transformation of ϵ_t, and then estimating and identifying (23) becomes equivalent to estimating (19). In the quantitative results, we confine ourselves to the $p = 1$ case.[27]

5.2 | Sign Restrictions and Bounds on the Forecast Error Variance Decomposition

Restricting the sign of impulse responses popularised by Uhlig [64] is being commonly used as an identification toolkit. Given a restricted shock s, with sign restrictions they can be written as $S(\bar{\tilde{A}}, \Sigma_e) q_s \geq 0$, where $S(\bar{\tilde{A}}, \Sigma_e)$ is a reduced-form $\tilde{s} \times n$ matrix collecting the gradient vectors of the $\tilde{s}$ sign restrictions imposed on shock s. To derive theory-driven sign restrictions, we follow Canova and Paustian [65] and collect the signs of the IRFs computed from simulating the SW model using 10,000 draws of the posterior estimates (see Table 1 in Supporting Information Appendix H). Sign restrictions (and FEV bounds) are imposed at $h = 0$.[28] In order to avoid sensitivity to the prior choice for Q, Algorithm 1 in Supporting Information Appendix I implements

TABLE 2 | Estimated FEV bounds for Smets and Wouters [7] shocks.

	$dlGDP_t$	$dlCON_t$	$dlINV_t$	$dlWAG_t$	HOU_t	$dlDEF_t$	FED_t
				Case 1 Perfect Info., $[lb_s^z, ub_s^z]$, $h = 0$			
Government	[0.23,0.50]	[0.00,0.07]	[0.00,0.02]	[0.00,0.01]	[0.21,0.47]	[0.00,0.01]	[0.00,0.06]
Monetary	[0.02,0.12]	[0.04,0.26]	[0.01,0.09]	[0.00,0.04]	[0.02,0.12]	[0.00,0.07]	[0.35,0.79]
Preference	[0.15,0.41]	[0.51,0.94]	[0.01,0.21]	[0.00,0.09]	[0.14,0.39]	[0.00,0.04]	[0.07,0.39]
Investment	[0.04,0.26]	[0.00,0.07]	[0.65,0.93]	[0.00,0.02]	[0.04,0.25]	[0.00,0.09]	[0.00,0.06]
PriceMarkup	[0.00,0.06]	[0.00,0.05]	[0.00,0.07]	[0.13,0.43]	[0.00,0.04]	[0.51,0.95]	[0.03,0.17]
				Case 2 Perfect Info., $[lb_s^z, ub_s^z]$, $h = 0$			
Government	[0.17,0.46]	[0.00,0.02]	[0.00,0.03]	[0.00,0.01]	[0.15,0.47]	[0.00,0.02]	[0.00,0.06]
Monetary	[0.02,0.13]	[0.01,0.17]	[0.01,0.19]	[0.00,0.04]	[0.02,0.12]	[0.00,0.17]	[0.66,0.96]
Preference	[0.08,0.51]	[0.16,0.68]	[0.01,0.62]	[0.00,0.15]	[0.08,0.50]	[0.00,0.12]	[0.00,0.13]
Investment	[0.03,0.28]	[0.00,0.02]	[0.18,0.94]	[0.00,0.01]	[0.03,0.32]	[0.00,0.10]	[0.00,0.06]
PriceMarkup	[0.00,0.04]	[0.00,0.02]	[0.00,0.05]	[0.00,0.25]	[0.00,0.02]	[0.07,0.89]	[0.00,0.17]
InflationObj	[0.00,0.05]	[0.00,0.03]	[0.00,0.07]	[0.00,0.02]	[0.00,0.05]	[0.00,0.14]	[0.00,0.06]

Note: The bounds of the FEVD are computed as the maximum and minimum value of the estimated FEVD simulated using the 10,000 parameter draws.

a distribution-free (robust-prior) approach for estimation and inference of impulse responses. Furthermore, as a measure of central tendency, we report the (estimated) set rather than the point-wise median, which is a combination of admissible models ([58, 59]).[29]

It is largely known that sign restrictions imply high identification uncertainty and misidentify structural shocks, for example, among others, see Kilian and Murphy [62] and Wolf [68]. Thus, to help further shrink the set of admissible structural parameters in our theory-driven sign-SVAR, we utilise Volpicella [8] and impose bounds on the FEV decomposition (FEVD) implied by the estimated DGP as an additional strategy of appropriating the impulse vectors and to eliminate any uncertainty about the specific values used for bounding the IRFs. In other words, we identify and estimate the SVAR restricted with both the sign restrictions and bounds on the FEV implied by the SW model. This approach, by complementing sign restrictions with a novel methodology, aims to further improve the estimation precision of our sign-restricted model and deliver informative results.

We generate the bounds by randomly drawing DSGE parameter vectors from the posterior distribution. The FEVD decomposes the variation in each endogenous variable into each shock to the system, thus providing information on the relative importance of each disturbance as a source of variation for each variable. In particular, it decomposes the FEV for the target $Y_{z,t+h}$ using information at time t into the percentage explained by each of the shocks s

$$FEVD_s^z(h) \equiv \frac{FEV_s^z(h)}{FEV^z(h)} \tag{24}$$

where $FEV_s^z(h)$ is the FEV of variable z due to shock s at h, $FEV^z(h)$ the total FEV of variable z at h, and $0 \leq FEVD_s^z(h) \leq 1$. Using the notation in Volpicella [8], we can write (24) as

$$FEVD_s^z(h) = q_s' \Gamma_h^z(\bar{\tilde{A}}, \Sigma_e) q_s = q_s' \frac{\sum_0^h c_{zh}(\bar{\tilde{A}}, \Sigma_e) c_{zh}'(\bar{\tilde{A}}, \Sigma_e)}{\sum_0^h c_{zh}'(\bar{\tilde{A}}, \Sigma_e) c_{zh}(\bar{\tilde{A}}, \Sigma_e)} q_s \tag{25}$$

We define the set of bounds on the FEVD for Y_z at h from shock s

$$lb_s^{zh} \leq q_s' \Gamma_h^z(\bar{\tilde{A}}, \Sigma_e) q_s \leq ub_s^{zh} \tag{26}$$

In particular, under the bounds on the FEVD, Step 2 of Algorithm 1 in Supporting Information Appendix I becomes

$$\begin{aligned} \min_{q_s} \quad \textit{and} \quad \max_{q_s} \quad & c_{zh}'(\bar{\tilde{A}}, \Sigma_e) q_s \\ \text{s.t.} \quad & S(\bar{\tilde{A}}, \Sigma_e) q_s \geq 0 \\ & lb_s^{zh} \leq q_s' \Gamma_h^z(\bar{\tilde{A}}, \Sigma_e) q_s \leq ub_s^{zh} \\ & \|q_s\| = 1 \end{aligned} \tag{27}$$

To derive the theory-driven restrictions using bounds on the FEVD, we simply compute the maximum and minimum value of the FEVD simulated by the 10,000 draws. Solutions to problem (27) then allow us to compute bounds of the identified sets of the impulse responses. Table 2 displays these FEV bounds at horizon $h = 0$ generated by the posterior estimation for the 7 variables of the SW model. These bounds show relatively small intervals for many variables. For instance, in the short run, a monetary policy shock explains a large share of unexpected movements in the interest rate and over 50% of the fluctuations of inflation can be attributed to the price mark-up shock. We therefore anticipate that these bounds, in conjunction with the impact sign constraints, can be very informative in terms of tightening the estimation precision and removing implausible effects of shocks from our set-identified IRFs. Having been able to maximise the ability of identifying assumptions to recover the DGP responses, we can turn our focus to the invertibility of the DGP.

