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Moduli space of $\mathcal{N} = 4$ super Yang-Mills from AdS/CFT

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Abstract: We study $\mathcal{N} = 4$ super Yang-Mills theory compactified on an S^1 at zero temperature, with VEVs for two scalar bilinears and three independent current sources. We show that type IIB supergravity provides a complete holographic description of this setup, admitting both supersymmetric and non-supersymmetric AdS-soliton solutions that are asymptotically AdS_5 and smooth in the IR. The current sources correspond in 2+1 dimensions to Q-ball charge densities for $U(1)^3 \subset SO(6)_R$, and are geometrically realized as twists along three angular directions of the S^5. We demonstrate that the bulk dynamics encodes the full vacuum structure of the dual field theory and explicitly reconstruct the supersymmetric moduli space.

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Contents

1 Introduction

Strongly coupled gauge theories are notoriously difficult to analyze from first principles. The AdS/CFT correspondence [1, 2] circumvents this difficulty by recasting questions in the gauge theory in terms of a tractable dual supergravity description. If the geometry is everywhere regular, it means that the large-N, strongly coupled behavior of the QFT is fully captured by the supergravity theory. However, for example, the standard holographic description of the Coulomb branch of gauge theories is singular in the infrared [3–5].

It was recently found that, by compactifying the gauge theory on an S^1 and giving a VEV to the R-symmetry current, the infrared geometry can end smoothly, yielding a supersymmetric ground state [6] of the AdS-soliton type [7]. These solutions describe the confining regime of the field theory, and the validity of the supergravity approximation has been explored in a large number of papers [8–26].

The existence of two solutions, singular in the infrared and exhibiting qualitatively different behavior while sharing the same boundary conditions, was noticed in the early holographic explorations of the Coulomb branch [5]. In [27], IR-regular geometries with a single supergravity scalar were constructed, and it was shown that the phase boundary between the different Coulomb branch behaviors corresponds to the point at which the scalar VEV vanishes. Imposing UV boundary conditions fixes the asymptotic data, but regularity in the IR further constrains the solutions, determining the VEVs as functions of the sources and the energy scale. This structure naturally gives rise to a moduli space at each energy, consisting of all solutions compatible with a given set of sources. The locus where the scalar VEV vanishes signals a phase transition in the theory, a phenomenon recently shown to take place for supergravity black holes as well [28, 29]. In particular, the requirement of IR

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regularity dynamically selects admissible vacua, thereby promoting what would naively be arbitrary integration constants into physically meaningful order parameters of the dual theory.

In this paper we begin by constructing a soliton solution in the STU model of type IIB supergravity compactified on the S^5, with two independent scalars and three Abelian vectors. The solution has nontrivial supergravity scalars and vectors. It generalizes all previous constructions of AdS solitons within this model. This construction provides the first fully regular SUSY multi-charge solitonic completion of the Coulomb branch within the STU truncation, offering a controlled holographic laboratory to study confinement, symmetry breaking, and vacuum selection in strongly coupled gauge theories.[1] The solution is found by means of double Wick rotation (together with a non-trivial diffeomorphism) and a reparametrization of the black holes of [31], in such a way that they admit a massless limit. The massless sector contains the supersymmetric solutions we are interested in. Unlike the previously known single-scalar constructions, our solution allows for the simultaneous backreaction of multiple scalar fields and independent "charge" densities, leading to a qualitatively richer phase structure and a non-trivial multi-parameter moduli space of supersymmetric vacua.

The uplift to ten-dimensional type IIB supergravity warps the five dimensional spacetime, and squashes and twists the S^5. This allows an interpretation in the dual $\mathcal{N} = 4$ SYM and its S^1 reduction to 2+1 dimensions, in terms of deformations by scalars in the $\mathcal{O}_{20'}$ multiplet and Q-ball charge densities.

The outline of this paper is as follows. In section 2 we describe the five-dimensional solution. We present the Lagrangian and its associated solutions. Then we compute the VEVs and sources by means of an asymptotic analysis. We characterize the solution space and show that there are at most five solutions for each choice of boundary conditions. We then describe how to recover the previously known solutions. Finally we compute the moduli space of supersymmetric solutions for our new general soliton and we carry out the calculation of the Killing spinors. In section 3 we uplift the solution to ten dimensional type IIB supergravity. In section 4 we interpret our new solution from the dual viewpoint of four-dimensional $\mathcal{N} = 4$ SYM and its S^1 twisted compactification to 2+1 dimensions. In section 5 we present conclusions and future directions.

2 Five-dimensional perspective

In this section, we present a new family of solutions to the gauged STU model, which we subsequently uplift to type IIB supergravity. In both the five-dimensional and ten-dimensional descriptions, we have explicitly verified that all equations of motion are satisfied. We perform a detailed asymptotic analysis of the five-dimensional solutions and establish their interpretation as deformations of $\mathcal{N} = 4$ SYM, regarded as the UV fixed point of the theory. We systematically explore the space of solutions, placing particular emphasis on the supersymmetry-preserving sector, for which we derive the explicit form of the preserved Killing spinors. For specific regions of parameter space, we demonstrate how our new solutions interpolate to previously known backgrounds.

[1] In this paper, we do not show the confining character of the new solutions. For the special cases of [6, 27] solid arguments for the confining character of the dual field theory were given. We carefully discuss this in [30].

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2.1 The model

We shall study a truncation of type IIB supergravity compactified over a deformed S^5 with action

$$S_0=\frac{1}{2\kappa}\int\sqrt{-g}\left(R-\frac{(\partial\Phi_1)^2}{2}-\frac{(\partial\Phi_2)^2}{2}+\sum_{i=1}^{3}4L^{-2}X_i^{-1}-\frac{1}{4}X_i^{-2}(F^i)^2+\frac{1}{4}\epsilon^{\mu\nu\rho\sigma\lambda}A^1_\mu F^2_{\nu\rho}F^3_{\sigma\lambda}\right)\mathrm{d}^5x, \tag{2.1}$$

where F^i are two forms, related with gauge fields in the standard way $F^i=dA^i$, $X_i=e^{-\frac{1}{2}\vec{a}_i\cdot\vec{\Phi}}$, $\vec{\Phi}=(\Phi_1,\Phi_2)$ and

$$\vec{a}_1=\left(\frac{2}{\sqrt{6}},\sqrt{2}\right),\qquad \vec{a}_2=\left(\frac{2}{\sqrt{6}},-\sqrt{2}\right),\qquad \vec{a}_3=\left(-\frac{4}{\sqrt{6}},0\right). \tag{2.2}$$

This is the gauged STU model in $D=5$. Let us present the new family of solutions.

2.2 New AdS solitons

We study a new family of solutions, depending on the parameters $(q_1,q_2,q_3,M,\mathsf{q})$. We activate the two scalars (Φ_1,Φ_2) and the three gauge fields A^i, exciting all possible fields in the STU model.

2.2.1 Three charge solutions

We find the following solution of the gauged STU model in $D=5$. The metric tensor is given by are

$$\mathrm{d}s^2=\frac{f(r)}{H(r)^{2/3}}\mathrm{d}\varphi^2+\frac{H(r)^{1/3}}{f(r)}\mathrm{d}r^2+\frac{H(r)^{1/3}}{L^2}r^2(-\mathrm{d}t^2+\mathrm{d}x^2+\mathrm{d}y^2). \tag{2.3}$$

The scalars and the vectors are

$$\Phi_1=\frac{\sqrt{6}}{6}\log\left(\frac{H_1(r)H_2(r)}{H_3(r)^2}\right),\quad \Phi_2=-\frac{1}{\sqrt{2}}\log\left(\frac{H_2(r)}{H_1(r)}\right),\quad A^i=\left(\frac{Q_i}{r^2H_i(r)}-\mu_i\right)\mathrm{d}\varphi\,. \tag{2.4}$$

The metric function and the harmonic functions are

$$f(r)=\frac{r^2}{L^2}H(r)-\frac{M}{r^2}-\frac{\mathsf{q}}{r^4},\qquad H(r)=H_1(r)H_2(r)H_3(r)\,,\qquad H_i(r)=1+\frac{q_i^2}{r^2}\,. \tag{2.5}$$

This configuration is a solution of the equations of motion provided

$$Q_i^2=-Mq_i^2+\mathsf{q}. \tag{2.6}$$

Compared to [32], one novelty of this set of coordinates is that $\mathsf{q}\neq 0$, which has the property that the massless limit, $M=0$, yields a metric that might smoothly close at the origin, provided there is a r_0 where $f(r_0)=0$. This limit yields $Q_1^2=Q_2^2=Q_3^2$.

