Book chapter :

Belblidia, F., Haroon, T. & Webster, M. (2009). *The Dynamics of Compressible Herschel—Bulkley Fluids in Die-Swell Flows.*(pp. 425-434).

http://dx.doi.org/10.1007/978-90-481-2669-9_45

e Dynamics of Compressible Herschel–Bulkley
ids in Die-Swell Flows

elblidia[1], T. Haroon[2] and M.F. Webster[1]

stitute of non-Newtonian Fluid Mechanics, School of Engineering, Swansea University,
ngleton Park, Swansea, SA2 8PP, UK

siting researcher, COMSATS Institute of Information Technology, Abbottabad, Pakistan

ct In a variety of industrial applications, modelling compressible inelastic free-surface
emains a numerical challenge. This is largely due to the physical phenomena involved and
nputational cost associated with the simulations of such flows. In particular, the die-swell
ark problem is characterised by specific features. These are related to the presence of a
separation point at the die-exit, the location and the shape of the free-surface, and
nally the consideration of fluid compressibility under various forms of material modelling.
s article, a time-marching pressure-correction scheme is considered to solve both
pressible and compressible inelastic flows. This is achieved via a pressure-based approach
a finite element framework employing efficient high-order time-stepping schemes. A
pe model is utilised to express the equation of state that links density to pressure, so that
e is retained as a primary variable. Various material models are considered in this
cal study for the die-swell problem, where the material rheological characteristics have a
impact upon the location and form of the free-surface. Initially, unyielded material is
ered through Newtonian and power-law assumptions. Further complication is then
ced through the Bingham model, where fluid yield stress is taken into account. More
l rheological modelling is constructed via the Herschel–Bulkley model, combining
c behaviour with yield stress presence. This is complimented by relaxing incompressible
tions, allowing the effects of compressibility to enter the problem. Results are presented
ady and transient flow scenarios and numerical solutions are validated against published
here is Focus upon on the effect of variation in compressibility parameter setting, inertia
power-law index and yield stress level, with regard to the evolving shape/location of the
rface and the response in extrudate swell. Extrudate swell is observed to decline with
se in power-law index. With increase in Reynolds number, extrudate swell decreases
finally reaching a plateau at high Reynolds number, in agreement with experimental
Swelling also decreases with rise in yield stress levels. The combination of these
eters within the compressible Herschel–Bulkley model renders it difficult to predict, a
the outcome in terms of die-swell behaviour.

rds: Die-Swell, Inelastic, Power-Law, Bingham, Herschel–Bulkley Model, Yield Stress.

troduction

days, extruded materials include metals, ceramics, polymers, paints, coatings
od products. Much research has focused attention on low pressure, high flow

ion processes in an effort to better understand and prevent interfacial ilities. The die-swell problem naturally introduces free surface modelling, cative transient evolution states, material rheology and influence of com-bility. In spite of a variety of industrial applications, modelling compressible urface flows itself remains a numerical challenge, largely due to the physical mena involved. For example, the *free surface* of the jet introduces mixed-boundary conditions on the flow. Various slip (velocity) conditions may be ent on the tube wall, while the jet itself supports traction (or stress) boundary tions. This local and sudden change in the type of boundary conditions will ate a singular stress field, accompanied with steep velocity gradients. There any approaches to treat free surface computations, such as the volume of echnique (VOF), marker and cell method (MAC) and the arbitrary Lagrangian an (ALE) scheme. Most studies have assumed incompressible flow and ded material through Newtonian or power-law model. A yield stress response be adapted through the Bingham model, or with inelastic effects, through the hel–Bulkley model. To date, the extrusion problem has attracted much interest the literature. Beverly and Tanner [1] analysed the effects of yield stress on late swell in a tube, and found that yield stress inclusion reduced the degree ell. Mitsoulis and co-workers [2] studied entry and exit flows of Bingham , observing the presence of unyielded regions within the flow. Compressible elastic domain remains relatively uncharted in the literature, Georgiou [3] ddressed non-Newtonian inelastic fluid modelling for compressible flows, ssing interest in slip effects. In addition, compressible flow computations covered by Webster and co-workers [4, 5].