5.3 | Assessment

A clear message emerges from the results in this section is that, with respect to the sign restrictions, the identification uncertainty decreases because of the additional restrictions (Table 2), significantly improving the precision of our estimated IRFs in line with

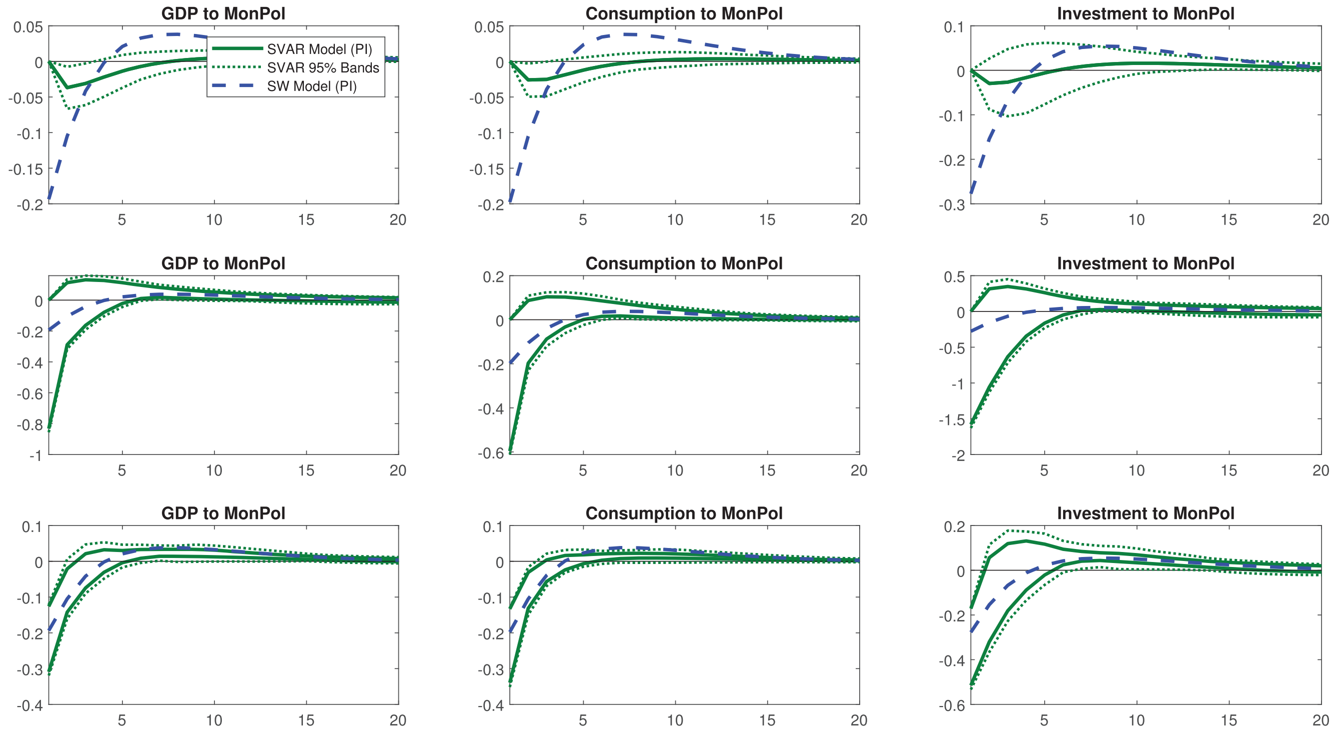


FIGURE 4 | The Real Effect of Monetary Policy Shock for Invertible Case 1 (Cholesky vs Sign vs BoundsFEV) in an SVAR(1). Cholesky (top panels), Sign (middle panels) and BoundsFEV (bottom panels). The real variables are GDP (left), consumption (centre) and investment (right). The solid lines plot the posterior means of the VAR response set bounds for Sign and BoundsFEV with the corresponding 95% band of the set (dotted). The solid lines plot the mean responses for Cholesky with the corresponding 95% band of the point estimates (dotted). The dashed blue lines are the SW-PI responses. [Colour figure can be viewed at wileyonlinelibrary.com]

the theoretical SW model. Clearly, identification is what mostly matters when the system/shocks are exactly fundamental but the information frictions can still impact on the recoverability of the SW IRFs which is conditional on the RE solution for agents being consistent with A-invertibility or not.

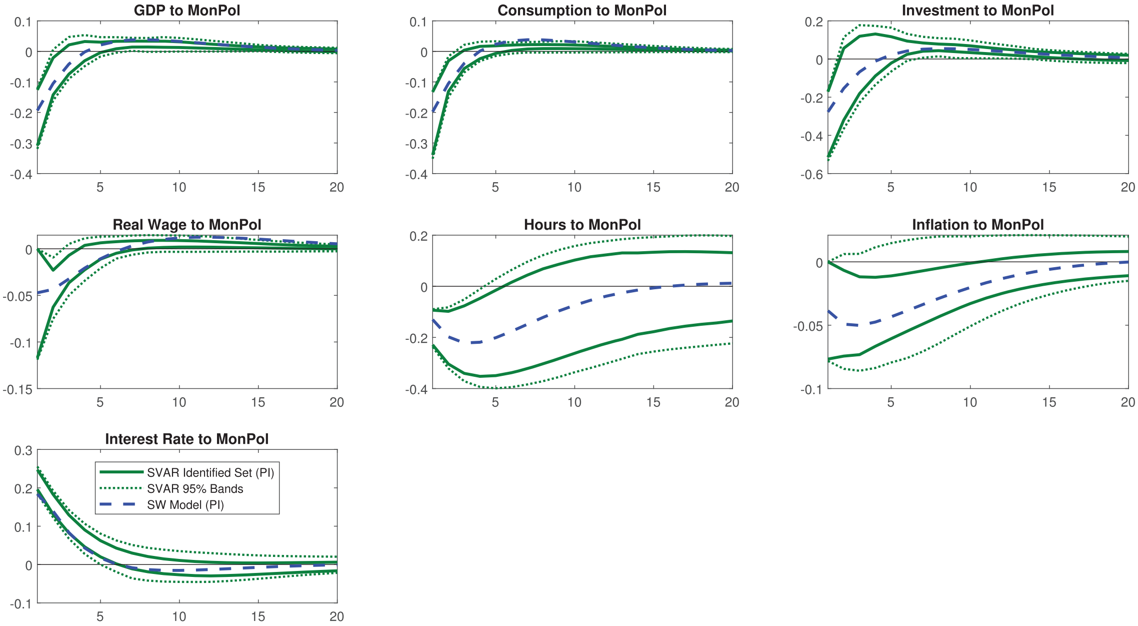
First, we can actually report some good news for the estimated SW model. For the original square Case 1, there is no invertibility problem so the divergence between the estimated model and SVAR(1) is entirely due to a combination of the finite order VAR assumption and the choice of the mapping matrix (the identification problem). For example, if we focus on the real effect of IRFs to a monetary policy shock for Case 1 and compare the outputs from the three identification schemes in Figure 4, it is very clear that BoundsFEV delivers the best estimation precision, removing the implausible responses and outperforming the Cholesky- and sign-VARs in replicating the responses in the assumed DGP.[30]

Our second finding reports more good news even for the non-square non-invertible Case 2. Namely, the monetary policy and government spending shocks are approximately fundamental as indicated by the IRF comparisons and their $\mathbb{F}_i$ measures. This is encouraging as many empirical researchers only focus on these two shocks. Indeed, our IRFs show that, for the monetary policy shock (with the smallest $\mathbb{F}_r^{II} = 0.0036$) for example, most of the SW-II responses capture the empirical responses very well, with most of them lying inside the 95% uncertainty bands and the mean of the identified sets (Figure 5).

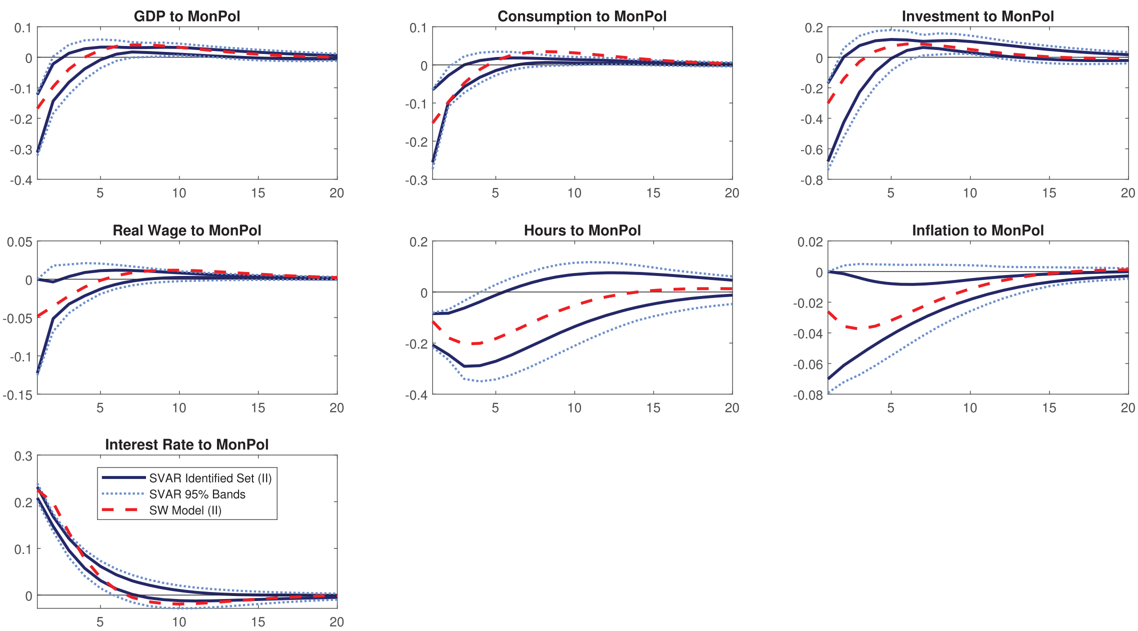
There is more evidence where the divergence in IRFs starts to appear and the IRFs from the SVAR may be badly misspecified for one particular set of IRFs if we just focus on comparing the investment responses in Figure 6 which highlight the IRF comparison between the two cases from a government spending shock. Note that $\mathbb{F}_g^{II} = 0.0194 > \mathbb{F}_r^{II}$. For Case 2 assuming II, there is a clear impact on the recoverability of the SW IRFs as there is considerable divergence in IRFs between the DGP and the posterior mean of the upper bound of the sets (and the dotted 95% credibility bands of the sets).