The regularity of the gauge fields at $r=r_0$, where by definition $f(r_0)=0$, fixes the constant μ_i in (2.4) to be

$$\mu_i=\frac{Q_i}{r_0^2H_i(r_0)}\,. \tag{2.7}$$

When an r_0 exists, the configuration is regular everywhere. The solution allows for a good $M\to 0$ limit, which yields new supersymmetric configurations, as we show further below. Now, let us focus on the asymptotic behavior of the family of backgrounds and relate this via holographic renormalisation [33, 34] to deformations of $\mathcal{N}=4$ SYM.

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2.3 The asymptotic analysis

To put the solution in standard asymptotically AdS form, we require that, at least,

$$H(r)^{1/3}\frac{r^2}{L^2} = \frac{\rho^2}{L^2} + O(\rho^{-3})\,. \tag{2.8}$$

The fall-off is fixed to gauge away all the possible contributions to the energy momentum tensor from the three-dimensional Minkowski part of the metric. We find that the asymptotic change of coordinate that implements this is

$$r = \rho - \frac{q_1^2+q_2^2+q_3^2}{6\rho} + (q_2+q_1-q_3)(q_1+q_2+q_3)(q_1+q_3-q_2)(-q_2-q_3+q_1)\frac{1}{24\rho^3} + O(\rho^{-5}). \tag{2.9}$$

This yields

$$g_{rr}\left(\frac{dr}{d\rho}\right)^2 = g_{\rho\rho} = \frac{L^2}{\rho^2} + \frac{1}{9}\frac{L^2(2q_1^2q_2^2+2q_1^2q_3^2+2q_2^2q_3^2-2q_1^4-2q_2^4-2q_3^4+9ML^2)}{\rho^6} + O(\rho^{-8}), \tag{2.10}$$

$$-g_{tt} = g_{yy} = g_{xx} = \frac{\rho^2}{L^2} + O(\rho^{-4}), \tag{2.11}$$

$$g_{\varphi\varphi} = \frac{\rho^2}{L^2} - \frac{M}{\rho^2} + O(\rho^{-4}), \tag{2.12}$$

and for the gauge fields

$$A_i = \left(\frac{Q_i}{\rho^2} - \mu_i + \frac{Q_i(q_1^2+q_2^2+q_3^2-3q_i^2)}{3\rho^4} + O(\rho^{-6})\right)\mathrm{d}\varphi. \tag{2.13}$$

We plug this asymptotic expansion in the scalar fields. We get

$$\Phi_1 = \frac{1}{\sqrt{6}}\frac{q_1^2+q_2^2-2q_3^2}{\rho^2} + O(\rho^{-4})\,,$$
$$\Phi_2 = \frac{1}{\sqrt{2}}\frac{q_1^2-q_2^2}{\rho^2} + O(\rho^{-4})\,. \tag{2.14}$$

These scalar fields saturate the Breitenlohner-Freedman bound, and therefore the source term is associated with the logarithmic mode, that is absent here.

2.4 Holographic renormalization

The analysis of the precise form of the VEVs, sources and the dual energy momentum tensor can be found in [27], which closely follow the standard references [33, 35, 36]. Here we provide the formulae for further reference and facilitate the verification of our results. The action plus counterterms is

$$S = S_0 + \frac{1}{\kappa}\int_{M^3\times S^1} K\sqrt{-h}\,\mathrm{d}^4x + \frac{1}{2\kappa}\int_{M^3\times S^1}\sqrt{-h}\left(-\frac{6}{L} + \frac{1}{2L}\left(\frac{1}{\ln(\rho/\rho_0)} - 2\right)(\Phi_1^2+\Phi_2^2)\right)\mathrm{d}^4x\,, \tag{2.15}$$

where S_0 is the bulk action, $g_{\mu\nu} = h_{\mu\nu} + N_\mu N_\nu$, and N_μ is the outward pointing normal to the boundary and $K_{\mu\nu} = \frac{1}{2}\nabla_\mu N_\nu + \frac{1}{2}\nabla_\nu N_\mu$ is the extrinsic curvature. The boundary geometry is that of a three dimensional Minkowski spacetime times a circle,

$$\mathrm{d}s^2 = \gamma_{ab}\mathrm{d}x^a\mathrm{d}x^b = -\mathrm{d}t^2 + \mathrm{d}y^2 + \mathrm{d}x^2 + \mathrm{d}\varphi^2\,, \tag{2.16}$$

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which is the background spacetime for the quantum field theory. The scalar fields have the general asymptotic expansion

$$\Phi_i = J_{\Phi_i}\frac{\ln(\rho/\rho_0)}{\rho^2} + \frac{\Phi_{0i}}{\rho^2} + O\left(\frac{\ln(\rho/\rho_0)}{\rho^4}\right) , \tag{2.17}$$

with the on-shell variation

$$\frac{\delta S}{\delta J_{\Phi_i}} = \frac{1}{2\kappa L^5}\Phi_{0i} . \tag{2.18}$$

The finite scalar source is related to the gravity one by the relation $J_{\Phi_i} = L^4 J_{\Phi_i}^{finite}$. Therefore, we obtain the following VEVs, which are order N^2 in the QFT

$$\langle \mathcal{O}_1 \rangle = \frac{q_1^2 + q_2^2 - 2q_3^2}{2\kappa\sqrt{6}L} , \tag{2.19}$$

$$\langle \mathcal{O}_2 \rangle = \frac{q_1^2 - q_2^2}{2\kappa\sqrt{2}L} . \tag{2.20}$$

The vacuum expectation value of the energy momentum tensor of the dual field theory is

$$\begin{aligned}\langle T_{ab} \rangle &= \frac{-2}{\sqrt{-\gamma}}\frac{\delta S}{\delta\gamma^{ab}} \\ &= \lim_{\rho\to\infty}\frac{\rho^2}{L^2}\frac{-2}{\sqrt{-h}}\frac{\delta S}{\delta h^{ab}} \\ &= \lim_{\rho\to\infty}\frac{\rho^2}{L^2\kappa}\left(h_{ab}K - K_{ab} - \frac{3}{L}h_{ab} - \frac{1}{2L}h_{ab}(\Phi_1^2 + \Phi_2^2)\right) .\end{aligned} \tag{2.21}$$

The scalars contribute to the dual energy-momentum tensor in a conformal invariant form and the results of [37] apply. The dual energy-momentum tensor is

$$\langle T_{tt} \rangle = -\frac{M}{2L^3\kappa}, \qquad \langle T_{xx} \rangle = \langle T_{yy} \rangle = \frac{M}{2L^3\kappa}, \qquad \langle T_{\varphi\varphi} \rangle = -\frac{3M}{2L^3\kappa}. \tag{2.22}$$

Let us now carefully study the space of solutions, parameterizing different quantities in terms of the boundary values of fields.