overning equations

ompressible Newtonian fluid flow under isothermal setting, the governing ons may be expressed in non-dimensional form as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \qquad (1)$$

$$\mathrm{Re}\frac{\partial u}{\partial t} = \nabla \cdot t - \mathrm{Re}\, u \cdot \nabla u - \nabla p \qquad (2)$$

field variables are ρ, u, τ, p, for density, velocity, stress and pressure, tively. $\mathrm{Re} = \rho_0 U \cdot l / \mu_0$ represents the conventional dimensionless Reynolds er.

ess is related to field kinematics through a constitutive law, which is defined

$$\tau_{ij} = \mu\left(d_{ij} - 2/3\,(\nabla \cdot u)\delta_{ij}\right) \tag{3}$$

μ is the viscosity (constant or function of shear-rate, see on), δ_{ij} is the ecker tensor, $2d = \nabla u + \nabla u^T$ is the rate of deformation tensor, and the 2/3 vanishes under incompressible assumptions. Further equations are necessary below), relating to free surface computation, material modelling and ressibility considerations.

Compressibility considerations

compressible flow settings, the modified two-parameter (B,m) Tait equation considered to relate density to pressure. Thus,

$$(p + B)/B = \rho^m \tag{4}$$

differentiating the equation of state, one gathers [4]:

$$\frac{\partial p}{\partial \rho} = \frac{m \cdot (p + B)}{\rho} = c_{(x,t)}^2 \tag{5}$$

$c_{(x,t)}$ is the derived speed of sound. Such dependencies have influence upon ent evolution.

Free surface considerations

the extrusion flow problem, no-slip boundary conditions are assumed along all, and free kinematic conditions on the free surface. This generates a singular field at the die-exit. On the free surface, zero normal velocity, zero shear and normal stress are set. We appeal to the evolving free surface equation:

$$\frac{\partial h}{\partial t} = u_r - \upsilon_z \frac{\partial h}{\partial z} \tag{6}$$

$u = (u_r, v_z) = (\partial r / \partial t, \partial z/\partial t)$ is the velocity vector and $h=h(x,t)$ is the radial . The die-swell ratio is defined as $\chi = h_f / h_0$, where h_f and h_0 are the final late radius and die radius, respectively (see Fig. 1).

ALE-technique is performed to radially adjust mesh.

Material modelling considerations

o-called 'yield stress τ_0' (first introduced by [7]), governs the transition from like to liquid-like response. It is the presence of yielded and unyielded ns across the domain, which provides the intrinsic discontinuity within the . To overcome this deficiency, several modifications have been proposed n). The power-law model allows for a degree of deviation from Newtonian iour ($n = 1$). Thus, shear-thinning is observed for $n < 1$, and $n > 1$ corresponds ar-thickening. The generalised Herschel–Bulkley (HB) model provides further gical richness, incorporating both power-law type and Bingham type.

Flow equations

on-Newtonian fluids, the viscosity is considered as a nonlinear function of econd invariant (Π_d) of the rate-of-strain tensor (d_{ij}), which modifies (3) dingly. A Bingham material remains rigid when the shear-stress is below the stress τ_0, but flows like a Newtonian fluid when the shear-stress exceeds τ_0:

$$\tau = \left(\mu + \frac{\tau_0}{2|\Pi_d|^{\frac{1}{2}}} \right) \cdot \dot{\gamma} \;\text{ for } |\Pi_\tau| > \tau_0^2 \text{ ; and } \dot{\gamma} = 0 \;\text{ for }\; |\Pi_\tau| \le \tau_0^2 \quad (7)$$

panastasiou [8] proposed a modified Bingham model, by introducing a risation stress growth exponent (m) to control the rate-of-rise in stress, in the

$$\tau = \left(\mu + \tau_0 \frac{1 - e^{-m|\Pi_d|}}{2|\Pi_d|^{\frac{1}{2}}} \right) \cdot \dot{\gamma} \quad (8)$$

her rheological derivations to accommodate for shear characteristics, may be dered through power-law model:

$$\tau = \left(k \cdot |\dot{\gamma}|^{n-1} \right) \cdot \dot{\gamma} \quad (9)$$

k is the consistency parameter.