Now we turn to the remaining shocks that are not approximately fundamental based on our $\mathbb{F}_i$ indicators. There is a mixed outcome from matching IRFs and the $\mathbb{F}_i$ measures. The first interesting case to examine is the investment-specific shock ($\mathbb{F}_i^{II} = 0.5085$). As before, we compare the results in Figure 7 between the invertible (top panels) and non-invertible (bottom panels) models. In addition, if we impose the same parameter estimates in simulating the SW model, we find that II introduces more persistence compared with PI with the longer drawn-out responses following this particular shock. This implies that persistence is endogenously generated which should lead to a better fit of the data.[31]

However, if we focus on Figure 7, our main finding can be clearly revealed again. In particular, the bottom panels show how the IRFs from the SVAR may be badly misspecified and are therefore less able to recover the DGP. The latter generates more hump-shaped responses and endogenous persistence in IRFs (especially when comparing the left panels). When it comes to



(a) Case 1 (PI)



(b) Case 2 (II)

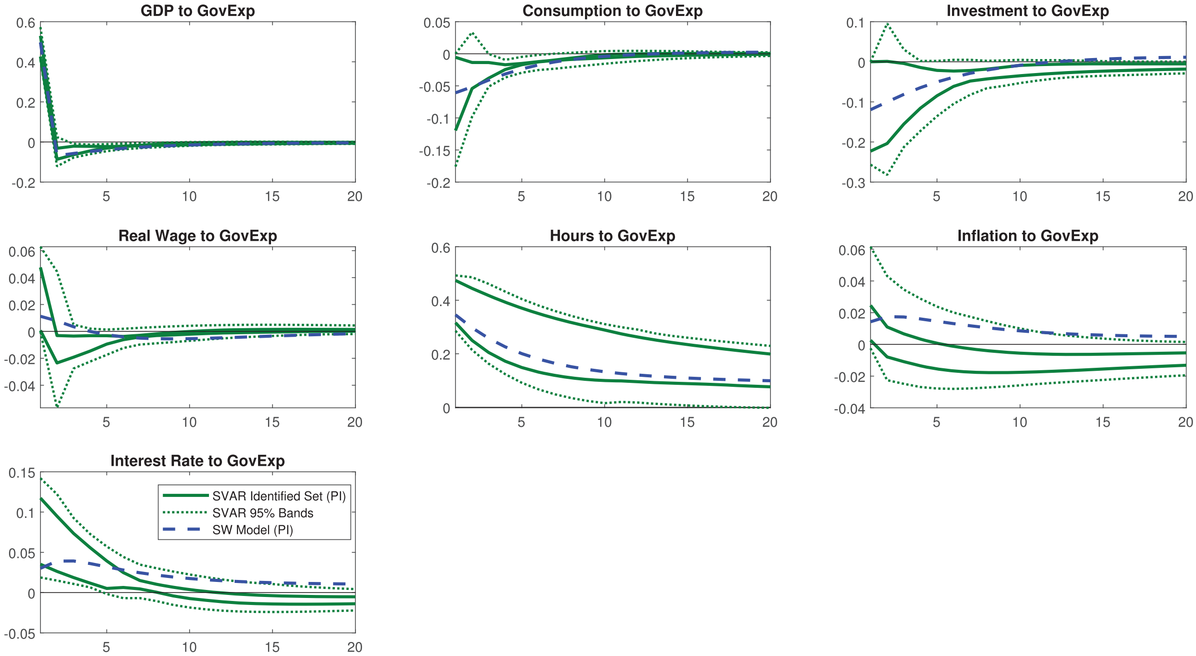
FIGURE 5 | Responses to Monetary Policy Shock (BoundsFEV) in an SVAR(1). In each panel, the solid lines plot the posterior means of the VAR response set bounds with the corresponding 95% band of the set (dotted). The dashed blue/red lines are the SW-PI/SW-II responses for Case 1/Case 2. $\mathbb{F}_r^{II} = 0.0036$. (a) Case 1 (PI), (b) Case 2 (II). [Colour figure can be viewed at wileyonlinelibrary.com]

matching the higher order responses, the evidence is even clearer, with the Case 1 SVAR generally fitting better the dynamics seen in the DSGE model, while the VAR responses produced by Case 2 match very poorly those implied by the DGP towards the end of the horizon.

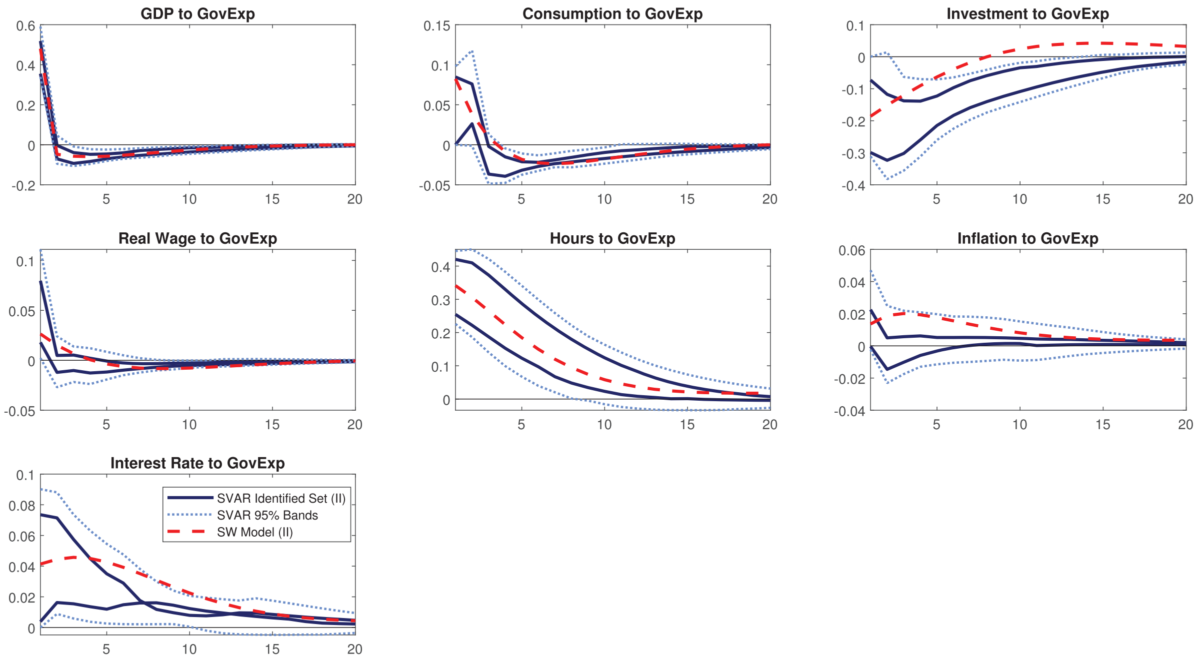
Finally, our findings are consistent across the different shocks we identify for the VAR and DSGE models but are not E- and A-invertible in the latter. For example, the same conclusions above can be drawn for the same variables that we have seen (i.e., inflation and the interest rate) if we look at the preference shock ($\mathbb{F}_b^{II} = 0.9526$) in Figure 8. In other words, the ability of identifying assumptions for the SVAR to recover the DGP response worsens with a non-invertible-fundamental system under II.