2.5 The solution space

From the gravity point of view, to construct the space of solutions is equivalent to parameterize the observables in terms of the boundary conditions. The main observable here is the energy, which is controlled by the parameter M in the metric, see eq. (2.22). On the other hand, the boundary conditions are the asymptotic values of the gauge fields μ_i and Δ, the period of the coordinate $\varphi \in [0, \Delta]$. This period is defined by the regularity condition (the space closes smoothly at $r = r_0$ where $f(r_0) = 0$)

$$\Delta = \frac{4\pi H(r_0)^{1/2}}{|f'(r_0)|} . \tag{2.23}$$

The absolute value in eq. (2.23) is important. Indeed,

$$f'(r_0) = \frac{2}{r_0^3 L^2}\left(h_2 h_3 + h_1 h_3 + h_1 h_2 - ML^2\right) , \tag{2.24}$$

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where we define

$$h_i \equiv H_i(r_0) r_0^2 \,. \tag{2.25}$$

The quantities h_i have the advantage of being well defined at $r_0 = 0$ and $h_i \in \mathbb{R}^+$. Therefore, $f'(r_0)$ can, in principle, be negative when M is large enough (although we shall see this is not the case). Hence, we use the definition

$$\Delta = \nu \frac{4\pi H(r_0)^{1/2}}{f'(r_0)} \,, \tag{2.26}$$

with $\nu = \pm 1$ to ensure that $\Delta > 0$.

From the QFT perspective, we would like to express the vacuum expectation values (VEVs) in terms of the sources, since this allows in principle for the calculation of higher n-point functions by derivation of the one-point function at nonzero source. To proceed with this program, we note that the regularity condition on the gauge fields implies that

$$Q_i = h_i \mu_i \,. \tag{2.27}$$

The existence of a region where $f(r_0) = 0$ implies that

$$\mathtt{q} = -M r_0^2 + \frac{h_1 h_2 h_3}{L^2} \,. \tag{2.28}$$

The mass parameter can be found replacing eq. (2.28) into eq. (2.26), using eq. (2.24). In fact,

$$M = \frac{h_2 h_3 + h_1 h_3 + h_1 h_2}{L^2} - 2\pi\nu \frac{\sqrt{h_1 h_2 h_3}}{\Delta} \,. \tag{2.29}$$

The remaining equations are $Q_i^2 - \mathtt{q} + M q_i^2 = 0$ (which are equivalent to the Einstein equations). These read

$$\mu_1^2 = -\frac{h_3 + h_2}{L^2} + 2\pi\nu \frac{\sqrt{h_1 h_2 h_3}}{h_1\, \Delta} = \frac{h_2 h_3}{h_1 L^2} - \frac{M}{h_1} \,, \tag{2.30}$$

$$\mu_2^2 = -\frac{h_3 + h_1}{L^2} + 2\pi\nu \frac{\sqrt{h_1 h_2 h_3}}{h_2\, \Delta} = \frac{h_1 h_3}{h_2 L^2} - \frac{M}{h_2} \,, \tag{2.31}$$

$$\mu_3^2 = -\frac{h_2 + h_1}{L^2} + 2\pi\nu \frac{\sqrt{h_1 h_2 h_3}}{h_3\, \Delta} = \frac{h_1 h_2}{h_3 L^2} - \frac{M}{h_3} \,. \tag{2.32}$$

We readily see that $h_i > 0$ implies that $\nu = 1$ and $\Delta < \infty$. The equations (2.30), (2.31), (2.32) can be decoupled to obtain $h_i = h_i(\mu_1, \mu_2, \mu_3, \Delta)$. These yield two linear equations for (let us choose h_1 and h_2) and a quintic equation $P(Z) = 0$ for the variable

$$Z = \frac{\Delta^2}{4\pi^2 L^4} h_3 .$$

The quintic equation for Z reads

$$\begin{aligned} P(Z) = {} & 4Z^5 + \left(8\psi_2^2 - 5 + 8\psi_1^2 - 4\psi_3^2\right) Z^4 + (14\psi_1^2\psi_2^2 - 2\psi_3^2 + \psi_3^4 - 6\psi_2^2\psi_3^2 + 5\psi_1^4 + 5\psi_2^4 - 6\psi_3^2\psi_1^2 \\ & -6\psi_1^2 + 1 - 6\psi_2^2) Z^3 + \Big(-6\psi_1^2\psi_2^2 - 8\psi_2^2\psi_1^2\psi_3^2 - 2\psi_2^2\psi_3^2 - 2\psi_3^2\psi_1^2 + \psi_1^2 + \psi_1^6 + \psi_2^6 + \psi_2^2 \\ & -2\psi_2^4\psi_3^2 - 2\psi_2^4 + \psi_2^2\psi_3^4 + 7\psi_2^4\psi_1^2 + \psi_1^2\psi_3^4 + 7\psi_2^2\psi_1^4 - 2\psi_1^4 - 2\psi_1^4\psi_3^2\Big) Z^2 + \psi_1^2\psi_2^2(-1 - 2\psi_3 \\ & + \psi_1^2 + \psi_2^2 - \psi_3^2)\left(-1 + 2\psi_3 + \psi_1^2 + \psi_2^2 - \psi_3^2\right) Z - \psi_2^4\psi_1^4 , \end{aligned} \tag{2.33}$$

where we redefined

$$\mu_i = \frac{2\pi L}{\Delta}\psi_i. \tag{2.34}$$

This completely determines the space of soliton solutions. Indeed, the polynomial equation $P(Z) = 0$, yields $Z = Z(\psi_i)$, which can be used to find $\langle T_{tt}\rangle$. The insight that we can obtain from this analysis is that there can be several different solutions for a given value of the sources. Each solution corresponds to a different positive root of $P(Z) = 0$.

Each of these solutions correspond to a branch of moduli space, or a certain quantum phase, and will be dual to a corresponding branch of solutions in the gravity dual.

With this general formalism, it is satisfactory to recover known solutions. Let us do this.

2.5.1 Recovering previously known solutions

By choosing different values of ψ_i, we find previously known solutions.

Einstein-Maxwell-AdS. When $h_1 = h_2 = h_3$ the scalar fields vanish everywhere and the solution is the Einstein-Maxwell-AdS-Soliton of [6]. When $\psi_1 = \psi_2 = \psi_3 = \psi$ the quintic polynomial yields

$$P(Z) = (-1+Z)\,(Z+\psi^2)^2\,(-Z+4Z^2+4Z\psi^2+\psi^4)\,, \tag{2.35}$$

which has two non-trivial solutions that exist provided $\psi^2 < 8^{-1}$. The root at $Z = 1$ is unphysical as yields $\psi^2 < 1$ when replaced in eqs. (2.30), (2.31), (2.32).

Truncation to one scalar. If $h_1 = h_2$, then $\Phi_2 = 0$ and we recover the solution of [27]. The quintic polynomial is now

$$P(Z) = (Z{+}\psi_1^2)^2\,(Z{-}5Z^2{+}4Z^3{-}2Z\psi_1^2{+}8Z^2\psi_1^2{-}\psi_1^4{+}4Z\psi_1^4{-}2Z\psi_2^2{-}4Z^2\psi_2^2{-}4Z\psi_1^2\psi_2^2{+}Z\psi_2^4). \tag{2.36}$$

To reproduce the cubic polynomial of equation (3.26) of [27] it is necessary to find the equation satisfied by the combination $X = \frac{h_1}{h_3}$, in the case $\psi_2 = \psi_1$. We find

$$0 = \psi_1^2X^3 + \left(4\psi_1^4 - 4\psi_3^2\psi_1^2 - \psi_1^2 + \psi_3^4 - \psi_3^2\right)X^2 - \psi_3^2\left(4\psi_1^2 - 1 - 2\psi_3^2\right)X + \psi_3^4\,, \tag{2.37}$$

which is indeed in full agreement with [27].

Einstein-AdS. The Horowitz-Myers solution has energy density

$$E_0 = -\frac{\pi^4L^3}{2\kappa\Delta^4}\,. \tag{2.38}$$

In our variables it corresponds to having no sources for the vectors, $\mu_i = 0$. From eqs. (2.30), (2.31), (2.32), this implies that $h_ih_j = ML^2$ for $i{\neq}j$. This, substituted in eq. (2.29), yields $M = \frac{\pi^4L^6}{\Delta^4}$, and we find that $\langle T_{tt}\rangle = E_0$.

It is algebraically hard to find other generic solutions. This is ameliorated if we consider the massless limit $M = 0$.