s the Herschel–Bulkley (HB) model that incorporates both a yield stress near behaviour. To address the shortcoming of infinite apparent viscosity at ing shear-rates, Mitsoulis [9], Alexandrou et al. [10] introduced the modified

$$= \left(k\,\Pi_d^{\frac{n-1}{2}} + \tau_0 \frac{1-e^{-m|\Pi_d|}}{|\Pi_d|^{\frac{1}{2}}} \right) \text{ for } |\Pi_\tau| > \tau_0^2 \text{; and } \dot{\boldsymbol{\gamma}} = 0 \text{ for } |\Pi_\tau| \le \tau_0{}^2 \quad (10)$$

umerical discretisation

e fractional-staged Taylor-Galerkin incremental pressure-correction (TGPC) work is considered (see [4, 5] for derivation). The *first phase* involves a tor-corrector doublet (Lax-Wendroff) for velocity and stress. The *second* is a pressure-correction scheme that ensures second-order accuracy in time. *d phase* recaptures the velocity field at the end-of-time step loop. Triangular ation is employed based on a quadratic velocity and linear pressure olations. For density, a piecewise-constant interpolation is employed, with ered density gradients, over an element. The discrete compressible TGPC be expressed via Eqs. (11–14). Note, the equations for compressible and pressible flows differences appear mainly under the continuity equation (13).

age 1a :

$$\left[\frac{M_\rho}{\Delta t/2} + \frac{1}{2} S_u\right]\left(\Delta U^{n+\frac{1}{2}}\right) = -\left[S_u U + N_\rho(U)U - L^{\mathrm{T}}\left\{P^n + \theta_1\left(P^n - P^{n-1}\right)\right\}P\right]^n \quad (11)$$

age 1b :

$$\left[\frac{M_\rho}{\Delta t} + \frac{1}{2} S_u\right]\left(\Delta U^*\right) = -\left[S_u U - L^{\mathrm{T}}\left\{P^n + \theta_1\left(P^n - P^{n-1}\right)\right\}P\right]^n - \left[N_\rho(U)U\right]^{n+\frac{1}{2}} \quad (12)$$

Stage 2 :

$$\left[\frac{M_C}{\Delta t^2} + \theta \cdot K\right]\left(\Delta P^{n+1}\right) = -\frac{1}{\Delta t} L_P \cdot U^* \quad (13)$$

Stage 3 :

$$\frac{M_\rho}{\Delta t}\left(U^{n+1} - U^*\right) = \theta\, L^{\mathrm{T}}\left(P^{n+1} - P^n\right) \quad (14)$$

umerical results and discussion

pproach is to start by analysing an incompressible Newtonian fluid and then natically introduce further complexity through inertia, compressibility, genera- Newtonian and HB representation. The capillary radius is held constant) and only half of the die-swell domain is analysed (symmetry). Applied lary conditions and die-swell dimensions are supplied in Fig. 1.

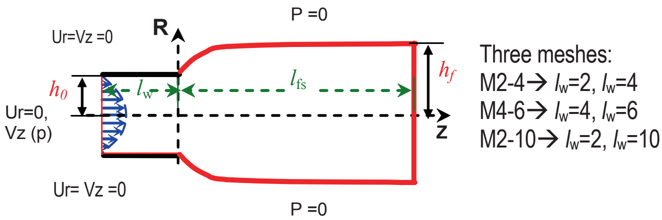


Fig. 1. Die-swell schema

Vewtonian case

Effect of die-swell design and mesh consistency analysis

s of mesh refinement, extrudate length and capillary length on the swelling are analysed. During the transient development, one observes a common num swelling at the same location followed by different swelling ratio steps jet, being larger with longer jet-lengths.

ce a steady development is achieved, swelling height reaches the ratio of independent of the level of refinement used. The transient pressure response ieved instantaneously, following a linear development trend in pressure-drop.