6 | Cumulative Mean Square Distance

We have established that the potential reasons for the IRF differences could be due to the problems of (1) approximate invertibility; (2) with the VAR identification; and (3) the lag length of the SVAR fitted to a large number of variables. In this section,



(a) Case 1 (PI)



(b) Case 2 (II)

FIGURE 6 | Responses to Government Spending Shock (BoundsFEV) in an SVAR(1). In each panel, the solid lines plot the posterior means of the VAR response set bounds with the corresponding 95% band of the set (dotted). The dashed blue/red lines are the SW-PI/SW-II responses for Case 1/Case 2. $\mathbb{F}_g^{II} = 0.0194$. (a) Case 1 (PI), (b) Case 2 (II). [Colour figure can be viewed at wileyonlinelibrary.com]

we address each of these points in turn in order to gain further insights into the IRF estimations. We do so by defining a metric to measure the square distance from the true responses. We also examine the performance from the several different identification schemes by focusing on the retrievable shocks. The results can be informative for the empirical researchers about the reliability of identification schemes.

6.1 | SW Model Case 2: Perfect Versus Imperfect Information

We first compute a measure of the cumulative difference that corresponds to the analysis in Section 4.2 and Figure 2. For the main shocks that we identify for the SVAR estimation, we focus on comparing the responses based on the same parameterization

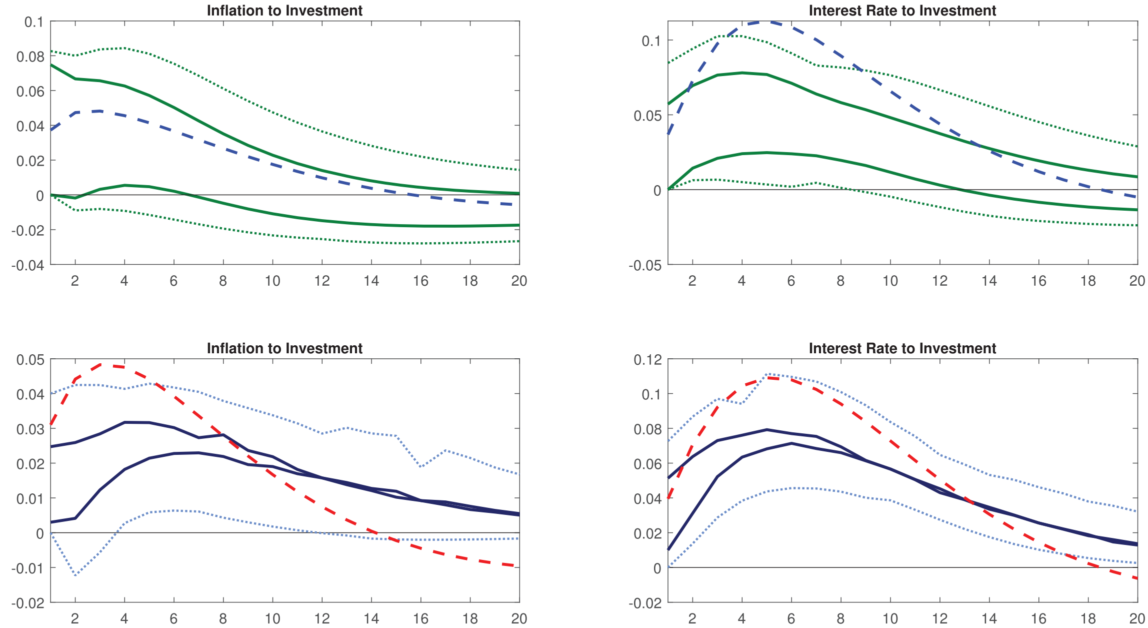


FIGURE 7 | Responses to Investment Specific Shock (BoundsFEV) in an SVAR(1). Case 1 (top panels) and Case 2 (bottom panels). The two variables are inflation (left) and the interest rate (right). In each panel, the solid lines plot the posterior means of the VAR response set bounds with the corresponding 95% band of the set (dotted). The dashed red lines are the SW-II responses for Case 2 and the dashed blue line are the SW-PI responses for Case 1. $\mathbb{F}_i^{II} = 0.5085$. [Colour figure can be viewed at wileyonlinelibrary.com]

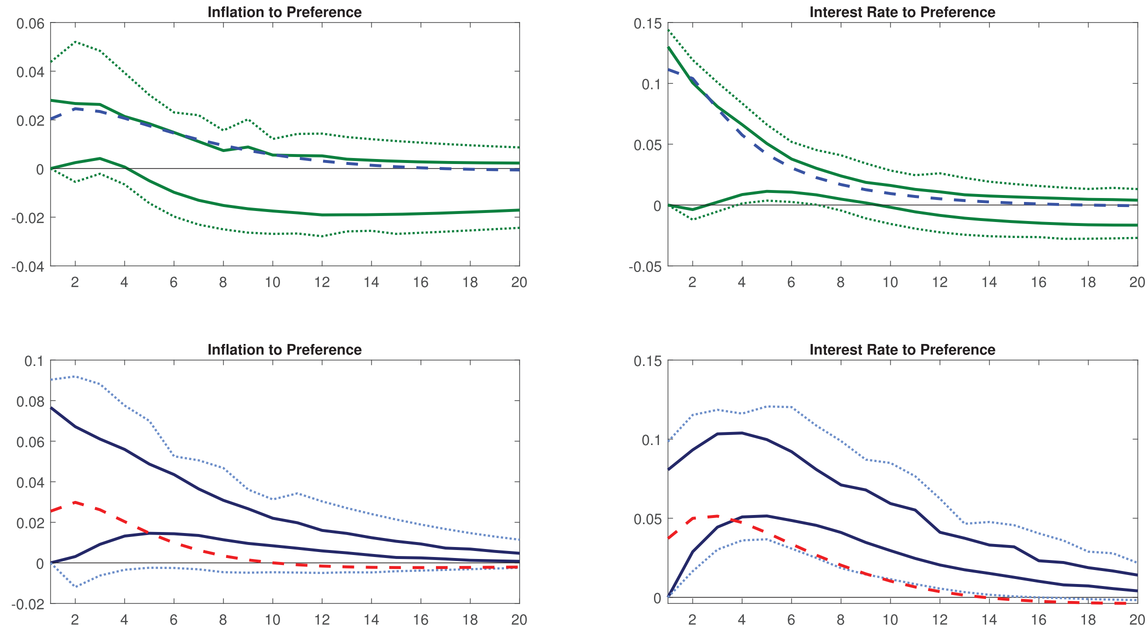


FIGURE 8 | Responses to Preference Shock (BoundsFEV) in an SVAR(1). Case 1 (top panels) and Case 2 (bottom panels). The two variables are inflation (left) and the interest rate (right). In each panel, the solid lines plot the posterior means of the VAR response set bounds with the corresponding 95% band of the set (dotted). The dashed red lines are the SW-II responses for Case 2 and the dashed blue line are the SW-PI responses for Case 1. $\mathbb{F}_b^{II} = 0.9526$. [Colour figure can be viewed at wileyonlinelibrary.com]

(i.e., II simulations with II estimates versus PI simulations with II estimates) to isolate the effects of information on IRFs. The cumulative difference is given by

$$d_H^m = \sum_{h=0}^{H} |[IRF_{SW}^{m=PI}(h,\theta)] - [IRF_{SW}^{m=II}(h,\theta)]| \qquad (28)$$

where d_H^m measures the distance between the IRFs accumulated from $h = 0$ to H. m is the informational assumption index ($m =$ PI or II). $|\cdot|$ stands for the Euclidean norm which we take to be the cumulative mean square distance (CMSD) between the two trajectories. Table 3 compares the results as an additional indication of A-invertibility and of the wedge between the IRFs arisen solely from the different informational assumptions.

TABLE 3 | Cumulative mean square distance–PI and II.

	$dlGDP_t$	$dlCON_t$	$dlINV_t$	$dlWAG_t$	HOU_t	$dlDEF_t$	FED_t	q_t	mc_t	d_H total
Technology ($\mathbb{F}_i^{II} = 0.0004$)	0.0003	0.0004	0.0004	0.0001	0.0005	0.0002	0.0001	0.0014	0.0003	0.0037
Monetary ($\mathbb{F}_i^{II} = 0.0036$)	0.0005	0.0005	0.0011	0.0003	0.0009	0.0003	0.0010	0.0026	0.0016	0.0087
Government ($\mathbb{F}_i^{II} = 0.0194$)	0.0107	0.0166	0.0090	0.0029	0.0197	0.0026	0.0042	0.0612	0.0109	0.1378
Investment ($\mathbb{F}_i^{II} = 0.5085$)	0.0185	0.0295	0.0199	0.0053	0.0328	0.0028	0.0070	0.1166	0.0185	0.2509
PriceMarkup ($\mathbb{F}_i^{II} = 0.6655$)	0.0256	0.0183	0.0637	0.0175	0.0611	0.0203	0.0244	0.0744	0.0634	0.3686
Preference ($\mathbb{F}_i^{II} = 0.9526$)	0.0487	0.0747	0.0365	0.0131	0.0942	0.0089	0.0222	0.2496	0.0521	0.6001

TABLE 4 | Cumulative mean square distance–BoundsFEV and SW.