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2.5.2 The general massless solution

Dealing with the full quintic polynomial $P(Z)$ is very involved. We focus on the massless solutions of $P(Z) = 0$. As we show in the next section, these are supersymmetric backgrounds. In this case, the zero energy condition $\langle T_{\mu\nu}\rangle = 0$ implies that $M = 0$ in eq. (2.29). We find that on this restricted *moduli space*

$$\sqrt{\frac{h_1 h_2}{h_3}} + \sqrt{\frac{h_1 h_3}{h_2}} + \sqrt{\frac{h_2 h_3}{h_1}} = \frac{2\pi L^2}{\Delta}\,. \tag{2.39}$$

It then follows from eqs. (2.30), (2.31), (2.32) that

$$|\mu_1| + |\mu_2| + |\mu_3| = \frac{2\pi L}{\Delta}\,. \tag{2.40}$$

The IR variables h_i in terms of the sources μ_i are then given by

$$h_1 = L^2|\mu_2\mu_3|\,, \qquad h_2 = L^2|\mu_1\mu_3|\,, \qquad h_3 = L^2|\mu_1\mu_2|\,. \tag{2.41}$$

The scalar VEVs are proportional to the following combinations of the same sources,

$$q_1^2 - q_2^2 = h_1 - h_2 = L^2|\mu_3|(|\mu_2| - |\mu_1|)\,, \tag{2.42}$$

$$q_1^2 + q_2^2 - 2q_3^2 = h_1 + h_2 - 2h_3 = L^2(|\mu_2\mu_3| + |\mu_1\mu_3| - 2|\mu_1\mu_2|). \tag{2.43}$$

Expressing $\mu_i = (2\pi L/\Delta)\psi_i$, we find that

$$\langle \mathcal{O}_1\rangle = \frac{q_1^2 + q_2^2 - 2q_3^2}{2\kappa\sqrt{6}L} = \frac{(2\pi)^2 L^3}{\kappa}\frac{1}{2\sqrt{6}\Delta^2}(|\psi_2\psi_3| + |\psi_1\psi_3| - 2|\psi_1\psi_2|)\,, \tag{2.44}$$

$$\langle \mathcal{O}_2\rangle = \frac{q_1^2 - q_2^2}{2\kappa\sqrt{2}L} = \frac{(2\pi)^2 L^3}{\kappa}\frac{1}{2\sqrt{2}\Delta^2}|\psi_3|(|\psi_2| - |\psi_1|)\,, \tag{2.45}$$

which are the strongly coupled, large-N, Yang-Mills VEVs in terms of the dimensionless variables ψ_i (which are independent of the number of colors N, it only appears through the overall factor $\frac{(2\pi)^2 L^3}{\kappa} = N^2$). This vacuum state is characterized by the VEVS of the scalar as shown in figure 1.

The diagram of figure 1 can be interpreted as the moduli space of supersymmetric solutions parameterized by the sources $(|\psi_1|, |\psi_2|)$. The lines and curves partition the space into regions characterized by different relative signs of the operator expectation values. Crossing a boundary does not lift the degeneracy of the vacuum but instead corresponds to a continuous change in the sign of a given VEV. All points in the diagram thus represent distinct supersymmetric vacua with identical vanishing energy at zero temperature and zero chemical potential.

This suggests another interpretation, as a phase diagram for quantum phases, with the blue and green lines (representing $\langle\mathcal{O}_2\rangle = 0$ and $\langle\mathcal{O}_1\rangle = 0$, respectively) separating qualitatively different quantum phases (defined by quantum fluctuations, not thermal fluctuations). The intersection is then a quantum critical point. Let us now study the Killing spinors preserved by these solutions with $M = 0$.

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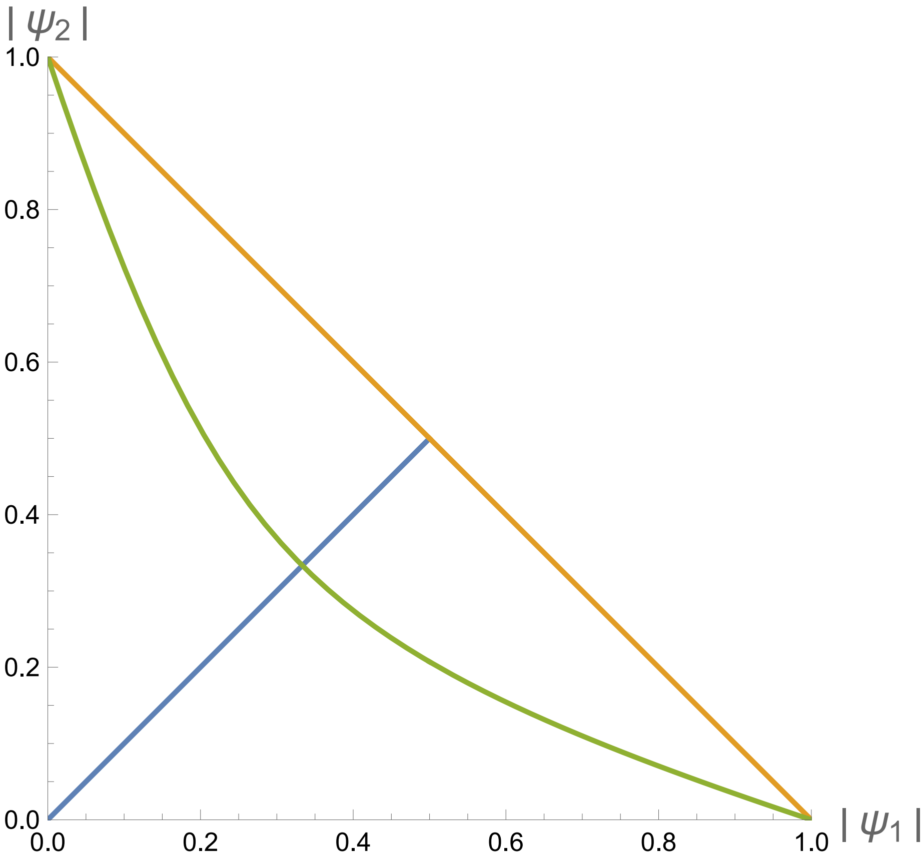

Figure 1. Moduli Space. Supersymmetric solutions exist below the orange line. Above the green line $\langle\mathcal{O}_1\rangle < 0$ and below the green line $\langle\mathcal{O}_1\rangle > 0$, $\langle\mathcal{O}_1\rangle = 0$ on the green line. The solutions that lie on the blue line have $\langle\mathcal{O}_2\rangle = 0$ and are the supersymmetric solitons of [27]. Above the blue line $\langle\mathcal{O}_2\rangle > 0$ and below the blue line $\langle\mathcal{O}_2\rangle < 0$. The point of intersection between the green and blue lines happens at $|\psi_1| = |\psi_2| = |\psi_3| = \frac{1}{3}$, which is the supersymmetric soliton of [6].