Effect of inertia

nerally accepted finding is that under incompressible assumptions, the jet ng reduces as inertia (Re) increases, see Georgiou and co-workers [11]. This arly illustrated in Fig. 2, when based on mesh M2-4, we detect two distinct ns: below Re = 7.5, there is expansion through jet swelling, whilst, above this vel, we observe compression of the jet, reaching a plateau of $\chi = 0.91$ at levels of Re > 40. There is a clear evidence of pressure-drop reduction as creases.

Influence of compressibility

ewtonian fluids and various compressible settings (Ma = 0.0 to Ma_{max} = 0.55), ee surface shape and extrudate swell-ratio are insignificantly affected by the proposed die-swell designs and mesh refinements. This fact was also high-d by Georgiou [12]. This is mainly due to the imposition of no-slip boundary tions on the wall. Detailed analysis of various slip conditions is the subject of re study.

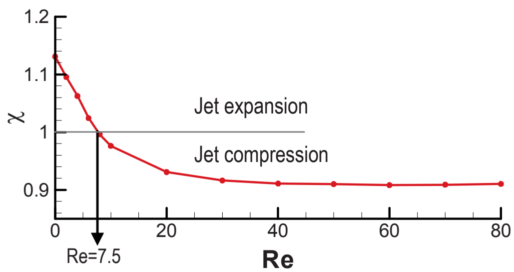


Fig. 2. Inertial effect on jet swelling; incompressible, Newtonian

nelastic power-law representation

Effect of power-index (n)

ariation of swelling ratio with power-index is depicted in Fig. 3. , under 0. Here, a longer channel length (M4-6) is selected to allow for the full opment of the velocity profile from its parabolic inlet state. In agreement Mitsoulis [13] findings, the general trend is that die-swell increases with the se of power-law index *n*, except in the range $0 < n < 0.2$, where a slight ction or negative swell is observed.

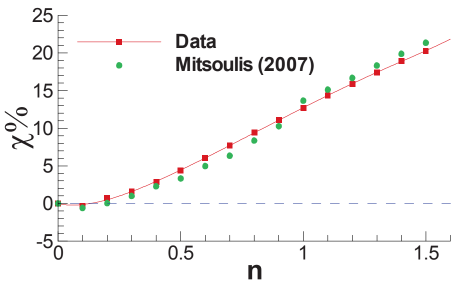


Fig. 3. Die-swell as a function of power-law index n for the power-law model

Viscoplasticity-Bingham yield stress

nalyse the effect of yield stress on the flow based on the modified Bingham . The (χ)-response against Bn is shown in Fig. 4.

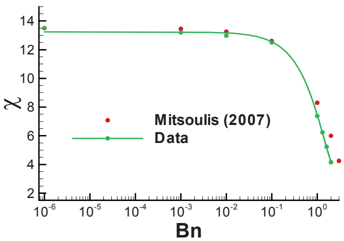
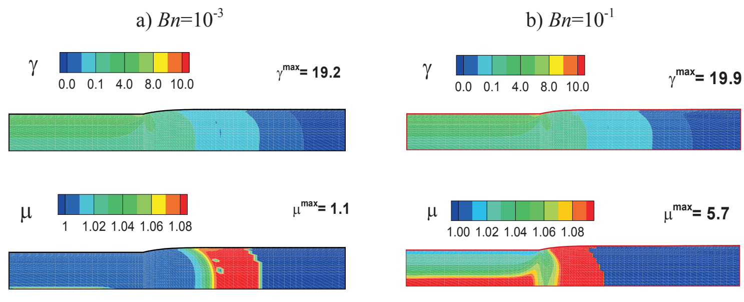


Fig. 4. Swelling ratio as function of yield stress with modified Bingham model

observe a substantial decrease of swelling for levels of yield stress above hilst there is a sustained plateau in swelling for *Bn* below 0.01, as in [13]. arly, the pressure-drop increase with increased yield stress, as the increase is pronounce for $Bn > 0.1$, whilst, pressure remains relatively constant over the n-range.