	$dlGDP_t$	$dlCON_t$	$dlINV_t$	$dlWAG_t$	HOU_t	$dlDEF_t$	FED_t	d_H total
				Case 1 d_H^{PI} ($H = 20$)				
Monetary	0.011	0.016	0.020	0.007	0.014	0.004	0.015	0.089
Government	0.008	0.006	0.016	0.007	0.054	0.018	0.021	0.131
Investment	0.020	0.032	0.156	0.010	0.157	0.010	0.036	0.420
PriceMarkup	0.032	0.029	0.056	0.030	0.239	0.052	0.073	0.511
Preference	0.024	0.048	0.021	0.009	0.108	0.010	0.019	0.239
				Case 2 d_H^{II} ($H = 20$)				
Monetary ($\mathbb{F}_i^{II} = 0.0036$)	0.014	0.016	0.037	0.006	0.022	0.005	0.016	0.115
Government ($\mathbb{F}_i^{II} = 0.0194$)	0.011	0.012	0.082	0.007	0.014	0.011	0.010	0.147
Investment ($\mathbb{F}_i^{II} = 0.5085$)	0.017	0.024	0.071	0.013	0.033	0.015	0.021	0.194
PriceMarkup ($\mathbb{F}_i^{II} = 0.6655$)	0.012	0.020	0.046	0.031	0.093	0.029	0.031	0.263
Preference ($\mathbb{F}_i^{II} = 0.9526$)	0.043	0.047	0.076	0.019	0.107	0.013	0.027	0.331

It clearly shows that the divergence in IRFs (measured by d_H Total) is consistent with our approximate $\mathbb{F}_i^{II}$ measures from the VAR-identified shocks. Quantitatively, the table reaffirms the usefulness of $\mathbb{F}_i^{II}$ for each shock: the good approximation of the structural shock to the innovation for the technology (e_t^a) and monetary policy shocks (e_t^r) produces a CMSD very close to 0 for every individual IRF. The intuition of the IRF results for the estimated SW model has been discussed in detail in Section 4.2.

6.2 | SVAR(1) and SW Model Cases 1 and 2

We further compute the CMSD between the responses to the VAR and SW shocks for the two cases under PI and II as the horizon increases ($H = 20$)

$$d_H^m = \sum_{h=0}^{H} |[IRF_{VAR}^m(h,\theta)] - [IRF_{SW}^m(h,\theta)]| \tag{29}$$

We can gain further understanding on (i) the difference of responses to shocks; (ii) how the CMSD changes over time with h after the shock hits the system. We begin with the estimated SVAR(1) identified by BoundsFEV. Table 4 reports the results.

The benefit of this exercise is allowing us to quantitatively study the degree and effects of invertibility and/or identification in response to a shock. Based on the statistics reported, not surprisingly, Table 4 shows that the CMSD measures between the VAR and DSGE responses are the smallest for e_t^r and government spending shocks (e_t^g)–that are approximately fundamental according to $\mathbb{F}_i^{II}$, even for the non-square Case 2. These values are close to being economically insignificant in terms of the CMSD of all the observables (e.g., d_H Total in the last column is less than 10 percentage points for e_t^r). The preference shock ($\mathbb{F}_b^{II} = 0.9526$), on the other hand, reports the largest CMSD generated by Case 2 estimated under II, which is again consistent with our IRF figures and invertibility indictors. In particular, Table 4 clearly indicates, for Case 2 solved and simulated under the relevant II assumption, a monotonic relationship between $\mathbb{F}_i^{II}$ and d_H Total.

Based on the estimated VAR responses with BoundsFEV, nearly all the CMSD measures to each shock (for $H = 20$) are close to being economically insignificant (i.e., $d_H < 10$ percentage points). In most cases, the introduction of II makes the identifying restrictions less able to recover the DGP. The two exceptions are the responses of hours to the investment-specific (with a large $d_H = 0.157$) and mark-up shocks ($d_H = 0.239$) for the invertible Case 1. One potential explanation for this is because $p = 1$ may not be enough for fitting the SVAR to this observable.[32]

6.3 | Identification of Retrievable Shocks

It is also useful to compare these statistics generated by the various identification schemes but only for the retrievable

TABLE 5 | Cumulative mean square distance–different identification schemes.

	$dlGDP_t$	$dlCON_t$	$dlINV_t$	$dlWAG_t$	HOU_t	$dlDEF_t$	FED_t	d_H total
Case 1 d_H^{PI} ($H = 20$)								
BoundsFEV								
Monetary	0.011	0.016	0.020	0.007	0.014	0.004	0.015	0.089
Government	0.008	0.006	0.016	0.007	0.054	0.018	0.021	0.131
Sign restrictions								
Monetary	0.051	0.029	0.127	0.052	0.107	0.038	0.056	0.459
Government	0.051	0.038	0.130	0.059	0.267	0.048	0.047	0.641
Zero restrictions								
Monetary	0.085	0.062	0.412	0.159	0.247	0.034	0.058	1.057
Government	0.358	0.027	0.038	0.024	0.163	0.144	0.031	0.784
Case 2 d_H^{II} ($H = 20$)								
BoundsFEV								
Monetary	0.014	0.016	0.037	0.006	0.022	0.005	0.016	0.115
Government	0.011	0.012	0.082	0.007	0.014	0.011	0.010	0.147
Sign restrictions								
Monetary	0.052	0.036	0.109	0.055	0.091	0.043	0.077	0.463
Government	0.045	0.053	0.111	0.061	0.193	0.048	0.048	0.561
Zero restrictions								
Monetary	0.150	0.051	0.289	0.146	0.239	0.030	0.077	0.982
Government	0.307	0.046	0.067	0.024	0.143	0.091	0.039	0.717

shocks: e_t^r and e_t^g. For the non-retrievable ones, the results are non-informative since no identification schemes can be successful. Table 5 can help make a clear recommendation about the performance of our most reliable identification scheme. BoundsFEV indeed produces the smallest and most acceptable d_H across the different assumptions and for all the observable variables whereas the Cholesky scheme fails considerably in recovering most of the IRFs especially for Case 1 which is invertible (e.g., d_H Total = 1.057).

It is interesting to note that, even for these retrievable shocks identified by the superior BoundsFEV, the fundamentalness problem worsens for the overall performance of VARs under II with the additional shocks (Case 2). This is clearly evident that, when we are able to minimise the identification uncertainty, the informational assumption plays a key role in our model's (in)ability to recover the DGP responses (i.e., d_H Total has risen to 0.115 and 0.147 for e_t^r and e_t^g, respectively). The sign-VARs also deliver very large set-estimates implying that the uncertainty around these estimates is mostly identification uncertainty (similar to the Cholesky case). In addition to the informational assumptions, not surprisingly, identification uncertainty also plays a key role in recovering the DGP responses. The key result here is that, further to our brief discussion about Figure 4, Table 5 quantifies and measures the superiority of BoundsFEV that we apply to tackle the information/invertibility issue when using SVARs for validation of a theoretical model.

7 | The Correct Comparison of the SVAR-SW Impulse Responses

We now turn to the application of our validation procedure set out in the paper. This addresses both the non-fundamentalness and identification problems for recovering the true responses of an assumed DGP which we take to be the SW model. Our empirical applications based on artificial data have established that the monetary policy and government spending shocks under II are approximately fundamental even when the overall model is non-invertible, and with appropriate identification, VARs can achieve a remarkably close approximation to the DSGE model. This means that, following the SVAR literature, we can actually validate the SW model by carrying out a comparison between the IRFs of the estimated DSGE model and an identified SVAR for the monetary policy and government spending shocks, linking our theory to practical macroeconomic implications.