2.6 Supersymmetry

Let us label the scalar by $\Phi_s = (\Phi_1, \Phi_2)$. The Killing spinor equations for gauged STU model can be written in terms of the complex spinor ϵ defined in terms of the symplectic Majorana spinor $\tilde{\epsilon}^a$ as follows $\epsilon = \tilde{\epsilon}^1 + \mathrm{i}\tilde{\epsilon}^2$. The pair of spin 1/2 and the spin 3/2 variations are

$$\delta\lambda_s = \sum_{i=1}^{3} \partial_s X_i \left[-\frac{1}{4X_i^2}\left(\not{F}^i + \mathrm{i}\not{d}X_i\right) + \frac{\mathrm{i}}{2L}\right]\epsilon = 0\,, \tag{2.46}$$

$$\delta\psi_\mu \mathrm{d}x^\mu = \mathrm{d}\epsilon + \frac{1}{4}\omega_{ab}\gamma^{ab} + \sum_{i=1}^{3}\left[\frac{\mathrm{i}}{4!X_i}[2\not{e}\not{F}^i - 6e^a(\iota_a F^i)_{/}] + \frac{1}{3!L}\not{e}X_i - \frac{\mathrm{i}}{2L}A^i\right] = 0\,, \tag{2.47}$$

where we have defined the slash of a generic p-form $F_p = \frac{1}{p!}F_{a_1\ldots a_p}e^{a_1}\wedge\cdots\wedge e^{a_p}$ as

$$\not{F}_p \equiv (F_p)_{/} \equiv \frac{1}{p!}F_{a_1\ldots a_p}\gamma^{a_1\ldots a_p}\,, \tag{2.48}$$

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and ι_a stands for the contraction operator. In what follows, we choose the vielbein basis

$$\begin{aligned} e^0 &= \frac{r}{L}H(r)^{1/6}\mathrm{d}t\,, \quad e^1 = \frac{r}{L}H(r)^{1/6}\mathrm{d}x\,, \quad e^2 = \frac{r}{L}H(r)^{1/6}\mathrm{d}y\,, \\ e^3 &= \frac{H(r)^{1/6}}{f(r)^{1/2}}\mathrm{d}r\,, \quad e^4 = \frac{\sqrt{f(r)}}{H(r)^{1/3}}\mathrm{d}\varphi. \end{aligned} \tag{2.49}$$

For the background (2.3)–(2.4), the vanishing of the determinants of the spin 1/2 matrices in (2.46) requires that $M = 0$. In this limit, the configuration is regular and there exist an r_0 such that $f(r_0) = 0$ when $\mathtt{q} > 0$, hence the configuration is still regular. The equations (2.46) for $s = 1, 2$ coincide and is given by

$$[L\sqrt{\mathtt{q}}\gamma^{34} + \mathrm{i}r^2 L f(r)^{1/2}\gamma^3 + \mathrm{i}r^3 H(r)^{1/2}]\epsilon = 0\,. \tag{2.50}$$

This can be written as a projector equation

$$\frac{1}{2}(1 + \mathrm{x}(r)\gamma^3 + \mathrm{y}(r)\gamma^4)\epsilon = 0\,, \tag{2.51}$$

with

$$\mathrm{x}(r) = \frac{r}{L}\frac{H(r)^{1/2}}{f(r)^{1/2}}\,, \qquad \mathrm{y}(r) = -\frac{\mathrm{i}\sqrt{\mathtt{q}}}{r^2 f(r)^{1/2}}\,, \tag{2.52}$$

satisfying $\mathrm{x}^2 + \mathrm{y}^2 = 1$, for any r.

Regarding the spin 3/2 equation, in the coordinate basis, the component r reads

$$\partial_r\epsilon + f(r)^{-1/2}\sum_i \frac{1}{H_i(r)}\left(\frac{\mathrm{i}\sqrt{\mathtt{q}}}{3r^3}\gamma^4 + \frac{H(r)^{1/2}}{6L}\gamma^3\right)\epsilon = 0\,. \tag{2.53}$$

By using the projector equation (2.51), it can be recast in the form

$$\partial_r\epsilon = [\mathrm{a}(r) + \mathrm{b}(r)\gamma^3]\epsilon\,, \tag{2.54}$$

with

$$\mathrm{a}(r) = -\frac{1}{3r}\sum_i \frac{1}{H_i(r)}\,, \qquad \mathrm{b}(r) = -\frac{H(r)^{1/2}}{2Lf(r)^{1/2}}\sum_i \frac{1}{H_i(r)}\,. \tag{2.55}$$

Following [38], the sufficient condition to find a common solution of (2.51) and (2.54) is that

$$\mathrm{x}'(r) + 2\mathrm{b}(r)\mathrm{y}(r)^2 = 0\,, \tag{2.56}$$

which is satisfied in our case. Then, the spinor that satisfies (2.51) and (2.54) is given by

$$\epsilon = \frac{f(r)^{1/4}}{2H(r)^{1/6}}\left(\alpha_+(r) - \alpha_-(r)\gamma^4\right)(1 - \gamma^3)\chi(t, \varphi, x, y)\,, \tag{2.57}$$

$$\alpha_\pm(r) \equiv \sqrt{1 \pm \frac{rH(r)^{1/2}}{Lf(r)^{1/2}}}\,. \tag{2.58}$$

Given the projector equation (2.51), it is straightforward to show that the spin 3/2 equations along t, φ, x, y reduce to

$$\begin{aligned} \partial_t \chi &= 0\,, \\ \partial_\varphi \chi + \frac{\mathrm{i}\sqrt{\mathtt{q}}}{2L}\sum_i \mu_i \chi &= 0\,, \\ \partial_x \chi &= 0\,, \\ \partial_y \chi &= 0\,. \end{aligned} \tag{2.59}$$

Therefore, the general solution to the Killing spinor equations is

$$\epsilon(r,\varphi) = \exp\left(-\frac{\mathrm{i}\varphi\sqrt{\mathtt{q}}}{2L}\sum_i \mu_i\right)\frac{f(r)^{1/4}}{2H(r)^{1/6}}\left(\alpha_+(r) - \alpha_-(r)\gamma^4\right)(1-\gamma^3)\epsilon_0\,,$$

where ϵ_0 is a constant complex spinor with four independent components. The projector in the solution reduces these to two, so the 5D background with $M = 0$ preserves four supercharges.

Now that we have established that the solutions with $M = 0$ preserve a fraction of the SUSY, we lift the solution to type IIB supergravity.

3 Uplift to type IIB supergravity

The uplift to type IIB supergravity of the gauged STU model in $D = 5$ was constructed in [32] as a compactification on a deformed S^5. The 10-dimensional metric and five-form field strength are given by[2]

$$\begin{aligned} \mathrm{d}s_{10}^2 &= \tilde{\Delta}^{1/2}\mathrm{d}s_5^2 + L^2\tilde{\Delta}^{-1/2}\sum_{i=1}^3 X_i^{-1}\left[\mathrm{d}\nu_i^2 + \nu_i^2\left(\mathrm{d}\phi_i + \frac{1}{L}A_i\right)^2\right]\,, \\ F_5 &= G_5 + \star G_5\,, \\ G_5 &= \frac{2}{L}\sum_i\left(X_i^2\nu_i^2 - \tilde{\Delta}X_i\right)\star_5 1 + \frac{L}{2}\sum_i X_i^{-1}\star_5 \mathrm{d}X_i\wedge \mathrm{d}(\nu_i^2) + \\ &\quad + L^2\sum_i X_i^{-2}\nu_i \mathrm{d}\nu_i \wedge\left(\mathrm{d}\phi_i + \frac{1}{L}A_i\right)\wedge \star_5 F_i\,. \end{aligned}$$

The Hodge dual in 10-dimension is denoted by $\star$, while the Hodge dual in five-dimensions with respect to metric $\mathrm{d}s_5^2$ is $\star_5$. We have defined

$$\tilde{\Delta} = \sum_i X_i\nu_i^2\,, \qquad \sum_i \nu_i^2 = 1\,. \tag{3.1}$$

Focusing on the metric, we define $\hat{f}(r) \equiv \frac{L^2}{r^2}f(r)$, so the ten-dimensional metric reads

$$\begin{aligned} \mathrm{d}s_{10}^2 =\ & \tilde{\Delta}^{1/2}\frac{r^2}{L^2}H(r)^{1/3}\left(\frac{\hat{f}(r)}{H(r)}\mathrm{d}\varphi^2 + \frac{L^4}{r^4}\frac{\mathrm{d}r^2}{\hat{f}(r)} + \mathrm{d}x_{1,2}^2\right) \\ & + L^2\tilde{\Delta}^{-1/2}\sum_{i=1}^3 X_i^{-1}\left[\mathrm{d}\nu_i^2 + \nu_i^2\left(\mathrm{d}\phi_i + \frac{1}{L}A_i\right)^2\right]\,, \end{aligned} \tag{3.2}$$

[2] We find the second term sign in G_5 is opposite respect to [32]. Our convention for the Hodge dual of a p-form $F_p = \frac{1}{p!}F_{\mu_1\ldots\mu_p}\mathrm{d}x^{\mu_1}\wedge\cdots\wedge\mathrm{d}x^{\mu_p}$ is $\star F_p = \frac{\sqrt{-g}}{(D-p)!p!}F^{\mu_1\cdots\mu_p}\epsilon_{\mu_1\ldots\mu_p\nu_1\ldots\nu_{D-p}}\mathrm{d}x^{\nu_1}\wedge\cdots\wedge\mathrm{d}x^{\nu_{D-p}}$, for a D-dimensional manifold with metric $g_{\mu\nu}$ and determinant g and $\epsilon_{12\ldots D} = +1$. We have checked the equations of motion for this type IIB configuration.