Herschel–Bulkley modelling

ing under the HB-model is governed by the variation of both power-law n and yield stress τ_0. At steady-state, the swelling under $n = 0.9$ and $\tau_0 = 0.1$ es a level of $\chi = 1.1$, as already observed under power-law modelling (Fig. is position is not affected by the yield stress level over this plateau range of lue (see Fig. 4). Steady-state shear-rate and viscosity contour plots for n=0.9 vo levels of Bn=10^{-3} and 10^{-1} are depicted in Fig. 5. The shear-rate contours milar for both levels of yield stress, whilst there is a clear increase in the num viscosity attained at larger τ_0 applied.



onclusions

lie-swell benchmark problem naturally introduces free surface modelling, separation point at the die-exit, provocative transient evolution states and ial modelling. Focus is placed on the jet shape dynamic evolution and the urface location. Steady and transient flow situations are presented under pressible and compressible assumptions, with findings validated against published data. Initially, unyielded material is considered through Newtonian ower-law assumptions. Further complication is then introduced through the am model, where fluid yield stress is taken into account. Subsequently, a rheological generalisation is built via the Herschel–Bulkley model.

e study demonstrated that extrudate swell is unaltered by compressibility lerations under no-slip wall conditions. However, it is expected that this position e altered under slip-wall settings. The swelling is observed to decline with ase in power-law index. Swelling also decreases with rise in yield stress . Therefore, predictions remain difficult, a priori, under the parameters nation within the Herschel–Bulkley model. Further challenges posed will be ed from its generalised Herschel–Bulkley model, to a novel visco-elasto- material modelling, permitting a direct comparison across regimes for ressible representation, ranging from viscoplastic, to viscoelastic, to visco- -plastic alternatives.

wledgments We gratefully acknowledge financial support under the EPSRC grant lex Fluids and Complex Flows – Portfolio Partnership', TH acknowledge research visit g from HEC of Pakistan.

rences

R. Beverly and R.I. Tanner. Numerical Analysis of Extrudate Swell in Viscoelastic aterials with Yield Stress. *J. Rheol.* 33 (1989) 989–1009.
S. Abdali, E. Mitsoulis and N.C. Markatos. Entry and exit flows of Bingham fluids. *J. heol.* 36 (1992) 389–407.
.C. Georgiou. The time-dependent, compressible Poiseuille and extrudate-swell flows of a arreau fluid with slip at the wall. *J. Non-Newt. Fluid Mech.* 109 (2003) 93–114.
.F. Webster, I.J. Keshtiban and F. Belblidia. Computation of weakly-compressible highly- scous liquid flows. *Eng. Comput.* 21 (2004) 777–804.
. Keshtiban, F. Belblidia and M.F. Webster. Numerical simulation of compressible scoelastic liquids. *J. Non-Newt. Fluid Mech.* 122 (2004) 131–146.
G. Tait, Physics and Chemistry of the Voyage of H.M.S. Challenger, 2, Part IV (1888), MSO, London, England.
C. Bingham. Fluidity and Plasticity. McGraw-Hill, New York, 1922.
C. Papanastasiou. Flows of materials with yield. *J. Rheol.* 31 (1987) 385–404.
Mitsoulis. Numerical simulation of planar entry flow for a polysobutylene solution using integral constitutive equation. *J. Rheol.* 37 (1993) 1029–1040.
N. Alexandrou, T.M. McGilvreay and G. Burgos. Steady Herschel–Bulkley fluid flow in

Taliadorou, G.C. Georgiou and E. Mitsoulis. Numerical simulation of the extrusion of
rongly compressible Newtonian liquids. *Rheol. Acta* 47 (2008).
.C. Georgiou. The compressible Newtonian extrudate swell problem. *Int. J. Num. Meth.
luids* 20 (1995) 255–261.
Mitsoulis. Annular extrudate swell of pseudoplastic and viscoplastic fluids. *J. Non-Newt.
luid Mech.* 141 (2007) 138–147.