We choose the best possible identification scheme which using artificial data has been shown to be the sign and bound restrictions on the FEV of these shocks for the SVAR which is a statistical representation of our non-invertible SW model under II (Case 2). We focus on the more interesting Case 2 with more shocks, 13, than observables, 7, which means that neither A- nor E-invertiblity can hold. The PI assumption becomes extreme as it then implies that agents cannot infer current realizations of shock processes from the data and must have PI as an endowment. A VAR(1) is estimated using the

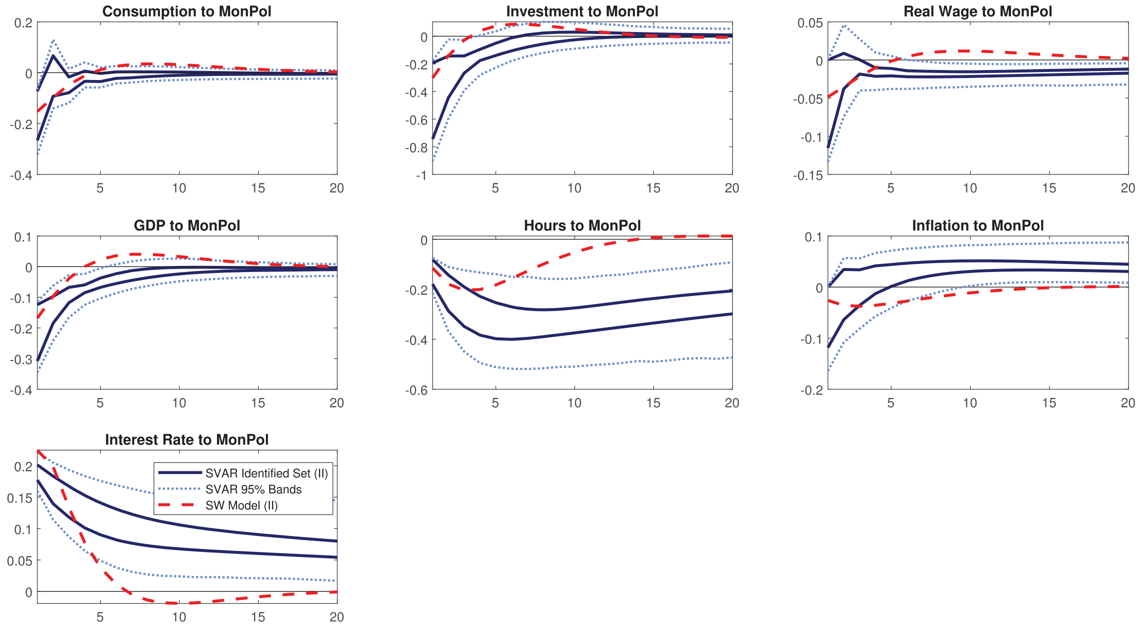


FIGURE 9 | Responses to Monetary Policy Shock (BoundsFEV) using Actual Data. The solid lines plot the posterior means of the VAR response set bounds with the corresponding 95% band of the set (dotted). The dashed red lines are the SW-II responses for Case 2. $\mathbb{F}_r^{II} = 0.0036$. [Colour figure can be viewed at wileyonlinelibrary.com]

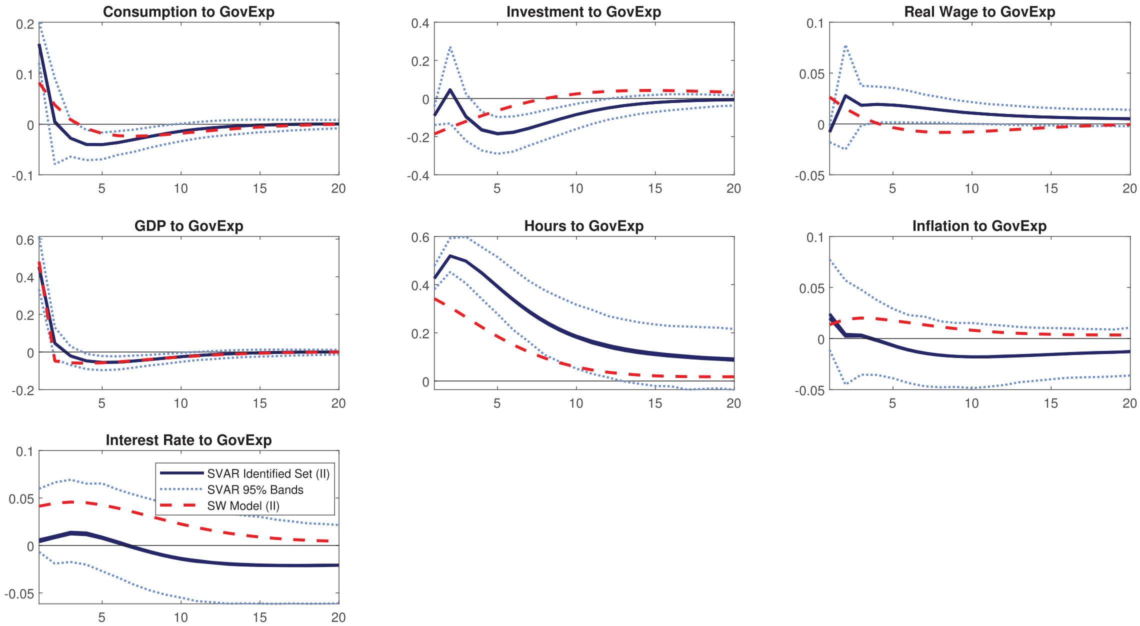


FIGURE 10 | Responses to Government Spending Shock (BoundsFEV) using Actual Data. The solid lines plot the posterior means of the VAR response set bounds with the corresponding 95% band of the set (dotted). The dashed red lines are the SW-II responses for Case 2. $\mathbb{F}_g^{II} = 0.0194$. [Colour figure can be viewed at wileyonlinelibrary.com]

same data sample (1966Q1-2004Q4) for the same observed variables ([7]).

Figures 9 and 10, respectively, compare the IRFs of the estimated data-SVAR with those of the assumed DGP to our approximately fundamental monetary policy and government spending shocks. Having followed our procedure using artificial data, the modeller would be hoping for responses close to those in Figures 5 and 6, Case 2. However, with real data, we see a failure of these responses to stay within both the identification sets and the 95% intervals after 5 periods for all the 7 data sets in the SVAR observables with the wedge particularly acute for the responses of hours and the nominal interest rate to a monetary shock. Table 6 reports a large $d_H > 10$ percentage points for the responses of hours. We have established that this failure is not caused by non-fundamentalness, a poor identification procedure nor to the length of lag in the SVAR. The model itself is not the true DGP and must be revisited. Faced with this result, the DSGE modeller

TABLE 6 | Cumulative mean square distance (BoundsFEV using actual data).

	$dlGDP_t$	$dlCON_t$	$dlINV_t$	$dlWAG_t$	HOU_t	$dlDEF_t$	FED_t	d_H total
Monetary	0.046	0.031	0.099	0.023	0.236	0.046	0.083	0.563
Government	0.024	0.023	0.099	0.017	0.142	0.022	0.033	0.359

could initially focus on the labour market using unemployment rather than hours (see, e.g., [69]) and different formulations of the interest rate rule (see, e.g., [70]). Of course model reconstruction could go further drawing upon a large literature on the general state of DSGE models (see, e.g., [71–75]). But these avenues of research all go beyond the scope of this paper.

8 | Local Projection Regression

Can the econometrician bypass SVARs and estimate IRFs directly? Using the method of local projections (LP) of Jorda [76], we can indeed bypass the intervening step of a VAR. The LP approach uses "external instruments" which are variables correlated with a particular shock of interest, but not with the other shocks. External instruments can then be used to directly estimate causal effects by direct IV regressions. This method does not require invertibility, but *does* require good instruments which, for many shocks, may not be available to the econometrician.

The choice of LP or SVAR for estimating IRFs to structural shocks each has its pros and cons which is studied in a growing recent literature. For example, Stock and Watson [77] compare the LP-IV approach with a more efficient SVAR-IV approach and propose a new test for invertibility which is applied to the study of Gertler and Karadi [78]. Plagborg-Møller and Wolf [21], building on Stock and Watson [77], show that the addition of an instrumental variable, whether external or internal, to the econometrician's information set may enable estimation of at least a scaling of the true IRF even when structural shocks are non-invertible. However, in the context of our paper which stresses the information problem of agents in the model, this then begs the question why agents are not able to observe the additional information as well. The consequences for LP of agents having this additional source of information is discussed in Levine et al. [24].