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Explicitly, the functions X_i read

$$X_1^3 = \frac{H_2(r)H_3(r)}{H_1^2(r)}\,, \qquad X_2^3 = \frac{H_1(r)H_3(r)}{H_2^2(r)}\,, \qquad X_3^3 = \frac{H_1(r)H_2(r)}{H_3^2(r)}\,. \tag{3.3}$$

We consider the following parameterization for ν_i as

$$\nu_1 = \cos\theta\,\cos\psi\,, \qquad \nu_2 = \cos\theta\,\sin\psi\,, \qquad \nu_3 = \sin\theta\,. \tag{3.4}$$

With this, the metric reads

$$\begin{aligned} ds_{10}^2 = {} & \frac{r^2}{L^2}\tilde{\Delta}^{1/2}H(r)^{1/3}\left(\frac{\hat{f}(r)}{H(r)}d\varphi^2 + \frac{L^4}{r^4}\frac{dr^2}{\hat{f}(r)} + dx_{1,2}^2\right) + \\ & + \frac{L^2}{\tilde{\Delta}^{1/2}}\left\{\mathtt{A}^2 d\theta^2 + \mathtt{B}^2 d\psi^2 + 2\mathtt{C}d\theta d\psi + \sum_{i=1}^{3}\frac{\nu_i^2}{X_i}\left(d\phi_i + \frac{1}{L}A_i\right)^2\right\}\,. \end{aligned} \tag{3.5}$$

Here we have defined

$$\begin{aligned} & \mathtt{A}^2 = \sin^2\theta\left(\frac{\cos^2\psi}{X_1} + \frac{\sin^2\psi}{X_2}\right) + \frac{\cos^2\theta}{X_3}\,, \quad \mathtt{B}^2 = \cos^2\theta\left(\frac{\sin^2\psi}{X_1} + \frac{\cos^2\psi}{X_2}\right)\,, \\ & \mathtt{C} = \cos\theta\,\sin\theta\,\cos\psi\,\sin\psi\left(\frac{1}{X_1} - \frac{1}{X_2}\right)\,, \quad \tilde{\Delta} = \cos^2\theta(X_1\cos^2\psi + X_2\sin^2\psi) + X_3\sin^2\theta\,. \end{aligned} \tag{3.6}$$

These expressions simplify in the limit $q_1 = q_2$, for which $X_1 = X_2$ and $\tilde{\Delta}$ is a function of θ and r. The Type IIB background is useful to explore the non-perturbative dynamics of $\mathcal{N} = 4$ SYM deformed by VEVs and flowing to a gapped QFT in $(2+1)$ dimensions. We briefly elaborate on the deformation in the next section. One can also calculate interesting observables in the non-perturbative QFT, for example, Wilson and 't Hooft loops, entanglement entropy, masses of glueballs, etc. We leave this for future work [30].

4 Field theory interpretation of the supersymmetric branch

In this section we interpret the massless ($M = 0$) soliton solutions constructed above from the viewpoint of the dual field theory.

We consider 4D $\mathcal{N} = 4$ Super Yang-Mills theory on $\mathbb{R}^{1,2} \times S^1$, where the circle is parameterized by the coordinate φ appearing in the boundary metric eq. (2.16). The compactification scale is set by the period Δ in eq. (2.23). In what follows, we also consider supersymmetry breaking spin-structure on the circle, that is, anti-periodic boundary conditions for the fermions. To restore 4D $\mathcal{N} = 1$ worth of supersymmetry, in addition to the geometric compactification, we turn on background flat connections for the Cartan subgroup $U(1)^3 \subset SO(6)_R$, thus breaking the global R-symmetry to[3] $U(1)_R \times U(1)_F^2$. In the bulk these correspond to the boundary values of the gauge fields,

$$A_i \longrightarrow \mu_i\, d\varphi. \tag{4.2}$$

[3] Since we restore 4D $\mathcal{N} = 1$ SUSY, it is convenient to work in such conventions. In 4D $\mathcal{N} = 1$ language, only the $U(1)_R \times SU(3)_F \subset SU(4) \cong SO(6)$ subgroup of the R-symmetry is manifest. Our $U(1)^3$ background gauge fields are linear combinations of the $U(1)_R$ and $U(1)_F^2 \subset SU(3)_F$ symmetries, specifically

$$A_R = A_1 + A_2 + A_3, \qquad A_F = A_1 - A_2, \qquad A_{F'} = \frac{2}{3}(A_1 + A_2 - 2A_3)\,. \tag{4.1}$$

From the dual field theory point of view, a priori there is no constraint relating the μ_i (besides (2.40)). On the other hand, the supergravity solution imposes an extra condition, due to eq. (2.6) and the regularity condition in eq. (2.7). This is to be contrasted with the supergravity solutions describing codimension-2 defects on the 4D $\mathcal{N}=4$ theory, see for example [39], where the three gauge fields of the 5D supergravity can be independently tuned to zero, leading to enhancement of supersymmetry. It would be interesting to understand whether there is a further generalization of the solution presented here that allows for such tuning.

The parameters μ_i are sources for the (conserved) R-symmetry currents J_i^μ. However, we note first that these are not quite normal sources, which would have been of the type $A_i \propto \mathrm{d}t$ (the sources for Noether currents would normally have electric-type charge), but these are of a new, different kind, similar to the case of a current in a neutral conducting wire: with overall current, but no overall charge. We will come back to this fact later. The dimension of the R-current in this four-dimensional theory is $\Delta_J = 3$, as in the standard case, and as dual to any (massless) gauge field in the bulk.

Since the theory is compactified on S^1, a (perhaps better) interpretation would be given in the reduced 2+1 dimensional field theory. In particular, the dimension of a Noether current in (2+1) dimensions is two. Yet, now, this is of a new kind, since we obtain $\langle J^a\rangle = 0$ for $a = 0, 1, 2$ (in *all* the field theory directions), and only $\langle J^\varphi\rangle \neq 0$, which is a "current component in an internal direction".

Twisted compactification and supersymmetry. Compactification of $\mathcal{N}=4$ SYM on a circle generically breaks supersymmetry (in our case, with periodic boundary conditions for bosons and anti-periodic for fermions, SUSY is broken). However, the presence of background R-symmetry Wilson lines implements a twisted compactification. The fermions pick up phases determined by their $\mathrm{U}(1)_\mathrm{R}\times\,\mathrm{U}(1)_\mathrm{F}^2$ charges when transported around the circle. For special values of the Wilson lines, these phases compensate the SUSY-breaking effect of the compactification [16].

On the gravity side, the SUSY solutions arise precisely in the massless branch $M = 0$. As shown in section 2.5.2, supersymmetry requires the relation

$$|\mu_1| + |\mu_2| + |\mu_3| = \frac{2\pi L}{\Delta}. \tag{4.3}$$

This condition ensures the existence of Killing spinors in the bulk, selecting a supersymmetric vacuum of the compactified theory. Since the five-dimensional STU model has eight supercharges, the projector found in section 2.6 reduces this by one half, leaving four preserved supercharges. The dual infrared theory therefore preserves $\mathcal{N}=2$ supersymmetry in three dimensions.

At energies E much less than $\frac{1}{\Delta}$, the Kaluza-Klein modes along the circle decouple, and the theory flows to an effectively $(2+1)$-dimensional supersymmetric theory characterized by the parameters μ_i.