Using Jorda [76]'s LP framework, we estimate our structural IRFs using the LP-IV approach set out in Stock and Watson [77] that use external instruments for the monetary policy shock and estimate the causal effects by direct IV regressions. As in Section 7 we focus on Case 2 where the estimated SW model is neither A-nor E-invertible. The estimates from the following single regression are direct projections rather than functions of reduced-form VAR parameters

$$Y_{i,t+h} = \theta_{i,h} Y_{1,t} + \gamma_h' W_t + \xi_{i,t+h}^h \tag{30}$$

where the observed variables in the estimation include the Federal Funds rate divided by four, the percentage growth rate of real GDP, and the first difference of the log of the Implicit Price Deflator of GDP: that is, $Y_{i,t} = \{FED_t, dlGDP_t, dlDEF_t\}$ includes a subset of the same variables as in our estimated models in Section 7. As far as the identified monetary shock is concerned, the idea is to directly regress future values of these macroeconomic variables for VAR-free IRFs.[33] The coefficient $\theta_{i,h}$ gives the response of Y_i at time $t+h$ to the shock at time t. (30) is estimated via two-stage least squares using an instrument for $Y_{1,t}$ and controls for lags. The serially correlated error term is $\xi_{i,t+h}^h = \{\varepsilon_{t+h}, \ldots, \varepsilon_{t+1}, \varepsilon_{2:n,t}, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots\}$.

As LP methods do not assume a specific DGP, they are more flexible, but suffer from higher estimation uncertainty, relative to our SVARs, in a structural application where time-series samples are short.[34] Therefore, we estimate our predictive regressions of a variable of interest based on a lower dimensional vector of macroeconomic variables in order to avoid over-parameterization in small samples which would result in higher variance and inefficiency of the LP estimator, relative to VARs.

We follow Gertler and Karadi [78] (GK), Ramey [79], Plagborg-Møller and Wolf [21], Miranda-Agrippino and Ricco [81] and Miranda-Agrippino and Ricco [23] that use the high-frequency identification (HFI) methods to deal with foresight about monetary policy changes: that is, the system uses changes in Federal Funds futures rates (FFF) around Federal Open Market Committee (FOMC) announcement dates as an instrument for estimating the dynamic effects of monetary policy. All the variables are publicly observed pre-dating the FOMC announcements themselves. For comparability of estimates, we consider the same data span as in Section 7 which encompasses several periods with increased variation in our monetary policy shock. The instrument $Z_t = FFF_t$ is available from 1990Q1.[35] The control variables to enforce the lead-lag exogeneity condition ($E(\varepsilon_{t+j} Z_t') = 0$ for $j \neq 0$) in the IV regression are $W_t = \{Y_{t-1}, Z_{t-1}\}$. The direct estimation of IRFs in (30) produces Figure 11.

For comparability of results, we also present the cumulative of the Euclidean norm for the CMSD for the IRF divergence in Figure 11 as the response horizon increases (measured by d_H). Table 7 summarises the divergence in IRFs and shows that our previous result is upheld with the LP estimation comparison. The IRFs to the HFI shock from LP are further from the IRFs of the DSGE model than those from the identified data-SVAR. This then further confirms that the DSGE model itself is misspecified and needs to be revisited following the literature discussed above and the literature studying misspecification problems in DSGE models (see, for example, Gorodnichenko and Ng [82], Inoue et al. [83], among others).

9 | Conclusions and Future Research

Can indeed SVAR methods be employed to recover the structural shocks and impulse response functions if the data-generating

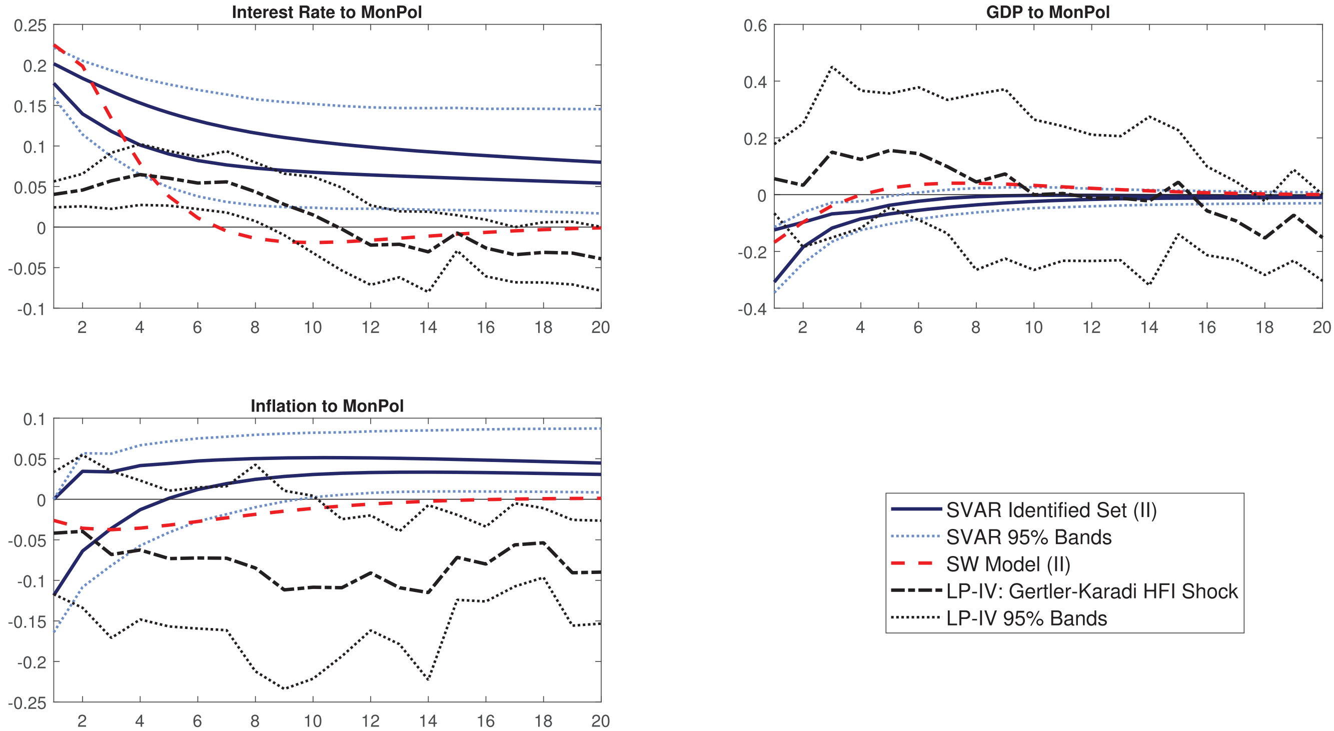


FIGURE 11 | Responses to Monetary Policy Shocks using Actual Data. The solid lines plot the posterior means of the VAR response set bounds with the corresponding 95% bands of the set (dotted). The dashed red lines are the SW-II responses for Case 2. The dashed black lines are the LP-IV responses to the HFI shock with the corresponding 95% bands (dotted). $\mathbb{F}_r^{II} = 0.0036$. [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 7 | Cumulative mean square distance (LP-IV regressions).

	FED_t	$dlGDP_t$	$dlDEF_t$	d_H **total**
$d_H = \sum_{h=0}^{H} \lvert [IRF_{LP}(h,\theta)] - [IRF_{SW}(h,\theta)] \rvert$	0.064	0.106	0.073	0.243
$d_H = \sum_{h=0}^{H} \lvert [IRF_{VAR}(h,\theta)] - [IRF_{SW}(h,\theta)] \rvert$	0.083	0.046	0.046	0.175

process is a DSGE model? In this paper, we tackled this question by addressing the invertibility issue under different information sets and employing model-consistent identifying assumptions, thus providing a novel procedure to uncover the potential (in)ability of an SVAR to match the structural IRFs of DSGE models. An important source of non-invertibility in our paper is the imperfect information set of agents in DSGE models, which we show leads to the existence of a Blaschke factor in the RE solution (the assumed DGP).

By generating artificial data using the appropriate DSGE assumptions and estimating several identified SVARs, we studied and revealed two sources of potential misspecification for the each shock that we identified for the SVAR in turn: (1) fundamentalness for each shock using our approximate $\mathbb{F}_i^{II}$ measure and (2) inappropriate identification restrictions highlighting our contribution using the identifying scheme employing theory-driven bounds on the FEV. Our application based on an industry standard DSGE model yielded very strong results that withstood a wide array of tests and checks and provided a clear-cut answer to the research question. In doing so, we provide a methodology which is completely general for the macroeconometrics literature and should precede any SVAR validation of a particular theoretical model using actual data.

There is some good news to report on both absolute and approximate invertibility. However, for some shocks, the results indicated that SVARs cannot be used to compare IRFs with those of a DSGE model. It is important to stress that our computational results are derived based on a well-established medium-sized NK-DSGE model and more (or less) severe invertibility and identification problems could well emerge with other examples. In particular, one feature of DSGE models that might prove important in this respect are uncertainty shocks which have driven an important literature on business cycles in recent years for which Fernandez-Villaverde and Guerron-Quintana [84] provide a very useful review. They show how stochastic volatility can be conveniently modelled in linear models by adding time-varying standard deviations as an ARMA (possibly an AR(1)) process. If we allow for such volatility for every shock process in our model, this then doubles the number of shocks and accentuates the non-invertibility problem. However, pursuing this research objective would require an II solution that captures nonlinearity and goes beyond the linear Kalman filter utilised in our paper.

Finally, NK-DSGE models with financial frictions often include a large number of financial shocks, not necessarily matched with data, thus moving further away from the invertible square structure. This and other sources of non-invertibility, coupled with our measure of approximate invertibility and the identification method of Volpicella [8], suggest possible areas for future research into the relationship between SVAR and DSGE models.

Endnotes

[1] This comparison, validation by SVARs, can be used to provide guidance to building or rejecting DSGE models. It can also be applied as a minimum distance estimator for estimating and testing DSGE models (see, for example, [2, 3], and [4]).

[2] Invertibility is a more general condition that implies fundamentalness, but in practice they are usually equivalent.

[3] For example, an empirical researcher may be interested in a smaller VAR in that its dimension is less than the number of structural shocks and focus only on those shocks that are relevant to the research question.

[4] The concept is related to "partial invertibility" proposed by Miranda-Agrippino and Ricco [23] who argue that only a subset of structural shocks needs to be recovered from a partially identified system for the VAR econometrician.

[5] Levine et al. [24] show that non-A-invertible structural shocks are not recoverable, and discuss, in the context of a DSGE model with II, the implications for mapping the true structural shocks to the innovations to the observables.

[6] Some of the theoretical restrictions of the DSGE are used to identify the VAR shocks in the validation procedure for the DSGE model.

[7] Standard text-books such as Kilian and Lutkepohl [55] provide an overview of this literature.

[8] Full details of the model are provided in Supporting Information Appendix A.

[9] For the empirically less plausible case $\psi_s > 1$, in terms of the innovations process $e_{s,t}$, we can write v_t as $v_t = \left(\frac{L-\psi_s}{1-\mu L}\right)\frac{\alpha}{\lambda_s}e_{s,t}$. Then to retrieve the structural shock, we would need to calculate $\epsilon_{a,t} = \left(\frac{1-\lambda_s L}{L-\lambda_s}\right)\left(\frac{L-\psi_s}{1-\psi_s L}\right)e_{s,t}$ thereby requiring *two Blaschke* factors creating an even greater identification problem for the VAR econometrician.

[10] The advantages of using the ABE state space form in what follows are (i) the Riccati equation is simpler than for any of the other formulations, (ii) the solution under II is much simpler to express and, most usefully, (iii) the representation of the model using the innovations process has the same structure as the original model (see [24] for further discussion). Note also that the ABCD state space form can be written as a VARMA process as follows: From (8) $s_t = (I - \tilde{A}L)^{-1}\tilde{B}\epsilon_t$ where L is the lag operator. Substituting into the expression for m_t^E, we then have $|I - \tilde{A}L|m_t^E = \tilde{C}(I - \tilde{A}L)^*\tilde{B}\epsilon_{t-1} + \tilde{D}\epsilon_t$ where $|X|$ and X^* denote the determinant and matrix of sub-determinants of matrix X respectively. This is of VARMA form $\Lambda(L)m_t^E = \Phi(L)\epsilon_t$.

[11] This result appears to date back at least to the work of Brockett and Mesarovic [56]. A slightly weaker condition than invertibility is fundamentalness which allows some eigenvalues to be on the unit circle. However, we use the two terms interchangeably and in fact, if we restrict our models to have only stationary variables, then the two concepts are equivalent.

[12] This is the second result in Fernandez-Villaverde et al. [10].

[13] Miranda-Agrippino and Ricco [57] propose a related concept of "partial invertibility" when only a subset of structural shocks is of interest and needs to be recovered for impulse response functions. Approximate fundamentalness can then be viewed as a generalisation to a continuous measure of the degree of invertibility-fundamentalness.

[14] If the theoretical model is estimated with constraints on B and with direct estimates of the shock variances $\sigma_1^2, \sigma_2^2, \ldots$, then the last term in (16) must be pre- and post-multiplied by the matrix $S = diag(1/\sigma_1, 1/\sigma_2, \ldots.)$.

[15] A perfect fit in the Forni et al. [18] case is $\mathbb{F}_i = 0,\ R_i^2 = 1$.

[16] The same comment applies as in the footnote to (16). This follows because P^A depends on $Bcov(\epsilon_t)B'$, so is invariant to whether the variances of the shocks are normalised to 1 or not.

[17] The linearized model is set out in Supporting Information Appendix E.

[18] These are equivalent to the noise in the signal in the "noisy news" papers by Blanchard et al. [6] and Forni et al. [12].

[19] According to the U.S. Bureau of Labor Statistics, national employment, hours and earnings statistics are surveyed and published very frequently (more so than GDP and CPI). The hours data is constructed based on these statistics. We do not assume a measurement error to the employment data and the reason for that is that the frequency in revising and publishing the employment data reduces measurement error, for hours to be observed.

[20] The complete set of empirical results is reported in Levine et al. [33].

[21] A section of our working paper (not reported here) shows similar plots for the other shocks. For example, for the monetary policy shock, $\mathbb{F}_r^{II} = 0.0036$, whereas for the investment-specific shock, $\mathbb{F}_i^{II} = 0.5085$. For the former, IRFs diverge very little, but for the latter, this is not the case and there is substantial divergence and, for consumption, an opposite sign. Furthermore, Section 6.1 below reports the cumulative mean square distance that provides an additional measure for the wedge between the blue and red lines for the shocks identified by the SVAR estimations in Section 5.

[22] Again, we focus our presentation here on the technology, monetary policy and preference shocks, for a more detailed presentation of the other IRFs, we refer to the longer, working paper version of this paper.

[23] IRFs have a standard interpretation with leisure as a normal good.

[24] For this reason, in what follows, we exclude the technology shock from our exercise as it does not display any invertibility issue and is completely fundamental in our example but include the monetary policy shock for completeness. Furthermore, results from Cholesky identification do not include the inflation objective shock as in the literature this is not identified with short-run zero restrictions for obvious reasons.

[25] Simulated using the DSGE posterior means, our artificial dataset consists of 1,000 periods (discarding the initial conditions), meaning that, in practice, there is no sample bias. This implies that the uncertainty around the VAR estimates is mostly identification uncertainty which we address systematically in the paper with set-identification. However, all the results shown here are robust once sample bias is taken into account.

[26] See, for further discussion, Supporting Information Appendix I.

[27] Supporting Information Appendix J carries out a robustness check for $p > 1$.

[28] As a robustness check, we extend the sign restrictions and bounds on the FEV up to 4 quarters. This does not change the results.

[29] The working paper of this article also shows i) that a uniform-prior approach for sign restrictions and BoundsFEV (as opposed to a robust-prior framework) à la Arias et al. [66] does not change the results and ii) the impulse responses under an identification strategy mixing zero and sign restrictions, for example, Arias et al. [67].

[30] To save space, here we report a limited set of responses. Since it is well-established that Cholesky decomposition and sign restrictions struggle in recovering the responses of the DGP even under invertibility, we focus on BoundsFEV. The longer working paper reports the full set of responses for Cholesky decomposition, sign restrictions and BoundsFEV. Furthermore, it should also be noted that there is no need

to increase the lag length p in the SVAR to achieve a good replication of the DGP.

[31] This finding has been extensively discussed in Collard et al. [30] and Levine et al. [32].

[32] We turn to Supporting Information Appendix J for examining the increased lags of the SVAR ($p > 1$) for all the observed variables.

[33] The specification of our LP-IV is consistent with that in several recent studies such as Ramey [79], Stock and Watson [77] and Bauer and Swanson [48] which produce estimates of the monetary policy effects on the macroeconomy comprising the interest rate, a measure of output and inflation, and on asset prices.

[34] See Li et al. [80] for a discussion about the bias-variance trade-off between the least-squares LP and VAR estimators.

[35] The GK data on futures rates surprises on FOMC dates are obtained from the Gertler and Karadi [78] replication materials.

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Supporting Information

Additional supporting information can be found online in the Supporting Information section. **Data S1.** Supporting Information.