Scalar operators and vacuum structure. The asymptotic analysis of section 2.3 shows that the bulk scalars Φ_1 and Φ_2 are dual to operators of dimension $\Delta_{\mathcal{O}_i} = 2$ in the four-dimensional theory, and as such the only difference between sources and VEVs in the asymptotic expansion ($\Delta = 2 \Rightarrow d - \Delta = 2$) is the presence or not of a log. But from eqs. (2.14) we see that the scalars admit only normalizable fall-offs: the logarithmic modes associated with sources are absent. Therefore, the dual operators acquire vacuum expectation values but are not explicitly sourced.

These operators belong to (are restrictions of) the $\mathbf{20'}$ operator $\mathrm{Tr}\left[X^I X^J - \frac{1}{6}\delta_{IJ}X^2\right]$ of $\mathrm{SO}(6)_{\mathrm{R}}$ and can be written in terms of the three complex scalars charged under the three U(1) R-symmetries (conversely under $\mathrm{U}(1)_{\mathrm{R}}\times\ \mathrm{U}(1)^2_{\mathrm{F}}$), namely

$$Z = X_1 + iX_2, \qquad W = X_3 + iX_4, \qquad V = X_5 + iX_6. \tag{4.4}$$

The operators in the QFT have VEVs proportional to $q_1^2 + q_2^2 - 2q_3^2$ and $q_1^2 - q_2^2$, implying a similar structure for the combination of scalars in $\mathcal{O}_1$ and $\mathcal{O}_2$, while the charges of the gauge fields A^i in the supersymmetric, $M = 0$, gravity solution are $Q_i^2 = \mathbf{q}$. This implies that the operator VEVs should only respect a diagonal U(1) symmetry (namely $\mathrm{U}(1)_{\mathrm{R}}$). These operators $\mathcal{O}_1$, $\mathcal{O}_2$ are

$$\mathcal{O}_1 = \mathrm{Tr}\Big(Z^2 + W^2 - 2V^2\Big)\,, \qquad \mathcal{O}_2 = \mathrm{Tr}\Big(Z^2 - W^2\Big)\,. \tag{4.5}$$

These operators are BPS protected. Their dimension is $\Delta = 2$ for all values of the coupling. In particular, at large N and strong coupling, this can be seen from the near-AdS expansion in eq. (2.14), see also eq. (2.20). Their non-zero expectation values characterize different vacua of the compactified theory. The compactification together with the Wilson lines breaks the global symmetry

$$\mathrm{SO}(6)_{\mathrm{R}} \;\longrightarrow\; \mathrm{U}(1)^3, \tag{4.6}$$

in agreement with the presence of three independent bulk gauge fields.

Importantly, since the scalars are not sourced, the deformation of the theory is entirely due to the compactification and the background R-symmetry holonomies. The scalar VEVs arise dynamically and are fixed by the requirement of regularity in the bulk. In particular, on the supersymmetric branch $M = 0$, the vacuum energy vanishes, $\langle T_{\mu\nu}\rangle = 0$, and the VEVs satisfy the relations derived in section 2.5.2.

Thus, the quintic structure uncovered in section 2.4 encodes the space of allowed vacua compatible with a given choice of Wilson lines ψ_i,

$$\psi_i = \lim_{\rho\to\infty} \frac{1}{2\pi L} \oint_{S^1} A_i = \mu_i \frac{\Delta}{2\pi L}\,. \tag{4.7}$$

Different positive roots correspond to distinct branches of the moduli space.

From the point of view of the reduced 2+1 dimensional field theory, the same story of scalars not being sourced is true: the dimension of the $\mathbf{20'}$ scalar operator is now 1 (it contains two scalars of dimension 1/2). As was found for holographic superconductors [40, 41], the case of $d = 3, \Delta = 1$ has no source, but rather its pair in the asymptotic expansion, with dimension $d - \Delta = 2$, just gives another kind of VEV (we have two VEVs instead of one VEV and one source).

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Heuristic effective infrared description. It is natural to attempt to describe the low-energy dynamics in terms of the composite operators $\mathcal{O}_1$ and $\mathcal{O}_2$, in the spirit of effective descriptions proposed in related holographic contexts [5, 27]. The gravity analysis indicates that the vacuum structure is non-trivial and admits multiple branches, suggesting that the effective potential for these operators possesses several extrema.

While a derivation of the precise three-dimensional effective action lies beyond the scope of this work, the holographic results strongly constrain its structure. The only explicit deformation parameters are the Wilson lines ψ_i. The scalar operators are not sourced but acquire VEVs determined dynamically. SUSY on the $M = 0$ branch enforces the linear relation among the ψ_i

$$|\psi_1| + |\psi_2| + |\psi_3| = 1. \tag{4.8}$$

These features indicate that the infrared theory should be viewed as a supersymmetric $(2+1)$-dimensional theory with background R-charge densities determined by ψ_i, and with vacuum expectation values for scalar bilinears in the $\mathbf{20'}$.

To gain a first insight into the structure of the low-energy (Wilsonian) effective action, we are inspired by [5], who also analyzed holographic Coulomb branch deformations (although their gravity solutions were singular, due to the absence of the gauge fields). These authors argued that the lowest dimension terms in the effective action were the canonical kinetic term, the classical scalar potential $\mathcal{V} = \text{Tr}\left[\sum_{I,J}[X^I, X^J]^2\right]$ and, in the generic case of a deformation by an operator corresponding to $\mathcal{O}_{20'} = \text{Tr}\left[X^I X^J - \frac{1}{6}\delta_{IJ}X^2\right]$, the deformation in the effective potential $\mathcal{V}_{\text{eff}}$ was by

$$\mathcal{O}_{\mathbf{20'}}^2 + \ldots = \left(\text{Tr}\left[X^I X^J - \frac{1}{6}\delta_{IJ}X^2\right]\right)^2 + \ldots \tag{4.9}$$

(plus higher orders).

Applying this to our case, we would have a deformation by $\bar{\mathcal{O}}_1\mathcal{O}_1 + \bar{\mathcal{O}}_2\mathcal{O}_2$. However, we also know that the effective potential must have a minimum for nonzero VEVs $\langle\mathcal{O}_1\rangle\neq 0, \langle\mathcal{O}_2\rangle\neq 0$ in the case of nonzero q_i, ψ_i, and that the minimum must be at zero potential, because of supersymmetry.

That means that the effective potential must contain the following lowest order terms in $\bar{\mathcal{O}}_i\mathcal{O}_i$,

$$\mathcal{V}_{\text{eff}} = m_1^2|\mathcal{O}_1| + m_2^2|\mathcal{O}_2| - |\lambda_1|\bar{\mathcal{O}}_1\mathcal{O}_1 - |\lambda_2|\bar{\mathcal{O}}_2\mathcal{O}_2 + |\lambda_1'|\left(\bar{\mathcal{O}}_1\mathcal{O}_1\right)^2 + |\lambda_2'|\left(\bar{\mathcal{O}}_2\mathcal{O}_2\right)^2 + \ldots\,, \tag{4.10}$$

where $\lambda_1, \lambda_2, \lambda_1', \lambda_2'$ must be related to the dual deformation parameters q_i, ψ_i. The terms with λ_i, λ_i' are needed in order to have a minimum at a nonzero VEV for $\mathcal{O}_i$, and the terms with m_i^2 are needed in order for the minimum to be at $\mathcal{V}_{\text{eff}} = 0$. For the relation between λ_i, λ_i' and q_i, ψ_i, we can find the VEVs at the minimum, and then use (2.43) and (2.45) to express them in terms of parameters. If $m_i\neq 0$, we obtain a cubic equation, but if $m_i = 0$, the relation becomes simply $(|\lambda_1|/(2|\lambda_1'|))^{1/2} = |\psi_2\psi_3| + |\psi_1\psi_3| - 2|\psi_1\psi_2|$ and $(|\lambda_2|/(2|\lambda_2'|))^{1/2} = |\psi_3|(|\psi_2| - |\psi_1|)$. We note that the above also respects the diagonal U(1) symmetry, as necessary.

Interpretation in terms of Q-ball charge densities. At this point we remember that from the point of view of a 2+1 dimensional field theory, J^φ was not a normal kind of current: its 0,1,2 components vanish, and only the internal φ component is nonzero. This is not a Noether current, and by the same token is also not a topological current. It could, however, be a current density for a *non-topological* solution, which is not so constrained.

In [27] it was proposed that the explanation for this current is in terms of Q-ball charge densities. Indeed, consider a Q-ball solution ansatz for the scalars that combines the fact that there is a φ extra direction with the standard Q-ball ansatz of Coleman [42],

$$\begin{aligned} Z(\varphi=\omega_1 t,\vec{x}) &= e^{ik_1\varphi}Z(\vec{x}) = e^{i\tilde{\omega}_1 t}Z(\vec{x}) \\ W(\varphi=\omega_2 t,\vec{x}) &= e^{ik_2\varphi}W(\vec{x}) = e^{i\tilde{\omega}_2 t}W(\vec{x}) \\ V(\varphi=\omega_3 t,\vec{x}) &= e^{ik_3\varphi}V(\vec{x}) = e^{i\tilde{\omega}_3 t}V(\vec{x}). \end{aligned} \tag{4.11}$$

where $\tilde{\omega}_i = k_i\omega_i$, and consider the Q-ball current densities

$$J_i^\varphi = i\left([(\partial_\varphi\bar{Z})Z - \partial_\varphi Z\bar{Z}],[(\partial_\varphi\bar{W})W - \partial_\varphi W\bar{W}],[(\partial_\varphi\bar{V})V - \partial_\varphi V\bar{V}]\right)\,, \tag{4.12}$$

which are nonzero. Of course, *on the solutions*, we could also have $J_i^0\neq 0$. But we could arrange for the total electric charge $\int J_i^0$ of a distribution to be zero, yet the Q-ball densities to be nonzero. This is similar to the case of a neutral current wire, when we can have a zero total charge, but nonzero current through the wire, by averaging over the positive static charges of the nuclei, and the negative moving charges of the electrons. Here, though, we have a new situation: both the overall charge and the overall current vanish, even though an individual Q-ball can have both, but only the "charge in the internal direction", i.e, Q-ball charge, is nonzero, by averaging over Q-balls of positive and negative electric charges, but same Q-ball charge.

Incidentally, this answers the question: how come the Q-ball ansatz is time-dependent, but the gravity solution is not? The point is that *one* Q-ball gives a relation between coordinates, $\varphi=\omega t$, just like *one* pointlike electron moving gives $x=vt$. When averaging over a continuum of solutions that dependence goes away.

Then, in order for the ansatz to generate actual Q-balls solutions, as Coleman showed [42],[4] we need the condition that $\omega_i < m_i$ (for stability against decay), which means a nonzero mass term, and that the potential divided by the modulus squared of the scalar fields, $\mathcal{V}_{\rm eff}/|Z|^2, \mathcal{V}_{\rm eff}/|W|^2, \mathcal{V}_{\rm eff}/|V|^2$, starts off at a nonzero value at zero fields (since the scalar masses need to be nonzero), and then has an absolute minimum at some nonzero value of the scalar VEVs. The simplest way to achieve this is to have a nonzero mass term, then the next power of the fields to have a negative coefficient, and the following power to have a positive coefficient. Of course, in terms of *independent* Q-ball charges, one should consider as fields not Z,W,V, but rather $\mathcal{O}_1,\mathcal{O}_2$, in which case the conditions are on $\mathcal{V}_{\rm eff}/|\mathcal{O}_1|, \mathcal{V}_{\rm eff}/|\mathcal{O}_2|$ instead.

But these are exactly the two conditions that we imposed on the effective potential in order to have a nonzero VEV and supersymmetry, leading to (4.10)! So we are guaranteed to have the Q-ball solutions, and thus the currents J_i^φ, as we wanted.

[4]Note that a nonabelian version of Q-balls is possible [43], though it is not needed here.

5 Conclusions and outlook

In this work we constructed a new family of AdS soliton solutions of the five-dimensional STU model with two independent scalar fields and three Abelian gauge fields, and uplifted them to full ten-dimensional type IIB supergravity. These backgrounds provide a holographic description of $\mathcal{N} = 4$ super Yang-Mills theory compactified on S^1 in the presence of three independent $\mathrm{U}(1)^3 \subset \mathrm{SO}(6)_\mathrm{R}$ Wilson lines and with vacuum expectation values for scalar bilinears in the $\mathbf{20}'$ representation. The resulting geometries are asymptotically AdS_5 and terminate smoothly in the infrared, realizing a confining and gapped phase of the compactified theory.

A central result of this paper is the explicit characterization of the space of regular solutions. By imposing regularity in the interior together with fixed boundary data for the Wilson lines and the compactification radius, we showed that the bulk equations reduce to a quintic polynomial governing the allowed solutions. Different positive roots correspond to distinct branches of vacua of the dual field theory. In this way the gravitational analysis provides a direct holographic realization of vacuum selection: integration constants that would naively appear arbitrary are dynamically fixed once the requirement of infrared regularity is imposed.

A particularly important sector arises in the massless limit $M = 0$. We demonstrated that these solutions preserve supersymmetry and admit a transparent field theory interpretation. The supersymmetric branch is characterized by a simple linear relation among the Wilson lines,

$$|\psi_1| + |\psi_2| + |\psi_3| = 1 \,, \tag{5.1}$$

which selects a moduli space of vacua with vanishing energy density. From the dual point of view, the compactification and R-symmetry twists induce vacuum expectation values for scalar operators while no explicit sources for these operators are present. The resulting moduli space admits a natural interpretation in terms of different quantum phases separated by lines where the expectation values of composite operators change sign. At the intersection of these loci the system exhibits a quantum critical point, which in the gravitational description corresponds to a special regular soliton solution.

The ten-dimensional uplift further clarifies the geometric origin of these deformations. The Wilson lines in the gauge theory correspond to twists along the angular directions of the internal S^5, while the scalar VEVs appear as squashing modes of the compact space. This ten-dimensional perspective provides a controlled framework to study non-perturbative phenomena in the strongly coupled theory, including confinement and symmetry breaking in the compactified $\mathcal{N} = 4$ system.

Our results open several interesting directions for future investigation.

- First, the geometries constructed here provide a natural laboratory to compute non-perturbative observables in the confining phase of the compactified theory. It would be particularly interesting to analyze Wilson and 't Hooft loops, glueball spectra, and entanglement entropy in these backgrounds. Such observables should encode detailed information about the vacuum structure and could help distinguish the different branches of solutions identified in the gravitational analysis. We do this in [30].

- Second, the quintic structure governing the solution space suggests the existence of a rich phase diagram in the dual field theory. A systematic study of the thermodynamics

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of these solutions, including finite temperature generalizations and possible black hole counterparts, could reveal phase transitions between different branches of vacua and clarify the role of the supersymmetric locus as a critical boundary in parameter space.

- Third, it would be interesting to understand the effective infrared description of the compactified theory more directly from the field theory side. Our holographic results strongly constrain the structure of the low-energy effective potential for the scalar bilinears in the $\mathbf{20'}$ multiplet. Deriving such an effective description explicitly would provide a valuable bridge between the gravitational picture and the microscopic dynamics of $\mathcal{N} = 4$ SYM.

- Fourth: finding a nonlinear embedding of the five-dimensional solutions into eleven dimensions, suggests a broader geometric framework in which these backgrounds may arise. Exploring the M-theory interpretation of these configurations, and understanding their relation to brane constructions and higher-dimensional compactifications, could shed light on the origin of the moduli space structure uncovered here.

In summary, the new class of multi-charge AdS solitons constructed in this paper provides a concrete holographic realization of vacuum structure and quantum phases in compactified $\mathcal{N} = 4$ super Yang-Mills theory. We expect that further exploration of these geometries will yield new insights into the interplay between supersymmetry, confinement, and holography in strongly coupled gauge theories.

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